Order Time, Multiple Shocks, and Short Selling in Security Price Adjustment

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Abstract

We investigate if order time and short selling have any bearing on how prices are formed and adjusted towards full information value in the securities market. We model a competitive dealership market in which complete and incomplete information traders arrive in a probabilistic fashion to trade a single risky security for cash with a market maker. The complete information traders receive a private information about the future value of a security and a signal from a public information release indicating a buy or a sell. The incomplete information traders trade for liquidity on the basis of the public information signal. Derived results from the model suggest that while buy or sell reveals private information, although asymmetrically, no-trade may provide information about the signal from public information depending on whether the market is optimistic (high buy) or pessimistic (high sell) market. Security prices adjust to their full information value at a faster rate when short selling constraints are low. The speed and the nature of adjustment of security prices differ if the market is optimistic vs. pessimistic. In an optimistic market adverse private information is instantaneously impounded into prices.

Introduction

The time series properties of security prices and bid-ask spreads are of common interest to a wide variety of financial economists. This interest has been fueled by the availability of transaction data on equity prices from NYSE, NASDAQ, and even some international exchanges. Transaction prices are widely studied to infer information available in the securities market, or its lack thereof. This information content in trades characterizes the stochastic process underlying the time series of security prices.

The literature on market microstructure focuses on how trades reveal information about future value of securities. In Kyle (1985) and Glosten and Milgrom (1985) time is exogenous to the price process and thus time has no information content. Hausman, MacKinlay, and Lo (1992) reject a direct test of the null hypothesis that the time interval between trades is exogenous to the price process. Hasbrouck (1991) documents a significant effect of order time and volume on bid ask spread.

Diamond and Verrecchia (1987), and Easley and O’Hara (1992) show how order time and no-trade reveal information to the market. In Diamond and Verrecchia (1987) short sell restrictions slow down the process of information revelation in the market; in Easley and O’Hara (1992) order time contains information since trading signals direction of trade, while absence of trade signals non-existence of information.

In this paper, we extend Diamond and Verrecchia (1987), and Easley and O’Hara (1992) to ask the following questions: How does order time affect price formation when there are multiple shocks? Does the effect of no-trade on price formation depend on the perception of a bull/bear market? And finally, how does short-sell prohibition affect the adjustment of prices to full information value during an optimist/pessimist (bullish/bearish) market? This paper complements a recent work of Chen, Hong, and Stein (2001) in which price formation is subject to both differences of opinion among investors and short sales constraints.

We model a competitive dealership market for securities trading with two sources of information shocks - a public and a private information shock. The private information shock occurs with some probability and signals the future value of securities. The public information shock determines the liquidity needs of the investors in terms of buy and sell. Two groups of risk-neutral traders- complete and incomplete information traders trade a single risky security for cash with the market maker. The complete information traders use both private and public information in their trading decisions, while the incomplete information traders use only public information to buy or sell securities. Competitive market makers set bid and ask prices as the expected value of the asset conditional on the order received.

Public information relates to among others interest rate change, corporate profits, business cycles (recession and recovery), regulation, and industry outlook. A public announcement regarding an upward movement in the interest rate and a positive outlook in corporate profits may induce some traders to interpret the public information release as a sell signal, while others treat it as a buy signal[1]. Harris and Raviv (1993) relate this divergence of opinion to the volume and volatility in stock markets. It's worth mentioning here that the public information analyzed here is quite different from firm-specific public information as dividend announcements, and earnings forecasts. Kim and Verecchhia (1991) explain the contraction in the volume of trading prior to announcements as due to expectations about such firm specific announcements.

The public and the private information signals may be contrary to each other. If the two sources of information deliver conflicting signals, a fraction of complete information traders, primarily the institutions short sell to mask their uncertainty. The individual traders, who are constrained from short selling due to institutional reasons e.g., the inability to borrow stock, do not trade at all. Incomplete information traders are also allowed to opt out of trading, if they want to. Information is long lived in this model, which allows multiple traders with different trading objectives to trade (or not trade) multiple rounds before all information is revealed.

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[1] For the purpose of this paper, a public information release at any time implies a whole set of economy and industry wide predictions. The fact that all economic indicators do not have the same level of impact on stock valuations, and do not affect stock value in the same direction, an aggregate measure often falls short of a consensus as to its overall impact on the stock market or on any individual stock. Nick Sargen, Chief Global Strategist, J.P.Morgan International mentioned how mixed signals about GDP and corporate profits from the Japanese market are interpreted differently by US analysts (On Wall Street Week with Lois Rukeyser, February 12, 2000).

The question(s) asked and the methods of analyses used in this paper are variants of Diamond and Verrecchia (1987), and Easley and O'Hara (1992). Nevertheless, the contribution of this paper is to combine the effect of private and public information on security prices in a very simple, sequential model to explain how short sell, and trading time convey information to the market and how such information are impounded in the prices of securities. We also provide some predictions related to the adjustment of security prices to new information in bull (high buy) and bear (high sell) markets.

In particular, the derived results from the model confirm that although buy and sell reveal similar information during bullish and bearish market conditions, no-trade and short-sell may reveal different information during bull and bear markets. The price effects of buys and sells are asymmetric. The rate and the nature of adjustment of security prices to full-information value are also dependent on whether the public information indicates optimism (bullish) or pessimism (bearish) in the market.

