Optimal pricing of endogenous congestion: a disaggregated approach

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Abstract

We design and estimate a game theoretic congestion pricing mechanism in which the regulator aims at reducing traffic congestion by discriminating travelers according to their willingness to travel on the network. He knows that travelers learn about their environment, that their preferences are affected by the reputation of each available mode of transportation and that congestion can be seen as a Bayesian game in which travelers impose externalities on each other. We derive individual optimal fares depending on each traveler's valuation of transportation. Welfare simulation results based on a French household survey show that the travelers' perception of the mode of transportation and income sensitivity differences are important determinants of welfare improvement.

Classification codes: R4, D8, D6.

Keywords: regulatory policy; endogenous congestion; incomplete information; reputation effects, welfare simulation.

1 Introduction

The predominant form of urban congestion pricing in Europe is flat rate pricing. For example, car travelers driving in Central London on weekdays between 7:00am and 6:30pm were required to pay 5 pounds, increasing to 8 pounds since 2005 (see also Mannheim, Oslo or Bergen, for similar experiments). This simple pricing scheme was chosen because it was relatively fast and easy

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to implement. The experiment has been successful in many ways, however, there are still concerns regarding the optimal price and/or price structure. In particular, once travelers pay the fee, they have no incentive to minimize driving. In this context, a variable road use fee that would reflect the heterogeneity of drivers, such as the type of vehicle, time and frequency, coupled with it is driven within the price-controlled area would seem more appropriate. This approach would most accurately reflect the external social costs imposed by driving and would give travelers an incentive to minimize their negative impact by stimulating drivers to make choices that maximize both their own utility and society's.

A core issue of concern to urban network users, transportation operators and economists is the constantly changing network conditions arising from aggregate decisions and behavior. The literature on road networks (initiated by Beckmann et al., 1956 and extended by Dafermos, 1973 to heterogenous travelers) and on second best optimal pricing (see Gmez-Ibez and Small, 1994; Arnott et al., 1994, 1998; Emmerinck, 1998; Parry and Small, 2005) is based upon the assumption that congestion is stochastic. In these studies, congestion is usually modeled by a function of time and traffic flow, where congestion externalities are a parameter to be estimated. Using this approach, the regulator is able to control the traffic flow "as is" using time varying congestion tolls. We argue that the network behavior depends on the aggregate load of the network - the result of many users' decisions on how to use the network. In this sense, it is important that the regulator is able to control the incentives of the travelers.
Our approach is closer to McKie-Mason and Varian who in 1995 proposed a "smart market" mechanism that suggests an auction based scheme to price internet congestion. Indeed, the urban European city network is similar, in many ways, to the internet network. The street network in the core areas that concentrate most activities (work, school, leisure and shopping) are rarely expanded.
Since urban congestion is caused by too many travelers competing for a limited road space, our objective is to find an economically efficient way to allocate network resources among travelers. Former studies show that incomplete information about aversion to congestion or Value Of Time (VOT) is significant and that urban travelers are willing to pay a non-negligible amount of money to improve their traffic conditions. However, this goal requires that we know the true value that each traveler places on transportation services. We propose a game theoretic mechanism that can reproduce the Bayesian game that travelers play to decide upon the number of trips they make and the resulting aggregation that forms congestion. To the best of our knowledge, this is the first paper to derive and estimate a congestion pricing mechanism that accounts for the endogeneity of congestion using travelers' anticipations of traffic and reputation of transportation modes. This paper differs from the analysis undertaken in Viauroux (2007) in three major ways. First, we assume that the marginal utility of income differs across individuals, while Viauroux (2007) assumed that it was fixed across travelers. In other words, the effect on subjective well-being and spending on transportation of a $1,000 increase in income is assumed to differ across individuals. It is known to each individual and unknown to the

analyst and the regulator of public transport. Second, we assume that the individual's preferences for travelling depend on two unobserved components: the aversion to traffic congestion and some unobserved errors on the choice of mode and type of payment used. These last error terms can be seen as a measure of the reputation of the mode of transportation, which can change the number of trips performed. Third, following the modeling assumptions above, the paper analyzes the outcome of the mechanism for the city of Montpellier (France), in particular the link between travelers heterogeneity and welfare generated (see also Small, 1992; Small and Yan, 2001). The optimal price per trip depends upon the odds of being a high valuation traveler, on the opportunity cost of using transportation and the rate of variation of the marginal utility of income. After giving the expressions of the pricing functions, we simulate and compare welfare over the two-day survey during peak and off peak periods using a nonlinear tariff for the underlying modelling of transportation demand using the following three assumptions: 1- No reputation unobservables, MU of income constant; 2- reputation unobservables, MU of income constant; 3- reputation unobservables, heterogenous MU of income. For the fourth case, we also consider the case of nonlinear homogenous congestion pricing. We show this "richer" preference specification with heterogenous marginal utilities of income and reputation unobservables results in larger efficiency gains. This further shows that optimal policy and its implied efficiency effects depend critically on the specification on how unobserved heterogeneity type enters preferences. Reputation effects improve the overall quality of the transportation demand model and acts as a regulator by increasing the overall welfare without the introduction of a nonlinear tariff. However, welfare can be improved further when heterogeneity in the marginal utility of income is introduced. The non linear pricing schedule accounting for the heterogeneity of travelers' sensitivity to price shows an improvement over the current pricing scheme of 6.2% and an improvement over the nonlinear pricing assuming constant MU of income of 10.9% during the peak period. These increases are respectively 4.3% and 4.5% during the off-peak period. Interestingly, the loss in welfare associated to a uniform nonlinear tariff, where the average estimated aversion to traffic congestion is introduced, doesn't substantially decrease welfare.

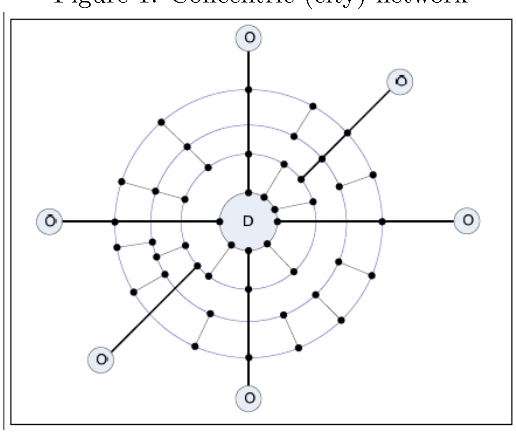
The paper is organized as follows. Section 2 briefly presents the network and the theoretical framework used. Sections 3 details the model estimation procedure. Section 4 derives the optimal pricing policies in the case of complete and incomplete information about each traveler's aversion to traffic congestion. Our work illustrates the gains that could be achieved if this mechanism were to be implemented. Estimation and simulation results are reported and analyzed in Section 5. Section 6 concludes.

2 Framework

2.1 Introduction

As described by Fudenberg and Tirole (1991), mechanism design can be viewed as a multi-step game of incomplete information where agent's "types" are private information. In the first step of the game the "principal" (here the regulator) designs a mechanism. The regulator, who maximizes social welfare, is a government authority who also provides public transportation. They have complete information about production costs, and possess a good estimate of the marginal cost of trips for individuals. In this paper, the mechanism is a pricing mechanism that accounts for the endogeneity of congestion, the reputation of the modes and the incomplete information of the regulator about each traveler's private VOT. The regulator will discriminate with respect to this VOT: travelers must pay the price (congestion fee) depending on the amount of congestion anticipated on the network. In the next step of the game, agents either accept or reject the mechanism designed by the principal. Agents that accept enter the third step and play the game of the mechanism. In the transportation network, the goal of the regulator is to design a mechanism that will produce a Pareto optimal distribution of traffic in equilibrium. Typically, the urban network in european cities can be illustrated in the Figure 1 below.

Figure 1: Concentric (city) network



Travelers' trips begin at one node and progress efficiently to a destination by traversing the path between the intervening nodes. Their goal is to find the route from the Origin node (O) to the Destination node (D), with lowest congestion and distance between the intervening nodes. Hence, the least cost path is not necessarily the shortest but rather the summation of the contiguous paths of lowest congestion. The problem is that travelers choose a path that is consistent with their best interest without regard for the interest of others traveling the network. Implementing the game above would entail the regulator to allow each traveler to select a quantity-subscription pair for each mode of transportation regulated. Note that the feasibility of this instrument is not obvious as travelers might find it difficult to give their bids and the underlying bureaucracy could be very costly. One step in that direction might be for the regulator to require the purchase of an RFID (Radio Frequency IDentification) card as a prerequisite to using toll roads and to link the traveler's identification to his/her observed characteristics. The traveler could then purchase a specific number of trips and choose a maximum amount of spending for the period. However, despite the limits in its practical feasibility, derivation of the mechanism gives an upper bound on the efficiency gains of a nonlinear endogenous congestion pricing schedule, ignoring transaction costs.

2.2 Assumptions

Our model relies on three major assumptions:

First, let h_i denote the marginal utility of income of individual i, which can be seen as a rate of change between money and transportation preferences: a 1 euro spending gives the individual a change in utility equal to h_i; or an increase by 1 euro in transportation costs implies a h_i euro decrease in the consumption of other goods, which quantity is denoted by $\nu_{i,p}$. When h_i is high, the individual is very sensitive to this increase, which translates in a sharp decrease in the consumption of $\nu_{i,p}$. One might think that this affects lower income individuals more. On the contrary, a low h_i means that the individual is hardly sensitive to an increase in transportation costs. Figures 2a-b summarize statistics on the number of car and bus trips by income[1] range. They show that the number of car trips increases with income up to a peak level for an income range between 2439 euros and 2897 euros, but decreases thereafter. The number of bus trips on the other end decreases down to a minimum for an income range of (3,354;3,811] and increases thereafter.

[1] Income is defined as the gross monthly income of the household, including the professional income of all members of the household including premiums such as a thirteen month, yield premium as well as other real estate income. For confidentiallity reasons, the household head was asked to give only a range of income. Despite this precautionary survey measure, 30% of household heads refused to answer. In this case, we replaced missing values of income by its prediction: observed income has been regressed on variables caracterising the individual, such as age, sex and professional status (executive, farmer, retired, factory worker).