This paper also addresses some questions relating to the ongoing debate on whether short sells convey a bearish or a bullish sentiment about the market. The conventional viewpoint is that short interest predicts bullish market ahead, since all short positions are eventually covered in the future with buys. Figlewski (1981) claim that a surge in short interests indicates a bearish market sentiment since short-sellers are essentially "pessimists". Asquith and Meulbrock (1999), and Dechow et al (1999) find evidence in support of Diamond and Verrecchia (1987) that "bad news" (private information) is responsible for surges in short interests. In our model short sales arise due to contrary private and public information signals. Thus both bearish or bullish market sentiments driven by public information with contrary private information may lead to short selling. We examine the effect of this assumption on the implicit cost of short selling under bullish or bearish market conditions.

The model

The sequential model of security trading in this paper builds on similar models of a competitive dealership market with informed and uninformed traders (investors) as in Diamond and Verrecchia (1987) and Easley and O'Hara (1992). In these models, potential buyers and sellers arrive in the market in a probabilistic fashion to trade one unit of a risky security with a competitive market maker. Market makers face no inventory costs or constraints and set prices to earn zero profits from each trade.

In this model, there are two sources of information in the securities market, public information and firm specific private information. Public information is periodically released containing future predictions about the economy and the industries e.g., interest rate, regulation, corporate profits, change in fiscal policies etc. A signal is generated from the public information releases and transmitted to all traders at the beginning of a trading day. The public information yields a buy or a sell signal. This signal is repeated until there are changes in the public information prompting another signal. Harris and Raviv (1993) argue for the difference of opinion about a public information release as the rationale for public information to cause volume and volatility in the securities market. All traders, but not the market makers, receive the same buy or sell signal. The market makers know the public information, but do not observe the signal that traders receive from the public information.

A private information signal regarding the true future value of the security follows an actual information event. Nature determines if there is an information event[2]. An information event occurs with a probability of e. If an event occurs, a private information signal about the future value of a security (high or low) is released to some traders in the market. Let q be the fraction of traders who observe the private information signal. We assume that the private information is long lived so that multiple rounds of trading occur before the signal changes. The future value of a security is a random variable v with two possible values, 1 and 0 denoting high and low values for the security. Nature determines whether the future value is high or low. A low value occurs with a probability of d that represents information uncertainty.

Investors observe either incomplete (only public information) or complete (both public and private) information signals. Incomplete information traders e.g., index funds and small retail traders trade on the basis of public information signals that drive demand for liquidity. They sell with probability m, and buy with probability $(1-m)$[3]. Incomplete information traders trade (buy or sell) according to the public signal with probability b. They choose not to trade at all with probability $(1-b)$.

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[2] This uncertainty about whether an information event has occurred is a key element in the price discovery process in Easley and O'Hara (1987, 1992). This feature also distinguishes their model from Glosten and Milgrom (1985) in which an information event is presumed to exist. In Dey and Kazemi (2000) where there are three types of traders including institutions as information-liquidity traders, event uncertainty is no longer relevant in the price discovery process.

[3] An alternative way of explaining how the public information is broken into buy and sell signals is to assume that on receiving public information a fraction of the population sells, while the rest buys. As long the public information release remains unchanged, this fraction of buyers and sellers in the market is constant.

On the other hand, complete information investors follow both private and public information signals to direct their trades. Both but not all institutions and individuals belong to the class of complete information traders that use a combination of public and private information in their trading decisions. The probability that any trade is from a complete (private-public) information trader is q , and the probability that the trade is from an incomplete (only public) information trader is $(1-q)$ where $q \in (0,1)$. There are infinitely many risk neutral traders in both categories.

Diamond and Verrecchia (1987) consider public information or lagged private information as public information as the source of liquidity shocks in their model. Note that if public information is considered to be the source of the liquidity shock, then the complete (private-public) information traders in this paper are *de facto* the information-liquidity investors in Dey and Kazemi (2000). However, in Dey and Kazemi (2000) the information-liquidity investors are institutions that trade large size only, which reduces the generality of the effect of information-liquidity shocks on security prices.

Complete information traders receive two information signals – a private information signal indicating low or high future value of the security, and a public information signal denoting a sell or a buy. On the basis of these two information signals four information quadrants are possible – (low value, sell), (low value, buy), (high value, sell), and (high value, buy). The complete information traders sell or buy one unit of a risky security with certainty i.e., with probability 1 when the signal set they receive is (low, sell) or (high, buy). They sell (buy) with certainty (probability 1) if the future value is low (high) and their signal from the public information is a sell (buy).

The trading behavior of the complete information traders can be quite complicated when they receive contrary signals from the private and the public information releases. However we choose a simple trading rule for the situation. The purpose is to demonstrate how short selling may increase trading activities and the expected efficiency with which information is revealed to the market.

Faced with contrary information signals, all complete information traders want to short-sell. However, among the complete information traders only a percent is allowed to short sell due to institutional reasons e.g., contractual prohibitions against shorting by corporate insiders, and the inability of individuals to borrow stock for shorting. Thus complete information traders short-sell with probability a , and choose not to trade with probability $(1-a)$. Note that the institutional prohibitions exclude mostly individuals from short selling. Therefore, a represents the fraction of institutions among complete information traders. After the respective traders receive the information signals, actual trading takes place as in Table 1 below.

Table 1: Summary of information events and trading by complete (private-public) information, and incomplete (only public) information traders in a securities market.