Figure 2a: Number of car trips by income range

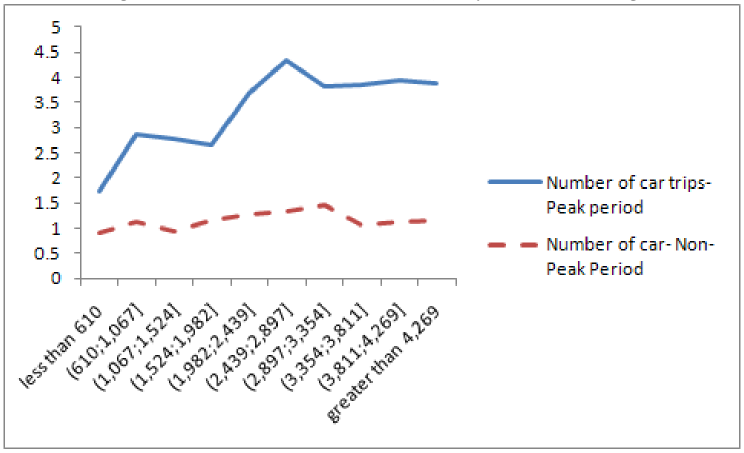
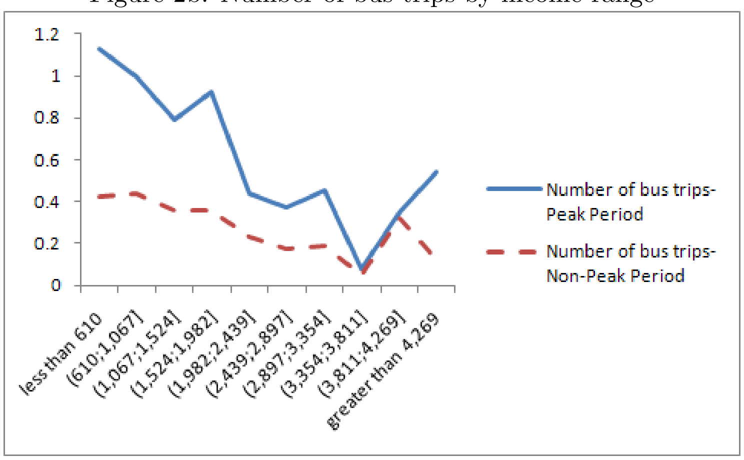


Figure 2b: Number of bus trips by income range



Hence, the figures show that the sensitivity to an increase in transportation costs reverses direction after a certain income level. In other words, the marginal utility of income is not broadly constant but a function of some unobserved phenomena, which becomes obvious after a certain level of income.

We specify the MU of income as a function of the unobserved aversion to traffic congestion: the more tolerant to traffic congestion, the more an individual uses transportation despite traffic conditions and the higher the marginal utility of transportation usage (as we expect k to be estimated to be positive). This is true for example for individuals, who have more flexibility in their transportation schedule, which more likely occurs among high ranges of income. As a first step, we will specify the MU of income as a linear function of the VOT of each traveler, i.e. $h(\theta) = k\theta$. Hence, we assume that high income travelers have a lower tolerence for traffic congestion than high income travelers. This is consistent with the literature stating that the MU of income decreases with income (see Frisch, 1964 and Clark, 1973 for the construction of empirical measures of MU of income in transportation).

The second assumption is about the choice between the alternative modes of transportation to travel during a two-day period. We assume that the traveler can either walk to his destinations, i.e use neither the car nor the bus; he can use the car at least once but not the bus; use the bus at least once but not the car. The choice of each of these modes is affected an unobserved factor, reflecting their reputation, and these factors will indirectly affect transportation usage.

Third, we assume that all roads leading to a central business district are congested but that the level of congestion varies; hence the selection is made across modes of transportation that could involve more traveling time (such as the bus for example). This assumption is in line with pricing experiments such as Singapore, London or Mannheim, for example, where the optimal regulatory policy was applied to an entire congested district. Travelers' preferences for travel depend on the anticipated level of congestion, which in turn was determined by the decisions of all other individuals. Travelers decide on the number of trips to take with a particular mode of transportation. They consider both car and public transportation use, hereafter referred to as "bus", which are interconnected by the inconvenience of traffic congestion created, mostly, by cars. Their decisions result from a Bayesian game in which travelers possess private information about their value of time (VOT) or 'type' as follows. [2]

2.3 The Model

We consider the Bayesian game (Harsányi, 1967–68) $\Gamma = (I, Q, \Theta, (p_i)_{i\in I}, \left(u^j_{i,p}\right)_{i\in I})$, where I is the set of individuals i traveling in the city ($i = 1, ...I$), Q is the set of possible number of car and bus trips performed in period p by individual i denoted by $q_{i,p} = (q^c_{i,p}, q^{bj}_{i,p})$, $\Theta = \Theta_i \times \Theta_{-i}$ is the set of possible types $\theta = (\theta_i, \theta_{-i})$ representing travelers' tolerance to traffic congestion or the value that they associate to the time lost in traffic, where $\Theta_{-i} := \underset{j\in I-i}{\times} \Theta_j$ (a low tolerance correspond to a high VOT). We will let θ_i index individual i VOT) and refer to θ_i as individual i's "type" (for $i = 1, \ldots, I$). $\theta_i \in \Theta_i = [\underline{\theta_i}, \overline{\theta_i}]$ with $\underline{\theta_i} > 0$. $\underline{\theta_i}$ is a taste parameter indexing the least tolerance (greatest aversion) for congestion while $\overline{\theta_i}$ represents the greatest tolerance (lowest aversion) for the externalities. The regulator belief about θ_i is reflected in the density $f_i(\theta_i)$ and a cumulative distribution function $F_i(\theta_i)$. We denote by $p_i(.|\theta_i)$ the probability distribution over Θ_{-i} and by σ is the randomized-strategy profile for the game,

[2] Given his type, a traveler determines the optimal probability distribution of use for each relevant mode of transportation and chooses the number of trips he wants to make for a given period of time. For example, if his value of time is low, he may lower his number of car trips and increase his probability of using an alternative mode of transportation, such as the bus. Note, that his best probability distribution depends on the probability of all other travelers (of different possible types) taking trips.

such that $\sigma = \left(\sigma_i\left(q_{i,p}|\theta_i\right)\right)_{i\in I, q_{i,p}\in Q_{i,p},\theta_i\in\Theta}$.

Travelling preferences are contingent upon the number of trips taken and upon the choice/decision of a mode of transportation. We assume that individuals' preferences for traveling are affected by their perception of the associated mode of transportation. This perception is negatively correlated to the individual's choice of consuming other goods in quantity $\nu_{i,p}$. In particular, we assume that there exists unobserved factors relevant to the decision of the individual toward one mode of transportation and other goods, that will impact the number of trips taken and the quantity $\nu_{i,p}$ of other goods consumed. We assume that these unobserved factors are correlated with the quality of the mode, the aversion to traffic congestion and the amount of traffic on the network. We use the random utility approach from Manski (1977) and consider a variation $\left(u_{i,p}^j\right)_{i\in I}$ of the form of utility functions introduced by Blackburn, 1970 and Hanemann, 1984 for $j = 1, ...J$,

$$\begin{aligned} U_{i,p}^j &= U_{i,p}^j(q_{i,p}^j, q_{-i,p}^{j*}, \nu_{i,p}, \theta_i, \varepsilon_{1i}, \varepsilon_{2i}^j, \varepsilon_{3i}, \varepsilon_{\nu i}) \\ &= \alpha\left(q_{i,p}^c + \varepsilon_{1i}\right)\left[1 + \psi_{i,p}^c + \ln\theta_i - \ln s_{-i,p} - \ln\left(q_{i,p}^c + \varepsilon_{1i}\right)\right] \\ &\quad +(1-\alpha)\left(q_{i,p}^{bj} + \varepsilon_{2i}^j\right)\left[1 + \psi_{i,p}^b + \ln\theta_i - \ln s_{-i,p} - \ln\left(q_{i,p}^{bj} + \varepsilon_{2i}^j\right)\right] \\ &\quad +h\left(\theta_i\right)\left(\nu_{i,p} + \varepsilon_{\nu i}\right) + \varepsilon_{3i} \end{aligned}$$

where $s_{-i,p} = s_{-i}\left(q_{-i,p}^c\right)$ is a mapping of number of automobile trips $q_{-i,p}^c$ made by individuals other than i, $\psi_{i,p}^c$ (respectively $\psi_{i,p}^b$) denotes a measure of comfort of traveling by car (respectively by bus) for individual i. The letter α (respectively $1-\alpha$) represents the marginal utility of using the car (respectively the bus), and $h(.)$ is the individual marginal utility of income assumed to be a function of the aversion to congestion θ_i. Both $h(.)$ and α are unknown parameters. We denote by ε_{1i}, ε_{2i}, ε_{3i}, and $\varepsilon_{\nu i}$ the unobserved factors associated respectively to the choice of car, bus (associated with a type of payment $j = 1, ...J$), other modes (walk or ride) and the consumption of other goods, such that $E(\varepsilon_{1i}) = 0$, $E(\varepsilon_{2i}^j) = 0$, $E(\varepsilon_{3i}) = 0$, and $E(\varepsilon_{\nu i}) = 0$, and by $F_1(.)$, $F_2(.)$, $F_3(.)$ and $F_\nu(.)$ the respective cumulative distribution functions.

The letter s_{-i} denotes the level of inconvenience in traffic met by individual i. We assume that it is represented by the average number of car trips made by individuals different from i, given by[3]

$$s_{-i} = \frac{1}{I-1}\sum_{j\in I-i}\int_{\varepsilon_{1l}} q_l^c\left(\varepsilon_{1l}\right)dF_1(\varepsilon_{1j})$$

in case of complete information of the regulator on the individual aversion to traffic congestion, and by

$$s_{-i} = \frac{1}{I-1}\sum_{l\in I-i}\int_{\Theta_l}\int_{\varepsilon_{1l}} q_l^c(\theta_l, \varepsilon_{1l})dF_1(\varepsilon_{1l})dF_\theta(\theta_l)$$

[3] Congestion initiated by buses is assumed negligible.

in case of incomplete information.

Each traveler i faces the following budget constraint:

$$a_i^c + p_i^c q_{i,p}^c + a_i^b + p_i^{bj} q_{i,p}^{bj} + p_\nu \nu_{i,p} \leq w_i \tag{1}$$

where p_i^c(resp. p_i^{bj}) denote the car unit price (resp. the bus unit price for all possible options of payment $j = 1, ...J$), a_i^c (resp. a_i^{bj}) denote the fixed charges for car use (resp. bus use for all $j = 1, ...J$) and p_ν represents the unit price of the composite good $\nu_{i,p}$ that we normalize to 1 without loss of generality.