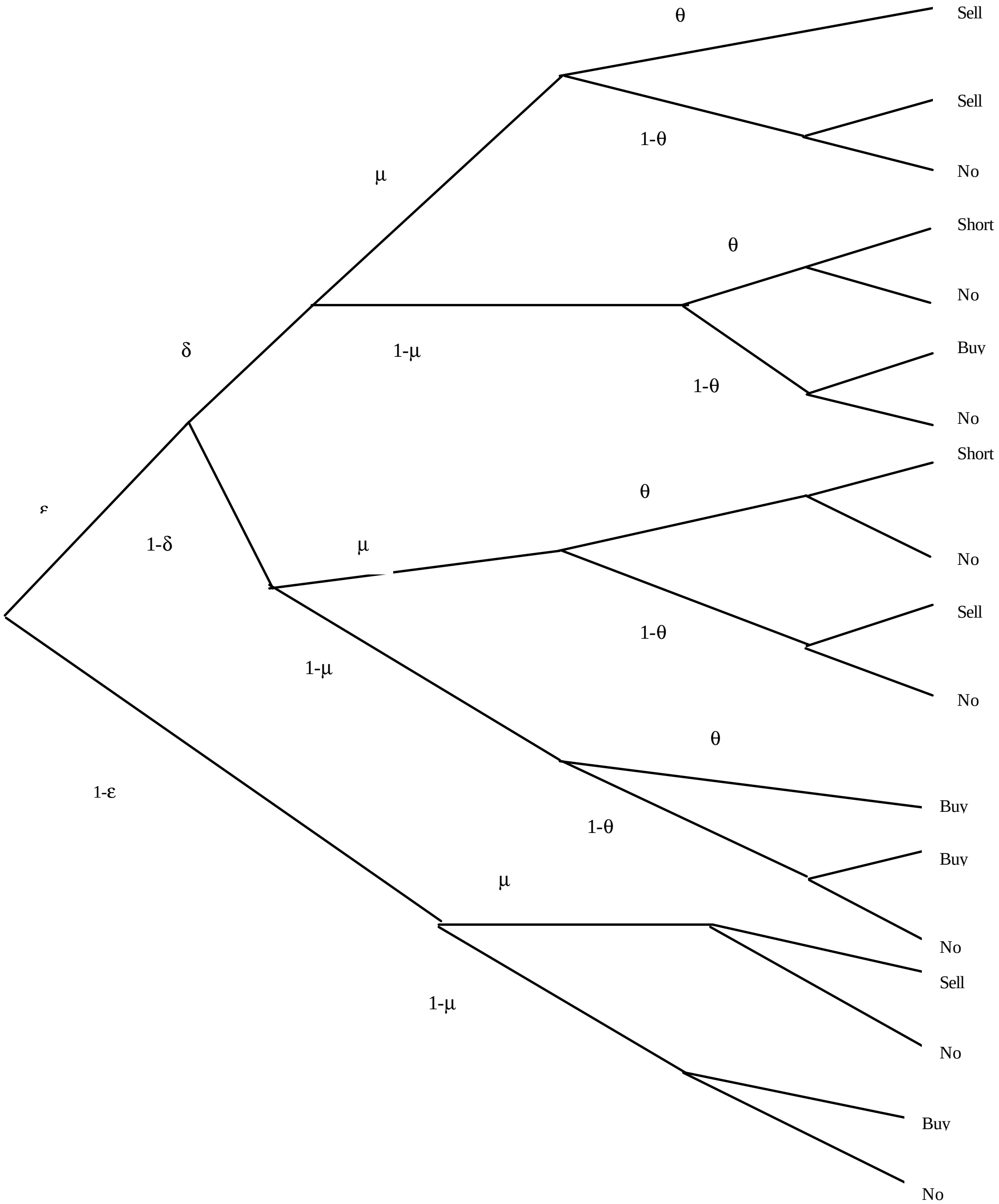
	Low value, sell	Low value, buy	High value, sell	High value, buy
Private & public information	Sell	Short-sell / No trade	Short-sell / No trade	Buy
Public information	Sell / No-trade	Buy / No-trade	Sell / No-trade	Buy / No-trade

Table 2: A summary of notations used in Figure 1 and elsewhere in the paper

Variable	Definition
v	Value of a traded security, 0 with probability d ; 1 with probability $(1-d)$.
d	Probability that the value of the security is low next period.
e	Probability of an information event happening that leads to a signal being generated about the value of the security next period.

m	Probability that a public information release signals sale. This probability represents fraction of sellers among those receiving public information.
q	Probability that a complete information trader selected for trading sells or buys according to the complete set of signals that she has. This probability represents fraction of complete information traders among all traders. A complete information trader has both private and public information. An incomplete information trader has only public information.
b	Probability that an incomplete information trader actually trades (buys or sells according to the public information signal that she receives).
a	Probability that a complete information trader selected to trade short-sells.
$T=1\ldots T$	Time of trade or quote
S	Signal set: includes L, H, and ϕ denoting low, high and no signal respectively

Figure 1: Securities market trading game with complete and incomplete information traders. Competitive market makers set bid ask prices at expected value conditional on buy or sell quotes.



Actual trades in this market move in the following sequence. An information event occurs with a probability of ε. In case there is an information event, bad news are likely with a probability of δ. A public information shock is likely to be a sell signal with probability μ, and a buy signal with probability (1-μ). A complete information trader is selected to trade with a probability of θ, and an incomplete information trader is selected to trade with a probability of (1-θ). An incomplete information trader sells or buys with a probability of β and does not trade with a probability of (1-β). A complete information trader short-sells with a probability of α and does not trade with a probability of (1-α).

Figure 1 above graphically describes how trading occurs in this market. At the beginning of a trading day, nature determines if there is a private information event. The probability that an information event happens is e . Simultaneously a public information signal is released (or repeated) at the beginning of every trading day.

If a private information event happens then a signal is released to some traders to indicate the future value of the traded security. 1 and 0 are the known high and low values of a security resulting from bad or good news respectively and d is probability of a low value signal. It follows then, that the value of the security, $v \sim \text{Bernoulli}(d)$. At the beginning of a trading day, the unconditional expected value and variance of the security are as follows:

$E(v) = d * 0 + (1-d) * 1 = (1-d)$ and

$Var(v) = d(1-d) = s_v^2 \text{ (say)}$.

A trading day is divided into discrete trading intervals denoted by $t = (1,2.\dots T)$. Each trading interval is long enough to accommodate exactly one trade. Dividing a trading day into trading intervals allows us to capture no-trading intervals within a trading day. At each trading interval a trader is selected to trade according to the probabilities mentioned above. If the trader selected belongs to the complete information class she observes a signal $s \in (LS, LB, HS, HB)$ indicating (low, sell), (low, buy), (high, sell), and (high, buy) denoting her private and public information signals. After observing the signal, a complete information trader chooses $Q_C \in (S/SS, B, N)$ denoting a sell/short, a buy, and a no-trade respectively.

The complete information (private-public information) trader sells (buys) one unit of the financial asset with certainty when the private-public information signals indicate low value-sell (high value-buy). They short-sell one unit of the security if the private-public information indicates mixed signals i.e., (low value, buy) or (high value, sell) with probability a and do not trade with probability $(1-a)$.

If the trader selected is an incomplete (only public) information trader, she sells (buys) one unit of the risky security with probability b, and opts not to trade with probability $(1-b)$. Thus for the incomplete information trader the actions belong to the set $Q_I \in (S, B, N)$ indicating sell, buy, or no-trade. The probability that the selected trader is a complete information trader is q, and the probability that the selected trader is an incomplete information trader is $(1-q)$.

A competitive market maker observes an incoming order belonging to $Q \in (S/SS, B, N)$ but not the information signal and makes inference about d. Note that the market maker observed action set does not differentiate between a short sell and a sell, i.e. $Q = Q_C = Q_I$ (Diamond and Verrecchia [1987]). Similarly although the signal set includes the public information signal, a market maker setting bid ask prices cares only about the private information signal, low, high, and no signal, since she observes the public information (although not the signal). Thus the true signal set $s \in (L, H, f)$ denotes low, high and no signal respectively.

Competitive market makers use the conditional probability $d(Q)$ to set the equilibrium bid and ask prices as $b^* = E(v|S)$ and $a^* = E(v|B)$ respectively where

$E(v|Q) = (1 - d(Q))$. Competitive market makers begin the day with a prior for d. If no information event occurs d remains unchanged. If an information event occurs then $\Pr(v = 0)$ is one if the signal is low, and zero if the signal is high. The market maker computes $d(Q)$, the conditional probability of a low signal given a quote for each element in Q as below:

$$\begin{aligned} d(Q) &= \Pr(v = 0|Q) \\ &= 1.\Pr(s = L|Q) + 0.\Pr(s = H|Q) + d\,\Pr(s = f|Q) \end{aligned}$$

for all s and Q. Further, by Bayes' rule

$$\Pr(s = x|Q) = \frac{\Pr(Q|s = x)\Pr(s = x)}{\sum_{s\in(L,H,f)} \Pr(Q|s)\Pr(s)}.$$

Price Effect and Equilibrium Bid-Ask Prices

Proposition 1: Probability of no-trade is not monotonically decreasing with the probability of an information event. The probability of no-trade conditional on no information event is higher than the probability of no-trade conditional on an information event only if one the following conditions hold:

a) The private and public information signals are directly correlated, or
b) The private and public information signals are inversely correlated and the probability of no-trade by incomplete information traders is greater than that of complete information traders.

In Easley and O'Hara (1992) the probability of no-trade is inversely related to the probability of an information event. However, in the present model, that is true only if one of the following happens:

a) The private and the public information signals are low value and sell or high value and buy respectively.

b) The private and the public information signals are low value and buy and high value and sell respectively, and $b > a$ i.e., the probability of no-trade by incomplete information traders is greater than the probability of no-trade by complete information traders.

Note that when the private information is revealed, and the market appears to be neither optimistic nor pessimistic i.e., the buy or the sell signal from the public information, $m = 0.5$, $1 - b > .5(1 - a)$. This direct relationship ($\frac{\partial b}{\partial a} = .5 > 0$) between a, and b is also indicative of an intuition forwarded by EKOP (1996) and further supported by Dey and Kazemi (2000) - as the relative proportion of short sellers or institutions (informed traders) among complete information traders increases, so does the incomplete information (liquidity) traders.

Proposition 2: Trades (sell or buy) convey private information. No-trade conveys a mixed signal and is inversely related to the public information. If public information causes optimism (bullish) then no trading is associated with a fall in prices; in case of pessimism (bearish) over public information, no trade is associated with a rise in prices. No-trade conveys no information at all if the public information is neither bullish nor bearish. Implicit cost of short selling (to the short-seller) is higher in a pessimist market than in an optimist market.