Proposition 1 *Traveler i's program of maximization is*

$$\max_{q_{i,p}^c, q_{i,p}^{bj}} U_{i,p}^j$$

subject to (1) gives the following demand functions for any $j = 1, ...J$:

$$q_{i,p}^{c*}(\theta_i, \varepsilon_{1i}) = \frac{\theta_i}{s_{-i}^*} e^{\psi_{i,p}^c - \frac{h(\theta_i) p_i^c}{\alpha}} - \varepsilon_{1i}, \tag{2}$$

$$q_{i,p}^{bj*}(\theta_i, \varepsilon_{2i}^j) = \frac{\theta_i}{s_{-i}^*} e^{\psi_{i,p}^b - \frac{h(\theta_i) p_i^{bj}}{(1-\alpha)}} - \varepsilon_{2i}^j, \tag{3}$$

with

$$s^* := \left(\frac{1}{I} \sum_{j \in I} \int_{\theta_j \in \Theta} \theta_j e^{\psi_j^c - \frac{h(\theta_j) p_j^c}{\alpha}} dF(\theta_l) \right)^{1/2} \tag{4}$$

in the case of incomplete information and

$$(s^*)^2 := \frac{1}{I} \sum_{j \in I - i} \theta_j e^{\psi_j^c - \frac{h(\theta_j) p_j^c}{\alpha}} \tag{5}$$

in the case of complete information.

Proof. See Appendix 1. ■

Note that $\frac{\alpha}{h(\theta_i)} = \frac{\partial U_{i,p}^j}{\partial U_{i,p}^c} \Big/ \frac{\partial U_{i,p}^j}{\partial w_i} = \frac{\partial w_i}{\partial U_{i,p}^c}$ (respectively $\frac{1-\alpha}{h(\theta_i)} = \frac{\partial w_i}{\partial U_{i,p}^b}$) where $U_{i,p}^c$ (respectively $U_{i,p}^b$) denote the separable portion of the direct utility function representing the individual taste for traveling by car (respectively by bus).

To obtain the expression of the indirect utility function, let us rewrite the equalities (45) and (46) in the form

$$1 + \psi_{i,p}^c + \ln \theta_i - \ln s_{-i}^* - \ln \left(q_{i,p}^{c*} + \varepsilon_{1i} \right) = 1 + \frac{h(\theta_i) p_i^c}{\alpha},$$

$$1 + \psi_{i,p}^b + \ln \theta_i - \ln s_{-i}^* - \ln \left(q_{i,p}^{bj*} + \varepsilon_{2i}^j \right) = 1 + \frac{h(\theta_i) p_i^{bj}}{(1-\alpha)} \text{ for } j = 1, ..., J.$$

Then, we obtain for $j = 1, ..., J$

$$\begin{aligned}
&V_i(w_i - a_i^c - a_i^{bj}, p^c, p_i^{bj}, \theta, \varepsilon_{1i}, \varepsilon_{2i}^j, \varepsilon_{3i}, \varepsilon_\nu) \\
&\quad = \alpha\left(q_{i,p}^{c*} + \varepsilon_{1i}\right)\left[1 + \frac{h(\theta_i)p_i^c}{\alpha}\right] + (1-\alpha)\left(q_{i,p}^{bj*} + \varepsilon_{2i}^j\right)\left[1 + \frac{h(\theta)p_i^{bj}}{(1-\alpha)}\right] \\
&\quad + h(\theta)(w_i - p_i^c q_{i,p}^{c*} - p_i^{bj} q_{i,p}^{bj*} - a_i^c - a_i^{bj} + \varepsilon_\nu) + \varepsilon_{3i} \\
&\quad = \alpha\left(q_{i,p}^{c*} + \varepsilon_{1i}\right) + (1-\alpha)\left(q_{i,p}^{bj*} + \varepsilon_{2i}^j\right) + \varepsilon_{1i} h(\theta_i) p_i^c + \varepsilon_{2i}^j h(\theta) p_i^{bj} \\
&\quad + h(\theta)(w_i - a_i^c - a_i^{bj} + \varepsilon_\nu) + \varepsilon_{3i}
\end{aligned}$$

as stated.

Assuming that each traveler chooses only one type of payment during the survey period so that $j := j(i)$, let $\varepsilon_{\nu i} = -\varepsilon_{1i} p_i^c - \varepsilon_{2i}^j p_i^{bj}$ where p_i^c and p_i^{bj} are points of exogenous price vectors. The indirect utility function becomes:

$$\begin{aligned}
&V_i(w_i - a_i^c - a_i^{bj}, p^c, p_i^{bj}, \theta, \varepsilon_{1i}, \varepsilon_{2i}^j, \varepsilon_{3i}) \\
&\quad = \alpha\left(q_{i,p}^{c*} + \varepsilon_{1i}\right) + (1-\alpha)\left(q_{i,p}^{bj*} + \varepsilon_{2i}^j\right) + h(\theta)(w_i - a_i^c - a_i^{bj}) + \varepsilon_{3i} \\
&\quad = \alpha q_{i,p}^{c*} + (1-\alpha) q_{i,p}^{bj*} + h(\theta)(w_i - a_i^c - a_i^{bj}) + \alpha\varepsilon_{1i} + (1-\alpha)\varepsilon_{2i}^j + \varepsilon_{3i}
\end{aligned}$$

3 Estimation

We use a household survey in the greater Montpellier area (south of France; 229,055 inhabitants) recording the transportation activity of 6341 individuals during a two day period. A trip is seen as a more-than-300-meters drive or run between two places on a public road. We focus on trips made for the purposes of work, school, shopping, leisure where the returns trips home are not accounted for. We specify vectors of comfort of traveling by $\psi_{i,p}^b = \beta^b X_i^b$, and $\psi_{i,p}^c = \beta^c X_i^c$ where X_i^b and X_i^c are vectors characterizing the trip based upon the time between the Origin and the Destination (O-D) as well as socioeconomic characteristics of individual i (see Tables 1 and 2).

We define the contribution to the likelihood as the probability of choosing a mode of transportation times the probability of making a positive number of trips given that this mode is being chosen. We assume that the observed number of trips corresponds to the equilibrium number of trips found in the theoretical section of the paper.

We model the choice of modes of transportation using McFadden's Multinomial Logit Model, while we model the observed number of trips using the Poisson distribution, the expectation of which depends continuously on θ and ε.[4] Since

[4] This is a reasonable assumption over a two-day sample period if we assume the number of trips made during nonoverlapping intervals of time are independent, the probability to make a trip does only depend on the length of the time interval, and the probability to make more than one trip during a very small interval of time is negligible.

a very small proportion of travelers chose both car and bus during the observation period (around 4%), which is consistent with our theoretical result that travelers reach a pure strategy equilibrium (see Viauroux, 2007), we can assume the corresponding Poisson distributions are independent.

3.1 Choice of mode of transportation

To travel during the two-day survey, each agent possesses $2J+2$ mutually exclusive and exhaustive alternative choices. During the two-day survey, we observe four possible trip schemes: (1) the traveler can travel neither by car nor by bus (the trip could then be by bike or by walk or there could be no trip at all), (2) by car only (with possible combined walk and/or bike trips), (3) by bus only (with possible combined walk or bike trips), (4) by car and bus (with possible combined walk and/or bike trips). In the case where the individual travels by bus, he can choose among J types of payment: more specifically, these include a unit ticket of FF7 (1.07 euros), a booklet of 3 tickets of FF20 (3.05 euros) or 10 tickets (discounted for handicapped or large families), a 7 day or 30 day lump sum (discounted for students, non students-employed, unemployed, scholars depending on district subventions, retired with and without "Carte Or"(retirement card) subscription) or a yearly pass (discounted for scholars, students and unemployed non students-scholars). Hence, these payment options depend on the characteristics of the traveler (See Viauroux, 2007 for details on the construction of these prices). The corresponding indirect utility functions are random with an additive error term, i.e. for $j=1,...,J$:

If the individual does not travel by bus but only by car, then $E(q_i^{bj*}+\varepsilon_{2i})=0$ and the indirect utility function is:

$$\begin{aligned} V_{1i}(w_i,a_i^c,p_i^c,\theta,\varepsilon_{1i}) &= \int_{\varepsilon_{3i}}\int_{\varepsilon_{2i}^j} V_i(w_i-a_i^c-a_i^{bj},p_i^c,p_i^{bj},\theta,\varepsilon_{1i},\varepsilon_{2i}^j,\varepsilon_{3i})dF_2(\varepsilon_{2i}^j)dF_3(\varepsilon_{3i}) \\ &= \alpha q_{i,p}^{c*}+h(\theta)(w_i-a_i^c)+\alpha\varepsilon_{1i} \end{aligned} \tag{6}$$

If the individual is not observed to travel by car, but only by bus, then $E(q_{i,p}^{c*}+\varepsilon_{1i})=0$, and the indirect utility function, net of possible additional trips by walk/ride becomes:

$$\begin{aligned} V_{2i}^j(w_i,a_i^{bj},p_i^{bj},\theta,\varepsilon_{2i}) &= \int_{\varepsilon_{3i}}\int_{\varepsilon_{1i}} V_i(w_i-a_i^c-a_i^{bj},p^c,p_i^{bj},\theta,\varepsilon_{1i},\varepsilon_{2i}^j,\varepsilon_{3i})dF_1(\varepsilon_{1i})dF_3(\varepsilon_{3i}) \\ &= (1-\alpha)q_{i,p}^{bj*}+h(\theta)(w_i-a_i^{bj})+(1-\alpha)\varepsilon_{2i}^j \end{aligned} \tag{7}$$

Finally, if the individual does not travel by car or by bus, but only by walking or riding a bike, we have

$$
\begin{aligned}
V_{i3}(w_i,\theta,\varepsilon_{3i}) &= \int_{\varepsilon_{1i}}\int_{\varepsilon_{2i}} V_i(w_i - a_i^c - a_i^{bj}, p^c, p_i^{bj}, \theta, \varepsilon_{1i}, \varepsilon_{2i}^j, \varepsilon_{3i}) dF_1(\varepsilon_{1i}) dF_2(\varepsilon_{2i}^j) \\
&= h(\theta_i) w_i + \varepsilon_{3i}, \qquad (8)
\end{aligned}
$$

Let $u_{1i} = \alpha\varepsilon_{1i}$, $u_{2i} = (1-\alpha)\varepsilon_{2i}^j$, and $u_{3i} = \varepsilon_{3i}$ be independant and identically distributed according to according to a Generalized type I distribution of location parameters 0 and of scale parameters respectively μ_1 and μ_2. The probability to choose an alternative k of joint type of payment and mode of transportation, is of logit form

$$
\text{Prob}_{ikj} = \frac{e^{\overline{V}_{ki}^j}}{\sum_{(k',j')} e^{\overline{V}_{k'i}^{j'}}}.
$$

where $\overline{V}_{ki}^j$ is the deterministic part of the indirect utility function associated with mode of transportation k and type of payment j.