The market maker observes either an order (buy or sell) or the absence of an order. As she observes an order or a no-order, she updates the probability of low value conditional on the order or its absence thereof. The conditional probability of low value conditional on a sell is lower than the unconditional probability. On the other hand, the conditional probability of low value given a buy order is higher than the unconditional probability. Thus the market maker considers any intention to sell to signal low value and thus reduces the bid price. On the other hand a buy order induces the market maker to increase the ask price.

The equilibrium bid and ask prices for the first buy or sell of the day are computed as follows:

$b_1 = E(v|sale/short) = 1-d(S_1)$, and $a_1 = E(v|buy) = 1-d(B_1)$. It follows that

$$b_1 = (1-d)\frac{emqa + emb(1-q) + (1-e)mb}{ed\left(mq + mb(1-q) + (1-m)qa\right) + em(1-d)\left(qa + (1-q)b\right) + (1-e)mb} < 1-d$$

and $a_1 = (1-d)\dfrac{emq + (1-m)(1-eq)b}{emq(1-d) + (1-m)(1-eq)b} > 1-d$.

Note that the bid and the ask prices divert away from the beginning of the day unconditional price, $1-d$. Easley and O’Hara (1992) find similar evidence of initial price movement with a sell or a buy order. However, unlike in Easley and O’Hara (1992) the movements in the bid and the ask prices are not symmetric since the conditional probabilities of buys and sells given any private information signal (low or high value) are not the same. This asymmetry in the price effect of sells vs. buys is documented in Chan and Lokonishok (1993) and Koski and Michaely (2000).

The effect of observing no-trade on the conditional probability of low value and thus on the bid and the ask prices is mixed depending on the market maker’s perception of the public information. If $m = .5$ indicating that the public information is evenly split between potential sells and buys there is no change in the conditional probability and hence the price. If $m < .5$ indicating optimism in the market, no-trade is a “bad” signal that drives down the price. If $m > .5$ indicating pessimism in the market, no-trade is construed as a “good” signal driving the price upwards.

By looking at the structure of our model, it is clear why no-trade moves prices. If the market maker perceives a bullish sentiment in the market and still observes no-trade, she understands that contrary private information i.e., low value of the security may exist in the market. Thus she increases the price. The opposite happens when she observes no-trade during a perceived bear market.

Interestingly, the model suggests no-trade signals either “bad” or “good” news with the implication that the market maker either decreases or increases the price. Note that in the sequential trade model, the market maker begins with the unconditional value of the asset and then moves on to set bid and ask prices based on the order she receives. When no-trade implies “bad news” the market maker lowers the price i.e., she lowers the expected value of the security. On the other hand, when no-trade implies “good news”, the market maker increases the price i.e., she increases the expected value of the security. Now since the bid and ask prices bracket the expected value of the security at each time, a no-trade moves bid and ask prices in the same direction. Note that a sell or a buy moves the bid or the ask price in the opposite direction i.e., away from the unconditional value.

Finally we discuss the effect of short selling in this market. Note that the market maker does not distinguish between a sell and a short sale. However, she knows from the structure of the market that complete information traders short sell due to contrary private and public information. Let us assume that the complete information traders can costlessly signal to the market makers that their short sales are not motivated by private information, but rather induced by the contrary signals. The conditional probability of low value conditional on a short sell is dependent on m. As in no-trade short sell conveys “bad news” when public information signals more buys than sells; it conveys “good news” when public information signal more sells than buys. When public information is evenly split between buyers and sellers, short sell is non-informative.

We define the difference between the bid prices for a sell and the bid prices for a short sell assuming the short seller can costlessly signal a short sale, as the implicit cost of short selling. Since $d(S) > d > d(SS)$ when $m > .5$ (pessimist market), and $d(S) > d$ and $d(SS) > d$, when $m < .5$ (optimist market) the cost of short selling is higher in a bear market than in a bull market.

Now consider the two different views about what induces short selling. Figlewski (1981) argues that “the pessimists sell short”, while Asquith and Muelbroek (1999) claim that “adverse private information induces short sell”. The first approach proposes a direct relation between short interest and the ratio $\frac{m}{1-m}$. The later approach suggests an increasing relation between short interest and d. In our model, short interest is provoked by an inverse although not continuous relation between the private and the public information signals - for $\frac{m}{1-m} > 1$ (pessimism about market) d is low, where as for $\frac{m}{1-m} < 1$, d is high. Note that the left hand side of the expression denotes optimism/pessimism based on public information releases, while the right hand side of the expression signifies private information signal. Hence we find support for both Figlewski (1981) and Asquith and Muelbroek (1999) in attempting to explain surges in short interests due to both public and private information.

Proposition 3: If there is no trade at time t then the probability of no information event either rises or falls; however, the probability of a high or a low signal falls.

If there is no trade at time *t* then at time *t+1* the probability of a high or low signal falls. The market maker learns from both trades and no-trades. No-trade signals not only the absence of any information event, but also the possibility of contrary information signals. While the market maker's belief about the absence of an information event causes the probabilities of a high or a low signal to fall, her uncertainty about contrary information signals either increases or decreases the probability of no-information event. Clearly, the probability of no-information event depends on m , the public information signal.

Proposition 4: Suppose there is no trade at time t *. Then at time* $t+1$

a) *If the bid (ask) is lower than the unconditional expected value of the security, the bid (ask) increases;*
b) *If the bid (ask) is higher than the unconditional expected value of the security, then the bid (ask) decreases;*

i.e., prices tend to converge to the unconditional expected value of the asset and hence spread approaches zero.

As shown in Proposition 3, since no-trade at time *t* causes the probability of low or high signal at *t+1* to fall, the bid and ask get closer to the unconditional expected value and thus spread diminishes. This result is quite similar to Easley and O'Hara (1992), although due to the asymmetry among the buy and sell sides in our model, the rate of convergence from the buy and the sell sides are different. In Easley and O'Hara (1992), the buy and the sell sides are symmetric and thus the rates of convergence from both sides are the same.

Speed of Adjustment of prices to full information value

In the following paragraphs we investigate how security prices adjust to the full information value, 0 or 1. For the convenience of exposition, and without reducing the generality of the model, we change the structure and some of the notations of the model to conform to Diamond and Verrecchia (1987).

Let the probability of a low or a high value for the security be 0.5[4]. Since in this part we are interested in how prices adjust to the full-information value of the security, we are implicitly assuming that an information event has actually occurred. Thus event uncertainty is resolved and $e = 1$. Now, we compute the conditional probabilities of actions, $Q \in \left(S/SS, B, N\right)$ directly observed given the state of nature *v=(0 or 1)* as follows:

Table 3: Conditional probabilities of observed actions given the true state of nature, *v=0* or *v=1*.

Actions directly observed	Conditional probability of an action when *v=1*	Conditional probability of an action when *v=0*
Buy	$(1-m)(q+(1-q)b)$	$(1-m)(1-q)b$
Sell or Short	$m(qa+(1-q)b)$	$m(q+(1-q)b)+(1-m)qa$
No-trade	$mq(1-a)+(1-q)(1-b)$	$(1-m)(1-a)q+(1-q)(1-b)$

Proposition 5: The process of adjustment of prices depends on market conditions i.e., bull (high buy) or bear (high sell) market. The expected number of periods required for the adjustment of prices to bad (good) news is a decreasing function of a*, the proportion of complete information traders who do not face short sell restrictions i.e., the institutions. However, when the market conditions are extremely bullish (no sell), the adjustment to low value of the security is instantaneous i.e., expected numbers of periods for price adjustment is zero. The adjustment to a high value is still an increasing function of the fraction of complete information traders who do not face short sell restrictions, the institutions.*

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[4] Easley et al (1996) empirically estimate δ, the unconditional probability of bad news to be 0.5.

The expected number of periods that will be necessary for security price to adjust to its full information value is a decreasing function of the institutional constraint (rather the lack of it) on short selling. As more short sellers or institutions in the context of our model enter into the market, prices adjust faster towards the full information value. This result is generally in agreement with the results from Diamond and Verrecchia (1987). Arnold et al (2000) provide empirical support for low short selling constraints to be facilitating price discovery.

Interestingly, in an extreme bull market (public information signaling buy only) low value of the security is reflected in the price instantaneously. The reason for such a phenomenon is due to the structure of our model in which a lot of sell/shorts during a bull market is clearly an indication of contrary private information in the market. This causes the market maker to immediately infer "bad news" from the sell/shorts and adjust the price as if the security value is "low". However, in an extreme bear market since buys do not explode (shorts and sells together increase the number of sell/short in a bull market) additional information about the "private signal" is not revealed and thus the adjustment is not instantaneous. Aitken et al (1998) find evidence of such instantaneous price adjustments in the Australian Stock Exchange.

Conclusion

In this paper, we investigate if the time to trade and short sell have any bearing on how prices are formed, and adjusted to their full information value in a market with multiple sources of information. We model a trading system where two groups of traders - complete information traders (having access to and using information signals both from private and public information) and incomplete information (only public information signal) traders trade. The public information relates to economy and industry wide information and yields either a buy or a sell signal. A private information signal relates to firm specific information. A fraction of the complete information traders, generally institutions are allowed to short sell if they receive contrary information signals from the public and the private information sources. The traders trade a single risky security with a competitive market maker.

The derived results from the model suggest that information content in lack of trades, and short selling are subject to the differences of opinion related to public information. The price effect due to sells and buys are found to be asymmetric. Optimism (anticipation of a bullish market) or pessimism (anticipation of a bearish market) in the market leads to different treatments of lack of trades and short selling in terms of their price effect. The adjustment of security prices to their full information value is a decreasing function of the fraction of short-sellers in the market. However, the speed of adjustment of prices towards a “low value” is instantaneous in an extreme bull market. In the future, we plan to look at the relative speed of adjustment to lend more insight into the adjustment process.