3.2 Choice of number of trips

Conditional on a mode of transportation being chosen, the probability to make k trips by car and n trips by bus is given by:

$$
r_{ikn} := P\left(Q_{i,p}^c = k, Q_{i,p}^b = n\right) = \frac{\exp\left(-q_{i,p}^{c*}\right)\exp\left(-q_{i,p}^{bj*}\right)\left(q_{i,p}^{c*}\right)^k \left(q_{i,p}^{bj*}\right)^n}{k!n!}
$$

for all nonnegative integers k and n.

Combining the above results, we obtain the following expressions for the likelihoods:

$$
\begin{aligned}
l_i &= \int_{\Theta_i}\int_{\varepsilon_{3i}} P(Q_{i,p}^c = 0, Q_{i,p}^{bj} = 0) dF(\theta_i) dF_3(\varepsilon_{3i}) \\
&= \int_{\Theta_i} \frac{e^{\overline{V}_{3i}}}{S}\, dF(\theta_i); \qquad (9) \\
l_{ik}^c &= \int_{\Theta_i}\int_{\varepsilon_{1i}} P(Q_{i,p}^c \geq 1, Q_{i,p}^{bj} = 0)\, dF(\theta_i) dF_1(\varepsilon_{1i}) \\
&= \int_{\Theta_i}\int_{\varepsilon_{1i}} \frac{\exp\left(-q_{i,p}^{c*}\right)(q_{i,p}^{c*})^k}{k!\left(1-\exp\left(-q_{i,p}^{c*}\right)\right)} \frac{e^{\overline{V}_{1i}}}{S}\, dF(\theta_i) dF_1(\varepsilon_{1i}); \qquad (10) \\
l_{in}^{bj} &= \int_{\Theta_i}\int_{\varepsilon_{2i}} P(Q_{i,p}^c = 0, Q_{i,p}^{bj} \geq 1, \bar{t}_i = \bar{t}_i^j)\, dF(\theta_i) dF_2(\varepsilon_{2i}^j) \\
&= \int_{\Theta_i}\int_{\varepsilon_{2i}} \frac{\exp\left(-q_{i,p}^{bj*}\right)(q_{i,p}^{bj*})^n}{n!\left(1-\exp\left(-q_{i,p}^{bj*}\right)\right)} \frac{e^{\overline{V}_{2i}^j}}{S}\, dF(\theta_i) dF_2(\varepsilon_{2i}^j), \quad j = 1,\ldots,J; \qquad (11)
\end{aligned}
$$

where $S = e^{\overline{V}_{1i}} + \sum_{j=1}^{J} e^{\overline{V}^{j}_{2i}} + e^{\overline{V}_{3i}}$.

Finally, the model is estimated using Maximum Likelihood. Let l_i denote the contribution to the likelihood associated with the joint probabilities. Then the unconditional likelihood function is given by the product

$$L = \prod_{i=1}^{N} l_i.$$

4 The mechanism

4.1 Introduction

Travelers must pay the price (congestion fee) depending on the amount of congestion anticipated on the network. The fee is calculated based on the Vickrey-Clarke-Groves Mechanism (see Vickrey, 1961; Clarke, 1971; Groves, 1973). The regulator allows each traveler to select a quantity-subscription pair for each mode of transportation regulated. For car trips, they select $(q^c_{i,p}, a^c_i)$, while for bus trips they select $(q^{bj}_{i,p}, a^b_i)$. The quantity of trips is defined as the number of trips from one origin to one destination within a two day period. The regulator will then set the associated prices (p^c_i, a^c_i) and (p^{bj}_i, a^{bj}_i) as a function of the individual's valuation of traveling. The regulatory policy must satisfy the constraint that the traveler should have an incentive to truthfully report his type.

For each possible set of observable characteristics, the regulator has some information for θ_i prior to any valuation report from the individual, which is common knowledge to the firm and all other travelers than i. The regulator's belief about θ_i is reflected in the density $f_i(\theta_i)$ and a cumulative distribution function $F_i(\theta_i)$. The regulator also has a priori information on the distribution associated to the individual perception of each mode of transportation, leading to their decision of chosing one. He knows that ε_{1i}, $\varepsilon^j_{2i}, \varepsilon_{3i}$ are respectively distributed according to a Gumbel distribution.

We describe a regulatory policy by six outcome functions $\left(p^c_i, p^{bj}_i, q^c_{i,p}, q^{bj}_{i,p}, a^c_i, a^{bj}_i\right)$ which can be interpreted as follows. For any $\widetilde{\theta}_i \in [\underline{\theta_i}, \overline{\theta_i}]$, the regulator proposes a subscription fee of a^c_i and/or a^b_i and a unit price per trip $p^c_i\left(\widetilde{\theta}_i\right)$ and $p^{bj}_i\left(\widetilde{\theta}_i\right)$ and $q^c_{i,p}$ and $q^{bj}_{i,p}$ is the corresponding quantity of trips satisfying respectively $p^c_i = P\left(q^c_{i,p}\left(\widetilde{\theta}_i, \varepsilon_{1i}\right)\right)$ and $p^{bj}_i = P\left(q^{bj}_{i,p}\left(\widetilde{\theta}_i, \varepsilon^j_{2i}\right)\right)$ where $P\left(q^c_{i,p}\left(\widetilde{\theta}_i, \varepsilon_{1i}\right)\right)$ and $P\left(q^{bj}_{i,p}\left(\widetilde{\theta}_i, \varepsilon^j_{2i}\right)\right)$ are the individual inverse demand curves resulting from the Bayesian game among travelers. Inverse demand functions for transportation usage are obtained from section 2 above:

$$p_i^c\left(q_{i,p}^c, \varepsilon_{1i}\right) = \frac{\alpha}{h\left(\theta_i\right)}\left[\psi_{i,p}^c + \ln\theta_i - \ln s_{-i} - \ln\left(q_{i,p}^c + \varepsilon_{1i}\right)\right],$$

$$p_i^{bj}\left(q_{i,p}^{bj}, \varepsilon_{2i}^j\right) = \frac{(1-\alpha)}{h\left(\theta_i\right)}\left[\psi_{i,p}^b + \ln\theta_i - \ln s_{-i} - \ln\left(q_{i,p}^{bj} + \varepsilon_{2i}^j\right)\right],$$

We assume that the functions $p_i^c(q_{i,p}^c, s_{-i}, \theta_i, \varepsilon_{1i})$ and $p_i^{bj}(q_{i,p}^{bj}, s_{-i}, \theta_i, \varepsilon_{2i}^j)$ are common knowledge. For ε_{1i} sufficiently small and such that $q_{i,p}^c + \varepsilon_{1i} > 0$, the *net* surplus for car and bus trips of each traveler are respectively

$$\begin{aligned} S_i^c &= S_i^c(q_{i,p}^c, s_{-i}, a_i^c, \theta_i, \varepsilon_{1i}) \\ &\approx \int_{-\varepsilon_{1i}}^{q_{i,p}^c} P\left(\widetilde{q}\right) d\widetilde{q} - p_i^c q_{i,p}^c - a_i^c \qquad (12) \\ &\approx \frac{\alpha}{h(\theta_i)}\left(q_{i,p}^c + \varepsilon_{1i}\right)\left[1 + \psi_{i,p}^c + \ln\theta_i - \ln s_{-i} - \ln\left(q_{i,p}^c + \varepsilon_{1i}\right)\right] - p_i^c q_{i,p}^c - a_i^c \end{aligned}$$

and recalling that each traveler chooses only one type of payment such that $j = j(i)$, we have

$$\begin{aligned} S_i^{bj} &= S_i^b(q_{i,p}^{bj}, s_{-i}, a_i^{bj}, \theta_i, \varepsilon_{2i}^j) \\ &\approx \int_{-\varepsilon_{2i}^j}^{q_{i,p}^{bj}} P\left(\widetilde{q}\right) d\widetilde{q} - p_i^{bj} q_{i,p}^{bj} - a_i^{bj} \qquad (13) \\ &\approx \frac{(1-\alpha)}{h(\theta_i)}\left(q_{i,p}^{bj} + \varepsilon_{2i}^j\right)\left[1 + \psi_{i,p}^b + \ln\theta_i - \ln s_{-i} - \ln\left(q_{i,p}^{bj} + \varepsilon_{2i}^j\right)\right] - a_i^{bj} - p_i^{bj} q_{i,p}^{bj}, \end{aligned}$$

Using the abbreviations $a_i^j := a_i^c + a_i^{bj}$, $a = (a_i)_{i\in I}$, $\theta = (\theta_i)_{i\in I}$, $q = (q_{i,p}^c, q_{i,p}^{bj})_{i\in I}$,,

$$S_i(q_{i,p}, s_{-i}, a_i, \theta_i, \varepsilon_{1i}, \varepsilon_{2i}^j) := S_i^c(q_{i,p}^c, s_{-i}, a_i^c, \theta_i, \varepsilon_{1i}) + S_i^{bj}(q_{i,p}^{bj}, s_{-i}, a_i^b, \theta_i, \varepsilon_{2i}^j),$$

the expected *total surplus* is defined as for $j = 1, ..., J$

$$S(q, a, \theta) := \sum_{i=1}^{I} \int_{\varepsilon_{1i}} \int_{\varepsilon_{2i}^j} S_i(q_{i,p}, s_{-i} a_i, \theta_i, \varepsilon_{1i}, \varepsilon_{2i}^j) dF_1(\varepsilon_{1i}) dF_2(\varepsilon_{2i}^j) \qquad (14)$$

in the case of complete information and by

$$S(q, a) := \sum_{i=1}^{I} \int_{\theta_i\in\Theta_i} \int_{\varepsilon_{1i}} \int_{\varepsilon_{2i}^j} S_i(q_{i,p}, s_{-i}, a_i, \theta_i, \varepsilon_{1i}, \varepsilon_{2i}^j) dF_i(\theta_i) dF_1(\varepsilon_{1i}) dF_2(\varepsilon_{2i}^j) \qquad (15)$$

in the case of incomplete information. Note that for a given value of θ, the expected total surplus is:

$$S(q, a, \theta) := \sum_{i=1}^{I} \int_{\varepsilon_{1i}} \int_{\varepsilon_{2i}^j} S_i(q_{i,p}, s_{-i}, a_i, \theta_i, \varepsilon_{1i}, \varepsilon_{2i}^j) dF_1(\varepsilon_{1i}) dF_2(\varepsilon_{2i}^j)$$

Furthermore, the *profit* of the transport authority is given by the formula

$$\begin{aligned}\pi &= \pi(q,a) \\ &= \sum_{i=1}^{I}\int_{\theta_i\in\Theta_i}\int_{\varepsilon_{1i}}\int_{\varepsilon_{2i}^{j}}(a_i\left(\theta,\varepsilon_{1i},\varepsilon_{2i}^{j}\right)+\left(p_i^c-c^c\right)q_{i,p}^{c}\left(\theta,\varepsilon_{1i}\right) \\ &\quad+\left(p_i^{bj}-c^b\right)q_{i,p}^{bj}\left(\theta,\varepsilon_{2i}^{j}\right))dF_i(\theta_i)dF_1(\varepsilon_{1i})dF_2(\varepsilon_{2i}^{j}),\end{aligned} \tag{16}$$

where c^c and c^b denote the marginal costs of car and bus trips, respectively. Finally, the *total welfare* is given by the formula

$$W(q,\theta):=\pi(q,a)+S(q,a,\theta). \tag{17}$$

Observe that the total welfare does not depend on a. Given a *welfare weight* $\alpha_1\in(0,1)$ placed on travelers' surplus, in the case of complete information the social planner's problem is to maximize the *social value function*

$$U(q,a,\theta):=\alpha_1 S(q,a,\theta)+(1-\alpha_1)\pi(q,a) \tag{18}$$

with respect to

$$q_{i,p}^{c}=q_{i,p}^{c}(\theta,\varepsilon_{1i}),\quad q_{i,p}^{bj}=q_{i,p}^{bj}(\theta,\varepsilon_{2i}^{j})\quad\text{and}\quad a_i^j=a_i^j(\theta) \tag{19}$$

under the *participation constraints*

$$S_i^j(q_{i,p},s_{-i},a_i,\theta_i,\varepsilon_{1i},\varepsilon_{2i}^{j})\geq 0 \tag{20}$$

for every i. In the case of incomplete information the social planner's problem is to maximize the function

$$\int_{\Theta}U(q(\theta),a(\theta),\theta)f(\theta)\ d\theta \tag{21}$$

under the participation constraints (20) and the *incentive compatibility constraints* for $j=1,...J$,

$$S_i^j(q_{i,p}(\theta,\varepsilon_{1i},\varepsilon_{2i}^{j}),s_{-i},a_i(\theta,\varepsilon_{1i},\varepsilon_{2i}^{j}),\theta_i)\geq S_i^j(q(\tilde{\theta}),s_{-i},a_i(\tilde{\theta},\varepsilon_{1i},\varepsilon_{2i}^{j}),\theta_i)\quad\forall\theta,\tilde{\theta}\in\Theta \tag{22}$$

for every i, where $\tilde{\theta}$ differs from θ only in its i component: $\theta_l=\tilde{\theta}_l$ for every $l\neq i$. By the revelation principle, it suffices to show that the truth telling strategy is a dominant strategy in a direct revelation mechanism. The revelation principle states that a dominant strategy equilibrium of any Bayesian game can be represented by an equilibrium in a direct revelation mechanism.(see Gibbard, 1973; Green and Laffont, 1977; Dasgupta et al., 1979; Myerson, 1979)

The timing of the regulation process is as follows: First, each traveler i learns his type θ_i. Second, the regulator announces the regulatory policy. Finally, the number of trips is taken by individuals and a new congestion level results from this policy.

4.2 Complete information

As previously mentioned, we have to maximize the function (18) with respect to the variables (19) under the constraints (20), creating the following result.

Proposition 2 *The regulator implements marginal cost prices. The maximum of the social value function is achieved by a unique triple $(q^c_{i,p}, q^{bj}_{i,p}, a_i)_{i\in I}$, given by the following formulae:*

$$q^c_{i,p} = \theta_i \left(\frac{1}{I} \sum_{j\in I} \theta_j e^{\psi^c_j - \alpha^{-1} h(\theta_j) p^c_j} \right)^{-1/2} e^{\psi^c_j - \alpha^{-1} h(\theta_j) c^c} - \varepsilon_{1i}, \tag{23}$$

$$q^{bj}_{i,p} = \theta_i \left(\frac{1}{I} \sum_{j\in I} \theta_j e^{\psi^c_j - \alpha^{-1} h(\theta_j) p^c_j} \right)^{-1/2} e^{\psi^c_j - (1-\alpha)^{-1} h(\theta_j) c^b} - \varepsilon^j_{2i}. \tag{24}$$

Let $0 < \alpha_1 < 1/2$. Then

$$\begin{aligned} a_i &= a_i(q^c_{i,p}, q^{bj}_{i,p}, \varepsilon_{1i}, \varepsilon^j_{2i}) \\ &= \frac{\alpha}{h(\theta_i)} q^c_{i,p} + \frac{(1-\alpha)}{h(\theta_i)} q^{bj}_{i,p} + \varepsilon_{1i}\left(p^c_i + \frac{\alpha}{h(\theta_i)}\right) + \varepsilon^j_{2i}\left(p^{bj}_i + \frac{(1-\alpha)}{h(\theta_i)}\right), \end{aligned} \tag{25}$$

$$S(q,a,\theta) = 0, \tag{26}$$

$$\begin{aligned} W(q,\theta) &= \pi(q,a), \\ &= \sum_{i=1}^{I} \frac{\alpha}{h(\theta_i)} q^c_{i,p} + \frac{(1-\alpha)}{h(\theta_i)} q^{bj}_{i,p} \\ &+ \int_{\varepsilon_{1i}}\int_{\varepsilon_{2i}} \left[\varepsilon_{1i}\left(p^c_i + \frac{\alpha}{h(\theta_i)}\right) + \varepsilon^j_{2i}\left(p^{bj}_i + \frac{(1-\alpha)}{h(\theta_i)}\right)\right] dF(\varepsilon_{1i}) dF(\varepsilon^j_{2i}), \end{aligned} \tag{27}$$

$$U(q,a,\theta) = (1-\alpha_1)\pi(q,a). \tag{28}$$

Let $\frac{1}{2} < \alpha_1 < 1$. Then,

$$a_i = 0, \tag{29}$$

$$\begin{aligned} W(q,\theta) &= S(q,a,\theta), \\ &= \sum_{i=1}^{I} \int_{\varepsilon_{1i}}\int_{\varepsilon_{2i}} \begin{pmatrix} \frac{\alpha}{h(\theta_i)} \left(q^c_{i,p} + \varepsilon_{1i}\right) [1 + \psi^c_{i,p} + \ln\theta_i \\ - \ln s_{-i} - \ln\left(q^c_{i,p} + \varepsilon_{1i}\right)] - p^c_i q^c_{i,p} \\ + \frac{(1-\alpha)}{h(\theta_i)} \left(q^{bj}_{i,p} + \varepsilon^j_{2i}\right) [1 + \psi^b_{i,p} + \ln\theta_i \\ - \ln s_{-i} - \ln\left(q^{bj}_{i,p} + \varepsilon^j_{2i}\right)] - p^{bj}_i q^{bj}_{i,p} \end{pmatrix} dF(\varepsilon_{1i}) dF(\varepsilon^j_{2i}), \end{aligned} \tag{30}$$

$$\pi(q,a) = 0, \tag{31}$$

$$U(q,a,\theta) = \alpha_1 W(q,\theta). \tag{32}$$

Proof. See Appendix 1. ■

If the regulator had complete information about each individuals' aversion to congestion, the optimal policy would be to set a unit price per trip equal to the marginal cost and leave travelers a surplus (if $\alpha_1 > \frac{1}{2}$) or charge them a subscription fee (if $\alpha_1 < \frac{1}{2}$) equal to the exact amount that they are willing to spend on transportation.

In case of $\alpha_1 = 1/2$, the social value function does not depend any more on a_i; otherwise, its maximum is attained for the same values of $q^c_{i,p}$ and $q^{bj}_{i,p}$ as in Proposition 1.

Of course, this policy is not feasible for the regulator when θ is unknown because it does not satisfy the incentive compatibility constraints. Each traveler would have positive incentives to misrepresent his/her aversion to congestion by reporting a valuation for transportation $\widetilde{\theta} \neq \theta$. Furthermore, this misrepresentation could take two possible directions. It could be higher if the traveler tries to take advantage of lower fares for making more trips or it could be lower if he anticipates being highly charged for the inconvenience that his trips may create on the network. Intuitively, he is likely to report $\widetilde{\theta} > \theta$ if he primarily uses public transportation because it does not generate congestion or if he anticipates that the reduction in fare he may get for traveling more overcomes his anticipated charge for creating congestion. On the other hand, one may expect $\widetilde{\theta} < \theta$ if the individual travels mostly by car; that is the individual knows that the portion of the road he uses or the time of the day during which he travels is highly congested.

4.3 Incomplete information

In this section, we analyze the case where the regulator has only incomplete information about the traveler's valuation for transportation. Travelers tend to announce a higher valuation to obtain a lower per trip price if they don't expect congestion, but tend to report a lower θ to avoid paying for congestion costs.

We recall from Section 2 that the regulator maximizes the function (21) with respect to the functions (19) under the participation constraints (20) and the incentive compatibility constraints (22).

We will use the envelope theorem to maximize the social value function. Hence, travelers' θ surplus will be maximized when he reveals his true type.[5]

Recall that the types $\theta_i \in \Theta$ are independently distributed according to the cumulative distribution function F_i of density f_i. We set $\Theta = \prod_{i \in I} \Theta_i$ and $f = \prod_{i \in I} f_i$. Furthermore, we assume that h is continuously differentiable, satisfying

$$1 + \frac{1 - 2\alpha_1}{1 - \alpha_1} \cdot \frac{1 - F_i(\theta_i)}{f_i(\theta_i)} \cdot \frac{h'(\theta_i)}{h(\theta_i)} \neq 0 \tag{33}$$

[5] From the envelope theorem, the derivative of the social value function with respect to θ takes into account only the direct effect of θ, and not the indirect effect stemming from the adjustment in quantity.

for all all i and θ_i.