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Appendix

Proof of Proposition 1:

$\Pr(N_t|e=0)=1-b$.
$\Pr\left(N_t|e=1,d=0\right)=qm(1-a)+(1-q)(1-b)$ and
$\Pr\left(N_t|e=1,d=1\right)=q(1-m)(1-a)+(1-q)(1-b)$.
Clearly,
$\Pr\left(N_t|e=0\right)>\Pr\left(N_t|e=1\right)$ iff
for $d=1, \dfrac{1-b}{1-a}>1-m$ and
for $d=0, \dfrac{1-b}{1-a}>m.$

Proof of Proposition 2:

By applying Bayes' rule, the conditional probability of a low value signal given a trade (buy or sell) or no trade is computed as follows:

When a sell is not distinguished from a short-sell,

$$d\left(S_1\right)=d\left[\frac{e\left(mq+m(1-q)b+qa(1-m)\right)+(1-e)mb}{ed\left(mq+m(1-q)b+qa(1-m)\right)+em(1-d)\left(qa+(1-q)b\right)+(1-e)mb}\right].$$

$d\left(S_1\right)>d$ iff $eq(1-d)\left(m+a(1-2m)\right)>0.$ Since e,q,d are all strictly positive, $d\left(S_1\right)>d$ if $m+a(1-2m)>0$. Now $m+a(1-2m)>0$ implies $\frac{1}{2}\left(\frac{1}{a}+\frac{1}{m}\right)>1.$ For any $m<1,$ and $a<1$ $\left(m+a(1-2m)\right)>0$.

Similarly, the probability of low value conditional on a buy order is computed as,

$$d\left(B_1\right)=d\,\frac{eb(1-m)(1-q)+(1-e)(1-m)b}{edb(1-m)(1-q)+e(1-d)\left(mq+(1-m)(1-q)b\right)+(1-e)(1-m)b}$$. Since e,d,q are strictly positive, $d\left(B_1\right)<d$.

$d(SS_1) = d\dfrac{eqa(1-m)}{edqa(1-m)+emqa(1-d)}$ is either greater than, less than or equal to d depending on m.

Similar calculations show that $d(N_1) > d$ iff $eq(1-d)(1-a)(1-2m) > 0$. Clearly, $d(N_1)$ can be either greater than or less than d. $d(N_1) = d$ iff $m = 0.5$.

The bid and ask prices for the initial sell (including short-sell) and buy orders, are computed as:

$b_1 = E(v|sale/short) = 1-d(S_1)$, and $a_1 = E(v|buy) = 1-d(B_1)$. It follows that

$$b_1 = (1-d)\frac{emqa + emb(1-q) + (1-e)mb}{ed\left(mq + mb(1-q) + (1-m)qa\right) + em(1-d)\left(qa + (1-q)b\right) + (1-e)mb} < 1-d$$

and $a_1 = (1-d)\dfrac{emq + (1-m)(1-eq)b}{emq + (1-m)(1-eq)b} > 1-d$.

Proof of Proposition 3:

Let the conditional probabilities of a low, high or no-signal conditional on a series of sale/short, buy, and no-trade be defined as follows:

$r_{0,t} = \Pr(s = f|Q^{t-1})$

$r_{L,t} = \Pr(s = L|Q^{t-1})$

$r_{H,t} = \Pr(s = H|Q^{t-1})$

By applying Bayes' rule, the probabilities of a low, high or no-signal conditional on a no-trade at time *t* are computed as follows:

$r_{0,t+1} = \dfrac{r_{0,t}}{r_{L,t}(1-mq) + r_{H,t}(1-q(1-m)) + r_{0,t}}$ is either greater than or less than $r_{0,t}$.

$$r_{L,t+1} = \frac{r_{L,t}(1-mq)}{r_{L,t}(1-mq) + r_{H,t}(1-q(1-m)) + r_{0,t}} < r_{L,t}.$$

$$r_{H,t+1} = \frac{r_{H,t}(1-q(1-m))}{r_{L,t}(1-mq) + r_{H,t}(1-q(1-m)) + r_{0,t}} < r_{H,t}.$$

Proof of Proposition 4:

By definition, $d_{t+1}\left(Q^{t+1}\right)=\Pr\left(s=L|Q^{t-1},N,S\right)+d*\Pr\left(s=f|Q^{t-1},N,S\right)$, where the t period outcome was no-trade. Computation shows:

$$d_{t+1}\left(Q^{t+1}\right)=\frac{r_{L,t}(1-q)\left(mq+m(1-q)b+(1-m)qa\right)+dr_{0,t}mb}{r_{L,t}(1-q)\left(mq+(1-m)qa\right)+r_{H,t}(1-q)\left(mqa+(1-q)(1-m)b\right)+mb\left(1-q\left(1-r_{0,t}\right)\right)+qr_{0,t}mb}.$$

$$\text{and}\, d_{t}\left(Q^{t}\right)=\frac{r_{L,t}\left(mq+m(1-q)b+(1-m)qa\right)+dr_{0,t}mb}{r_{L,t}\left(mq+(1-m)qa\right)+r_{H,t}\left(mqa+(1-q)(1-m)b\right)+mb\left(1-q\left(1-r_{0,t}\right)\right)}.$$

Now $b_t<E(v)=1-d$ implies $d_t\left(Q^t\right)>d$. We need to show that $b_{t+1}>b_t$ i.e., $d_{t+1}\left(Q^{t+1}\right)<d_t\left(Q^t\right)$. Now, $d_{t+1}\left(Q^{t+1}\right)<d_t\left(Q^t\right)$ iff $d_t\left(Q^t\right)>d$.

Similar computations are done for other bid and ask prices above or below the unconditional expected value at time *t*.

Proof of Proposition 5:

The details of this proof exactly follows a similar proof in Diamond and Verrecchia (1987). We first compute the expected number of periods until the price of a security reaches its full information price, either 0 or 1. In fact, instead of 0 and 1, we work with two hypothetical boundaries $\log(\Phi)$, and $\log(\Psi)$ where $\Psi>\Phi$. The threshold $\log(\Phi)$ and $\log(\Psi)$ are the possible values of the posterior log likelihood ratios and is similar to the Wald's sequential ratio test, $\log\left(\frac{p_t}{1-p_t}\right)$ for the hypothesis *v=0* against *v=1*, where p_t is the price or conditional expectation of *v* given low value of the security.