In order to state our result, let us introduce the following notation:

$$s^* := \left(\frac{1}{I} \sum_{j \in I} \int_{\Theta_j} \theta_j e^{\psi_j^c - \alpha^{-1} h_j c_{j1}^c(\theta_j)} f_j(\theta_j)\, d\theta_j \right)^{1/2},$$

$$m(\theta_i) := \left(1 + \frac{1 - 2\alpha_1}{1 - \alpha_1} \cdot \frac{1 - F_i(\theta_i)}{f_i(\theta_i)} \cdot \frac{h'(\theta_i)}{h(\theta_i)} \right)^{-1},$$

$$c_{i1}^c(\theta_i, \varepsilon_{1i}) := m(\theta_i) \left(c^c + \frac{\alpha}{h(\theta_i)} \cdot \frac{1 - 2\alpha_1}{1 - \alpha_1} \cdot \frac{1 - F_i(\theta_i)}{\theta_i f_i(\theta_i)} \right), \tag{34}$$

$$c_{i1}^b(\theta_i, \varepsilon_{2i}^j) := m(\theta_i) \left(c^b + \frac{(1-\alpha)}{h(\theta_i)} \cdot \frac{1 - 2\alpha_1}{1 - \alpha_1} \cdot \frac{1 - F_i(\theta_i)}{\theta_i f_i(\theta_i)} \right), \tag{35}$$

$$\begin{aligned} V_i &:= V_i(q_{i,p}(\theta, \varepsilon_{1i}, \varepsilon_{2i}^j), \theta_i) \\ &= \frac{1}{\theta_i h(\theta_i)} \left(\alpha q_{i,p}^c(\theta, \varepsilon_{1i}) + (1-\alpha) q_{i,p}^{bj}(\theta, \varepsilon_{2i}^j) \right) \\ &\quad - \alpha \frac{h'(\theta_i)}{h^2(\theta_i)} q_{i,p}^c(\theta, \varepsilon_{1i}) [1 + \psi_{i,p}^c + \ln \theta_i - \ln s_{-i} - \ln \left(q_{i,p}^c(\theta, \varepsilon_{1i}) - \varepsilon_{1i} \right)] \\ &\quad - \frac{(1-\alpha) h'(\theta_i)}{h^2(\theta_i)} q_{i,p}^{bj}(\theta, \varepsilon_{2i}^j) [1 + \psi_{i,p}^b + \ln \theta_i - \ln s_{-i} - \ln \left(q_{i,p}^{bj}(\theta, \varepsilon_{2i}^j) - \varepsilon_{2i}^j \right)]. \end{aligned}$$

We assume that s^*, as defined above is finite.

The solution of the social planner's problem is different again for $\alpha_1 < 1/2$ and for $\alpha_1 > 1/2$. Let us begin with the first case:

Proposition 3 *Assume that*

$$\alpha \left(\frac{1}{\theta_i h(\theta_i)} - \frac{h'(\theta_i)}{h^2(\theta_i)} \right) - \frac{h'(\theta_i)}{h(\theta_i)} c_{i1}^c(\theta_i, \varepsilon_{1i}) \geq 0 \tag{36}$$

and

$$(1-\alpha) \left(\frac{1}{\theta_i h(\theta_i)} - \frac{h'(\theta_i)}{h^2(\theta_i)} \right) - \frac{h'(\theta_i)}{h(\theta_i)} c_{i1}^b(\theta_i, \varepsilon_{2i}^j) \geq 0 \tag{37}$$

for all i and θ_i. If $0 < \alpha_1 < 1/2$, then the regulator implement prices c_{i1}^c and c_{i1}^b and the maximum of the social value function is achieved by a unique triple $(q_{i,p}^c(\theta, \varepsilon_{1i}), q_{i,p}^{bj}(\theta, \varepsilon_{2i}^j), a_i(\theta, \varepsilon_{1i}, \varepsilon_{2i}^j))_{i \in I}$, given by the following formulae for all $i \in I$ and $\theta \in \Theta$:

$$q_{i,p}^c(\theta, \varepsilon_{1i}) = \frac{\theta_i}{s^*} e^{\psi_{i,p}^c - \alpha^{-1} h(\theta_i) c_{i1}^c(\theta_i, \varepsilon_{1i})} - \varepsilon_{1i}, \tag{38}$$

$$q_{i,p}^{bj}(\theta, \varepsilon_{2i}^j) = \frac{\theta_i}{s^*} e^{\psi_{i,p}^b - (1-\alpha)^{-1} h(\theta_i) c_{i1}^b(\theta_i, \varepsilon_{2i}^j)} - \varepsilon_{2i}^j, \tag{39}$$

$$\begin{aligned} a_i(\theta, \varepsilon_{1i}, \varepsilon_{2i}^j) = \Big(\frac{\alpha}{h(\theta_i)} \left(q_{i,p}^c(\theta, \varepsilon_{1i}) + \varepsilon_{1i} \right) + \Big(\frac{(1-\alpha)}{h(\theta_i)} \left(q_{i,p}^{bj} + \varepsilon_{2i}^j \right) \\ - \int_{\underline{\theta_i}}^{\theta_i} V_i(q_{i,p}(\theta_{-i}, \tilde{\theta}_i), \tilde{\theta}_i)\, d\tilde{\theta}_i. \end{aligned} \tag{40}$$

Proof. See Appendix 1. ■

To understand why this regulatory policy may be optimal, observe that the regulator wants to encourage the traveler to admit that he has a high traveling valuation whenever this is true, so that pricing accounts for the congestion costs generated. To prevent a traveler from misrepresenting his true valuation, the regulator punishes him when reporting a low valuation. This punishment takes the form of a per-trip price higher than the marginal cost of a trip. The traveler's fee is a linear function of the MC of using the mode of transportation where both the slope and the intercept increase with the odds of being a high type and the opportunity costs of using car transportation.

This fee increases with the likelihood of frequent travel that creates congestion if the time spent in traffic prevents him from performing highly paid tasks. In other words, the regulator's intervention is primarily intended for individuals who are less sensitive to transportation costs and whose probability of generating congestion is large. Moreover, the rate at which the congestion fee increases is proportional with MC by an amount that decreases with the variation in MU of income of individual travelers. Intuitively, the less information the regulator has about the individual MU of income, the lower the distortion. Note that the intercept shift is decreasing with the true valuation of the individual; in other words, the more obligated the traveler is to his schedule, the less punishment imposed. Finally, the subscription fee is designed to offset the low valuation traveler's incentive to report a higher type and incur lower punishment. It is equal to the traveler's willingness to pay for total transportation usage minus the surplus that he gets by announcing any type that is lower than his own. Hence, if the traveler's aversion is high (θ_i is low), then he pays the maximum subscription fee. But the higher θ_i is, the smaller is the subscription fee that a traveler pays. Thus, the subscription fee is inversely related to the traveler's valuation.

The effect of the regulatory policy on urban congestion is threefold: by raising the price above the marginal cost, it reduces the number of trips made by high valuation travelers. This in turn will decrease the level of congestion on the network, and will increase the incentive to make a trip.

Turning to the case $\alpha_1 > 1/2$, let us add again the extra condition $a_i(\theta, \varepsilon_{1i}, \varepsilon_{2i}^j) \geq 0$ for all i and θ.

Proposition 4 *Let $\frac{1}{2} < \alpha_1 < 1$. Then the maximum of the social value function is achieved by a unique triple $(q_{i,p}^c(\theta, \varepsilon_{1i}), q_{i,p}^{bj}(\theta, \varepsilon_{2i}^j), a_i(\theta, \varepsilon_{1i}, \varepsilon_{2i}^j))_{i \in I}$, given by the following formulae for all $i \in I$ and $\theta \in \Theta$:*

$$q_{i,p}^c(\theta, \varepsilon_{1i}) = \frac{\theta_i}{s^*} e^{\psi_{i,p}^c - \alpha^{-1} h(\theta_i) c_{i1}^c(\theta_i, \varepsilon_{1i})} - \varepsilon_{1i}, \tag{41}$$

$$q_{i,p}^{bj}(\theta, \varepsilon_{2i}^j) = \frac{\theta_i}{s^*} e^{\psi_{i,p}^b - (1-\alpha)^{-1} h(\theta_i) c_{i1}^b(\theta_i, \varepsilon_{2i}^j)} - \varepsilon_{2i}^j, \tag{42}$$

$$a_i(\theta, \varepsilon_{1i}, \varepsilon_{2i}^j) = 0. \tag{43}$$

Proof. See Appendix 1. ■

Finally, $V_i(q_{i,p}(\theta, \varepsilon_{1i}, \varepsilon_{2i}^j), \theta_i)$ simplifies to

$$V_i(q_{i,p}(\theta, \varepsilon_{1i}, \varepsilon_{2i}^j), \theta_i) := \frac{1}{\theta_i h(\theta_i)} \left[\alpha q_{i,p}^c(\theta, \varepsilon_{1i}) + (1-\alpha) q_{i,p}^{bj}(\theta, \varepsilon_{2i}^j) \right],$$

so that (36) and (37) are redundant.

In this case, the regulator rewards travelers for announcing a high valuation. The subsidy is higher if (1) the odds of being a high type are large, (2) the opportunity cost of using transportation is low. This subsidy is then increased if the marginal utility of income varies substantially with the valuation, and this increase is higher if the odds in favor of a low valuation traveler are large. Finally, since there is no incentive for high valuation travelers to report a lower type, no fixed grant is necessary. Note that the optimal level of congestion, although endogenous, does not directly enter the regulator's pricing policy.

As in the preceding case, when $\alpha_1 = \frac{1}{2}$, conditions (54) and (55) simplify and give $p_i^c = c^c$ and $p_i^b = c^b$, profit $\pi = \sum_{i=1}^{I}(a_i^c + a_i^b) = 0$ and by simplification of (40) the individual surplus becomes

$$S_i(q_{i,p}(\theta), s_{-i}, a_i, \theta_i) = \frac{1}{h(\theta_i)} \left(\alpha \left(q_{i,p}^c(\theta, \varepsilon_{1i}) + \varepsilon_{1i} \right) + (1-\alpha) \left(q_{i,p}^{bj}(\theta, \varepsilon_{2i}^j) + \varepsilon_{2i}^j \right) \right). \tag{44}$$

Observe that the validity Proposition 4 depends on the specification of travelers' marginal utility of income.

Let us consider two special cases of the proposition. In the case of unobservable heterogeneous marginal utility of income, where for example, $h(\theta_i) = k\theta_i$, then s^* is finite for all α_1 and Proposition 4 applies. Indeed, the integrals can only blow up for $\theta_i \to 0$. In this case we have

$$m(\theta_i) \approx k_1 \theta_i^2 \quad \text{and} \quad c^c + \frac{\alpha}{k\theta_i} \frac{1-2\alpha_1}{1-\alpha_1} \frac{1-F_i(\theta_i)}{\theta_i f_i(\theta_i)} \approx \frac{k_2}{\theta_i^2},$$

so that c_{i1}^c and c_{i1}^b remain bounded. Therefore, $\theta_i \exp(\psi_{i,p}^c - \alpha^{-1} h \theta_i c_{i1}^c(\theta_i))$ is bounded and thus s^* is finite.

On the contrary, in the case of constant marginal utility of income $h(\theta_i) = k$ we have $m(\theta_i) = 1$ and the formulae of $c_{i1}^c(\theta_i, \varepsilon_{1i})$ and $c_{i1}^b(\theta_i, \varepsilon_{2i}^j)$ simplify to

$$c_{i1}^c(\theta_i, \varepsilon_{1i}) := c^c + \frac{\alpha}{k} \cdot \frac{1-2\alpha_1}{1-\alpha_1} \cdot \frac{1-F_i(\theta_i)}{\theta_i f_i(\theta_i)},$$
$$c_{i1}^b(\theta_i, \varepsilon_{2i}^j) := c^b + \frac{(1-\alpha)}{k} \cdot \frac{1-2\alpha_1}{1-\alpha_1} \cdot \frac{1-F_i(\theta_i)}{\theta_i f_i(\theta_i)}.$$

It follows from these formulae that

$$c_{i1}^c(\theta_i, \varepsilon_{1i}) \begin{cases} \text{tends to } \infty \text{ if } \alpha_1 < \frac{1}{2}, \\ \text{tends to a finite value if } \alpha_1 = \frac{1}{2}, \\ \text{tends to } -\infty \text{ if } \alpha_1 > \frac{1}{2} \end{cases}$$

and then that s^* is finite for $\alpha_1 \leq 1/2$ and infinite for $\alpha_1 > 1/2$. Thus, Proposition 4 does not apply in the latter case.

5 Results

Using data from the city of Montpellier (southern France), we compare the structural parameters of transportation demand when assuming a constant marginal utility of income, $h(\theta) = k_1$ to the case where $h(\theta) = k_2\theta$, where k_1 and k_2 are parameters to be estimated. We assume that individual valuations θ are simulated according to a $Beta(1, \mu)$ distribution, where μ is a parameter to estimated. Note that the Gumbel distribution assumptions imposed in subsection 3-1 on u_{1i} and u_{2i} imply that $\varepsilon_{1i} \sim G(0, \alpha\mu_1)$ and $\varepsilon_{2i}^j \sim G(0, (1-\alpha)\mu_2)$ where α, μ_1 and μ_2 are parameters to be estimated. Estimation results of the structural parameters in case of incomplete information on θ, ε_{1i}, ε_{2i}^j are reported in Tables 1 and 2 of Appendix 2. Table 1 reports the results assuming constant marginal utility of income across travelers. We measure the impact of introducing the unobservables ε_{1i}, ε_{2i}^j by comparing the structural parameters to those in Viauroux (2007). Introduction of these reputation effects improve the overall quality of the model with a mean log likelihood rising from -7.66 to -3.94 during the peak period and from -6.61 to -6.68 during the off-peak period. We also see a decrease in the effect of the marginal utility of using the car (as opposed to the bus). The favorable reputation of public transport shifts the relative preferences between car and bus. We observe a decrease of the average tolerance to traffic congestion going from 0.76 (1/1.3) to 0.63 (1/1.589) during peak period and from 0.72 (1/1.386) to 0.58 (1/1.718) during the off-peak period. This might be explained by the fact that part of the tolerance to traffic congestion is due to the reputation of the mode. For example, if a mode's reputation is considered slow, then the traveler places himself in that situation knowingly (presumably because it offers other advantages that are more important to him), consequently showing more tolerence to traffic congestion. On the other hand, if a traveler chooses a mode with a reputation of being fast, chances are that there is less congestion, which makes him better off. Introduction of reputation unobservables also translates into a lower effect of all factors except for the distance and the Origin-Destination time. The constant marginal utility of income is found to increase slightly: reputation can been seen as a cost of additional "quality" of traveling. All other effects have a similar sign, except for the distance O-D of a bus trip which now affects positively the probability of traveling: this is consistent with the decrease in α above. Table 2 allows to identify the effect of introducing heterogeneity in the marginal utility of income. It shows an additional decrease in the marginal utility of car usage, an attenuation of the average aversion to traffic congestion, with an average θ increasing from 0.63 to 0.68 during the peak period and from 0.58 to 0.72 during off peak period. The average marginal utility of income stays constant during peak times (from 2.03 to 3/1.454=2.06) but increases from 2.43 to 3.17 during off peak times. Other things being equal, travelers are more sensitive to an increase in transportation costs during more flexible hours; during these times, transportation opportunity cost is felt to a greater extent because of the foregone leisure or shopping activities. Other factor effects are very similar.

Tables 3 to 6 presents the welfare simulation results that would prevail if the pricing scheme was designed assuming that all travelers have an a-priori belief about each mode of transportation and, either the same MU of income, i.e. $h(\theta) = k$ (Tables 3-4) or an heterogenous MU of income $h(\theta) = k\theta$.[6] The first specification verifies the conditions of Proposition 3 for $\alpha_1 \leq 1/2$, since when $\alpha_1 > 1/2$, s^* is infinite (see above comments on proposition 4), while the second specification verifies the conditions associated to Proposition 4. Note that these functions are special cases of the more general functional form $h(\theta) = k_3\theta^{\xi}$, which would verify both propositions assumptions for $0 < \xi < 1$. For confidentiality reasons, we arbitrarily set a value for the transportation operator's marginal cost , namely 1.07 euros (FF7) for the unit cost of a car trip, 0.76 euros (FF5) for a bus trip, slightly lower than the average cost of a trip made with the respective modes of transportation. Simulations of welfare is performed as follows. For each period, estimated parameters are used to compute the optimal individual functions presented in Propositions 3 and 4. It is possible to compute an expected optimal number of trips as well as an average unit price, subscription fee, individual surplus, profit and welfare during each period for the two-day survey. The sum over all individuals traveling in each period gives a total average surplus, profit and welfare while the average over individuals gives the average price and subscription. The average surplus is computed as the sum over all individuals of the expected value (according to the distribution of unobservables ε_{1i} and ε_{2i}^{j} and of θ) of the individual surplus as defined by equations (51) and (56) in Proposition 3 and equation (44) in Proposition 4. In these tables, the "Current pricing" column displays the simulated number of car and bus trips, the average subscription (reported for a two-day period), the total profit, average total surplus and average total welfare using the estimated parameters of Model 1 (in Tables 3-4) and Model 2 (Tables 5-6). The current average price of car usage per trip (on average 2 euros) is equal to the price per kilometer multiplied by a distance from the centroid of the Origin area to the centroid of the destination area. The price varies according to the number of horsepower of the vehicle and the type of fuel used.[7]

In Tables 3-4, the number of car trips performed over the two-day survey (excluding returns home) is estimated at about 2.2 during peak times and 0.93 during off-peak times. Columns 4 ,5 and 6 reports the same welfare statistics when the regulator implements heterogenous prices as defined in equations (34) and (35). Results are reported for the consumer surplus weights of 1/8, 1/4, and

[6] For welfare results with no reputation effects, the reader is refered to Viauroux 2007)

[7] When the household possessed more than one car, the survey did not indicate which car was used for the trip considered. Consequently, we assumed that the trips made by the household head were with the most powerful car, trips made by the second household member were made with the second most powerful car, and so on.

1/2. During both periods, and despite higher average prices, implementation of non linear heterogenous prices does not represent any improvement over the current situation. Indeed, in this case the increase in average subscription tends to increase profit in lower proportion than it decreases the surplus, leading to a decrease to overall highest welfare (for $\alpha_1 = 1/2$) about 6.4% during peak period (19.8% during off-peak period). This is in contrast with Viauroux (2007) who observed an improvement in welfare. Assuming that all travelers have the same MU of income, the fact that they anticipate congestion is sufficient to increase their surplus.

In contrast, the non linear pricing accounting for the heterogeneity of travelers' sensitivity to price (Tables 5-6) shows an improvement over the current pricing scheme of 6.2% and an improvement over the nonlinear pricing assuming constant MU of income case of 10.9% during the peak period. These increases are respectively 4.3% and 4.5% during the off-peak period. The higher increase during peak hours reflects the increase in surplus of those travelers whose tolerence to traffic congestion is determined by their rather fixed schedule, i.e. travelers in the low income category who are more sensitive to a change in transportation costs. Indeed, recall that in that case, the subsidy is higher if the odds of having a high tolerance for congestion is large and the opportunity cost of using transportation is low. Finally, as a last experiment, assume that the regulator knows the baysian game played by travelers, the distribution of their aversion to traffic congestion (and hence the mean aversion for each time period), the distribution of reputation parameters for both car and bus modes. In other words, Model 2 structural parameters are available but the regulator decides to impose a homogenous pricing, at the estimated mean value of θ. In this case, Tables 7-8 show that the welfare loss is of 2.7% during the peak period and 0.6% during the off peak period.

6 Conclusion

We undertake a disaggregated approach to estimated transportation demand structural parameters, accounting for the fact that traffic congestion is endogenous and can be represented as a Bayesian Nash game between travelers. In this framework are introduced two types of private information: information on the reputation of each mode of transportation and information on the VOT, e.g. aversion to traffic congestion. Using a cross-sectional two-day period data set, we estimate and compare the results where the marginal utility of income is assumed constant to the case where it is assumed heterogenous and proportional to the VOT. Comparing the constant marginal utility of income case to the results of Viauroux (2007), we find that the introduction of reputation effects improve the overall quality of the model.

We investigate the efficiency gain that can be achieved by offering different tariffs according to travelers observed and unobserved characteristics, discriminating them by their VOT. Our tariff derivations show that the per trip price differs from the marginal cost by a distortion term depending on both the odds of being a high valuation traveler and the opportunity cost of using transportation. We show that the reputation effects act as a regulation device: its introduction increases the estimated welfare,while non linear tariffs do not improve efficiency. However, welfare can be improved further when heterogeneity in the marginal utility of income is introduced. The non linear pricing schedule accounting for the heterogeneity of travelers' sensitivity to price shows an improvement over the current pricing scheme of 6.2% and an improvement over the nonlinear pricing assuming constant MU of income of 10.9% during the peak period. These increases are respectively 4.3% and 4.5% during the off-peak period. Interestingly, the loss in welfare associated to a uniform nonlinear tariff where the average estimated aversion to traffic congestion is introduced doesn't substantially decrease welfare.