Let $\tilde{N}$ be a random variable denoting the number of periods until the price first reaches either the lower boundary $\log(\Phi)$ or the upper boundary $\log(\Psi)$.

$$E\left(\tilde{N}\right)=\frac{E\left[Log\left(\tilde{\Lambda}_N\right)\right]}{E\left[\tilde{Z}\right]}\quad\text{where}$$

$\Lambda_N = \frac{p_n}{1-p_n}$ and $Z^A = \log\left(\frac{q_1^A}{q_0^A}\right)$.

Further we define, the expected number of periods till the price reaches a boundary conditional on the true value of the security as follows:

$$\tilde{N}_0 = E\left(\tilde{N} | v = 0\right) = \frac{\left(\frac{1-\Phi}{\Psi-\Phi}\right)\log(\Psi) + \left(\frac{\Psi-1}{\Psi-\Phi}\right)\log(\Phi)}{\sum_{A\in(Q)} q_0^A \log\left(\frac{q_1^A}{q_0^A}\right)} \text{ and}$$

$$\tilde{N}_1 = E\left(\tilde{N} | v = 1\right) = \frac{\Psi\left(\frac{1-\Phi}{\Psi-\Phi}\right)\log(\Psi) + \Phi\left(\frac{\Psi-1}{\Psi-\Phi}\right)\log(\Phi)}{\sum_{A\in(Q)} q_1^A \log\left(\frac{q_1^A}{q_0^A}\right)}.$$

Since the numerators are fixed we consider the effect of short-sell constraint, a on the denominator. Let the denominators be called D_0 and D_1 corresponding with N_0 and N_1 respectively. Since our interest is to show the effect of a on N_0 and N_1 we postulate the following relationships:

$\frac{\partial N_0(a)}{\partial a} = \frac{\partial N_0}{\partial D_0}\frac{\partial D_0}{\partial a} < 0$ which implies $D_0' > 0$ since $\frac{\partial N_0}{\partial D_0} < 0$. Similarly,
$\frac{\partial N_1(a)}{\partial a} = \frac{\partial N_1}{\partial D_1}\frac{\partial D_1}{\partial a} < 0$ implying $D_1' > 0$.

We derive explicitly the denominators as follows:

$$D_0 = q_0^B \ln(1+y) + q_0^S \ln(1+x) + q_0^N \ln(1+z) \text{ and}$$

$$D_1 = (1+y)q_0^B \ln(1+y) + q_0^S(1+x)\ln(1+x) + (1+z)q_0^N \ln(1+z)$$

where,

$$q_0^B = (1-m)(1-q)b$$
$$q_0^S = m(q + b(1-q)) + (1-m)qa$$
$$q_0^N = (1-q)(1-b) + (1-m)(1-a)q$$
$$y = \frac{q}{(1-q)b}$$

$$x = \frac{qa(2m-1) - mq}{mq + m(1-q)b + (1-m)qa}$$

$$z = \frac{(1-a)q(2m-1)}{(1-m)(1-a)q + (1-q)(1-b)}.$$

We first turn our attention to the general case where $1 \geq m \geq 0$. Notice that both D_0 and D_1 become undefined if either *x, y, or z* becomes less than or equal to –1. It turns out that each of *x, y, and z* is greater than –1 for $1 \geq m > 0$. For $m = 0$, x=-1. However the signs of the first order conditions $D_0'(a)$ and $D_1'(a)$ for the general case can not be determined without a numerical exercise. Therefore we break down the proof at three discrete points over possible values of m.

a) If m =0.5

$$D_0(a) = q_0^S \ln(1+x) = .5(q + (1-q)b + qa)\ln\left[\frac{qa + (1-q)b}{q + qa + (1-q)b}\right].$$

$$D_0'(a) = .5q \ln(1+x) + q^2 q_0^S \left[\frac{1}{g(a)h(a)}\right] > 0$$

where, $g(a) = qa + (1-q)b$

and $h(a) = q + g(a)$

$$D_1(a) = (1+x)q_0^S \ln(1+x) = \frac{g(a)}{h(a)} q_0^S \left[\ln(g(a)) - \ln(h(a))\right]$$

$$D_1'(a) = x\ln(1+x)q_0^S \frac{q}{h(a)} + (1+x)D_0' > 0$$ (Since both x and $\ln(1+x)$ are negative).

b) If $m = 1$

$$D_0(a) = q_0^S \ln(1+x) + q_0^N \ln(1+z)$$

$$D_0'(a) = \frac{q + (1-q)b}{qa + (1-q)b} - \frac{(1-q)(1-b)}{(1-a)q + (1-q)(1-b)} > 0$$

and $D_1(a) = (1+x)q_0^S \ln(1+x) + (1+z)q_0^N \ln(1+z)$

$$D_1'(a) = \ln\left(\frac{1+x}{1+z}\right) > 0.$$

c) If $m = 0$

$D_0(a)$ is undefined since x=-1 and $\ln(1+x)$=undefined.

$$D_1(a) = (1+z)q_0^N \ln(1+z)$$
$$D_1'(a) = \frac{q(1-q)(1-b)}{(1-q)(1-b)+(1-a)q} > 0.$$