Among the directions to future research, the use of panel data could improve the outcome even further. It would allow the individuals' preferences for traveling to depend not only on the current anticipation of congestion but also on all other travelers' experience of congested areas via their optimal behavior from any point in time onward.

7 Appendix 1. Derivation of the optimal tariff

We denote by σ the randomized-strategy profile for the game

$$\Gamma = \left\{ I, Q, \Theta, (p_i)_{i \in I}, \left(u_{i,p}^{j} \right)_{i \in I} \right\},$$

such that

$$\sigma = \left(\sigma_i \left(q_{i,p} | \theta_i \right) \right)_{i \in I, q_{i,p} \in Q, \theta_i \in \Theta},$$

where $\sigma_i \left(q_{i,p} | \theta_i \right)$ represents the conditional probability that traveler i would do $q_{i,p}$ trips if his type were θ_i (see Myerson, 1991). At a Nash equilibrium σ, one may compute for each $i \in I$ and $\theta_i \in \Theta$ the expected number of trips for traveler i by the formula

$$q_{i,p}^{*} \left(\theta_i \right) = \sum_{q_{i,p} \in Q} q_{i,p} \sigma_i (q_{i,p} | \theta_i).$$

Henceforth, the parameter of aversion to traffic is assumed to be independently and identically distributed in the population according to a Beta $(1, \mu)$ distribution for its simplicity of use and the variety of forms it can represent (exponential, uniform). We recall that, by definition of the equilibrium (see also Viauroux, 2007), for any fixed i and $\theta \in \Theta$, putting

$$s_{-i}^{*} = s_{-i} \left(q_{-i,p}^{c*} \right)$$

for brevity, the probabilities $\sigma_i(q_{i,p}|\theta_i)$ have to maximize the expression

$$\sum_{q_{i,p}\in Q} \sigma_i(q_{i,p}|\theta_i) u^j_{i,p}(q_{i,p}, q^*_{-i,p}, \theta),$$

where the functions $u^j_{i,p}$ are given by the formula for $j = 1, ..., J$

$$\begin{aligned} u^j_{i,p} &= u^j_{i,p}(q_{i,p}, q^*_{-i,p}, \theta, \varepsilon_{1i}, \varepsilon^j_{2i}, \varepsilon_{\nu_{i,p}}, \varepsilon_{3i}) \\ &= \alpha \left(q^c_{i,p} + \varepsilon_{1i}\right) \left[1 + \psi^c_{i,p} + \ln\theta_i - \ln s^*_{-i,p} - \ln\left(q^c_{i,p} + \varepsilon_{1i}\right)\right] \\ &+ (1-\alpha)\left(q^{bj}_{i,p} + \varepsilon^j_{2i}\right)\left[1 + \psi^b_{i,p} + \ln\theta_i - \ln s^*_{-i,p} - \ln\left(q^{bj}_{i,p} + \varepsilon^j_{2i}\right)\right] \\ &+ h(\theta)\left(w_i - a^c_i - p^c_i q^c_{i,p} - a^{bj}_i - p^{bj}_i q^{bj}_{i,p} + \varepsilon_{\nu_{i,p}}\right) + \varepsilon_{3i}. \end{aligned}$$

This definition leads to a pure multistrategy equilibrium corresponding to the value of $q_{i,p}(\theta, \varepsilon_{1i}, \varepsilon^j_{2i})$, which maximizes $u^j_{i,p}(q_{i,p}, q^*_{-i,p}, \theta)$.

Proof. In order to simplify the computation, let us admit that the variable $q_{i,p}(\theta, \varepsilon_{1i}, \varepsilon^j_{2i})$ can be changed continuously, and let us write down the first-order conditions associated with the above maximization. Both partial derivatives with respect to $q^c_{i,p}$ and $q^{bj}_{i,p}$ must vanish at the equilibrium point, namely,

$$\alpha\left[\psi^c_{i,p} + \ln\theta_i - \ln s^*_{-i,p} - \ln\left(q^{c*}_i + \varepsilon_{1i}\right)\right] - h(\theta)p^c_i = 0, \tag{45}$$

$$(1-\alpha)\left[\psi^b_{i,p} + \ln\theta_i - \ln s^*_{-i,p} - \ln\left(q^{bj*}_{i,p} + \varepsilon^j_{2i}\right)\right] - h(\theta)p^{bj}_i = 0 \tag{46}$$

Solving for q^{c*}_i and $q^{bj*}_{i,p}$, we obtain the first two equalities of Proposition 1:

$$q^{c*}_{i,p}(\theta_i, \varepsilon_{1i}) = \frac{\theta_i}{s^*_{-i,p}} e^{\psi^c_{i,p} - \frac{h(\theta)p^c_i}{\alpha}} - \varepsilon_{1i}, \tag{47}$$

$$q^{bj*}_{i,p}(\theta_i, \varepsilon^j_{2i}) = \frac{\theta_i}{s^*_{-i,p}} e^{\psi^b_{i,p} - \frac{h(\theta)p^{bj}_i}{(1-\alpha)}} - \varepsilon^j_{2i}. \tag{48}$$

In order to determine the value of $s^*_{-i,p}$, integrate both parts of (47) with respect to θ_i of density $f(\theta_i)$; we obtain

$$s^*_{-i,p}(q^{c*}_{-i}) = \frac{1}{I-1}\sum_{j\in I-i}\int_{\theta_j\in\Theta}\int_{\varepsilon_{1j}} q^{c*}_j(\theta_l, \varepsilon_{1j}) dF_1(\varepsilon_{1j}) dF_\theta(\theta_j)$$

Assuming that one individual is negligible in the continuum of individuals so that $s^*_{-i,p} = s^*_{-j}$, we may denote this common value by s^*_p. Given that $E(\varepsilon_{1i}) = 0\ \forall i$, we have that

$$\begin{aligned} s^*_p &= \frac{1}{I-1}\sum_{j\in I-i}\int_{\theta_j\in\Theta}\int_{\varepsilon_{1j}}\left(\frac{\theta_j}{s^*} e^{\psi^c_j - \frac{h(\theta)p^c_j}{\alpha}} + \varepsilon_{1j}\right) dF_1(\varepsilon_{1j}) dF_j(\theta_j) \\ \left(s^*_p\right)^2 &\approx \frac{1}{I}\sum_{j\in I}\int_{\theta_j\in\Theta} \theta_j e^{\psi^c_j - \frac{h(\theta)p^c_j}{\alpha}} dF_j(\theta_j) \end{aligned}$$

which is equivalent to the formula of Proposition 1. In the case of complete information, the proof always remains the same, except the determination of s^*, where we do not have to integrate over Θ. Then we obtain

$$\begin{aligned} \left(s_p^*\right)^2 &= \frac{1}{I-1}\sum_{j\in I-i}\theta_j e^{\psi_j^c - \frac{h(\theta)p_j^c}{\alpha}} \\ &\approx \frac{1}{I}\sum_{j\in I}\theta_j e^{\psi_j^c - \frac{h(\theta)p_j^c}{\alpha}}. \end{aligned}$$

■

Proof of Proposition 2. Case 1: $0 < \alpha_1 < 1/2$.

This problem enters the convex optimization framework of the Kuhn–Tucker theorem. However, we can also solve it directly as follows.

Let us rewrite the social value function in the form

$$U(q,a,\theta) = (2\alpha_1 - 1)S(q,a,\theta) + (1-\alpha_1)W(q).$$

For any given $q_{i,p}^c$ and $q_{i,p}^{bj}$, since $2\alpha_1 - 1 < 0$, the maximum is achieved if the numbers a_i are chosen so as to make $S(q,a,\theta)$ as small as possible. The best choice is obtained by requiring $S_i^c = S_i^b = 0$ for all i, proving (26). Then we have to maximize $W(q)$ with respect to $q_{i,p}^c$ and $q_{i,p}^{bj}$. Since this function is concave and differentiable, the maximum is achieved if and only if its first partial derivatives all vanish, i.e.,

$$\begin{aligned} \frac{\alpha}{h(\theta_i)}\left[\psi_{i,p}^c + \ln\theta_i - \ln s_{-i,p} - \ln\left(q_{i,p}^c + \varepsilon_{1i}\right)\right] - c^c = 0, \\ \frac{(1-\alpha)}{h(\theta_i)}\left[\psi_{i,p}^b + \ln\theta_i - \ln s_{-i,p} - \ln\left(q_{i,p}^{bj} + \varepsilon_{2i}^j\right)\right] - c^b = 0 \end{aligned}$$

for every i. This implies (23), (24) and that $p_i^c\left(q_{i,p}^c\right) = c^c$ and $p_i^b\left(q_{i,p}^{bj}\right) = c^b$. It follows that

$$\begin{aligned} S_i^c &= \left[\frac{\alpha}{h(\theta_i)} + p_i^c - c^c\right]\left(q_{i,p}^c + \varepsilon_{1i}\right) + \varepsilon_{1i}p_i^c - a_i^c \\ &= \frac{\alpha}{h(\theta_i)}\left(q_{i,p}^c + \varepsilon_{1i}\right) + \varepsilon_{1i}p_i^c - a_i^c \end{aligned}$$

$$\begin{aligned} S_i^b &= \left[\frac{(1-\alpha)}{h(\theta_i)} + p_i^{bj} - c^b\right]\left(q_{i,p}^{bj} + \varepsilon_{2i}^j\right) + \varepsilon_{2i}^j p_i^{bj} - a_i^{bj} \\ &= \frac{(1-\alpha)}{h(\theta_i)}\left(q_{i,p}^{bj} + \varepsilon_{2i}^j\right) + \varepsilon_{2i}^j p_i^{bj} - a_i^{bj} \end{aligned}$$

for every i, and therefore the condition $S_i = 0$ implies (25). Equations (27) and (28) then follow by definition.

Case 2: $1/2 < \alpha_1 < 1$.

Writing

$$U(q, a, \theta) = (2\alpha_1 - 1)S(q, a, \theta) + (1 - \alpha_1)W(q, \theta)$$

again, we see that for any given q this expression takes its largest value when S is biggest. This leads to the conditions (29) and to the equality

$$U(q, 0, \theta) = \alpha_1 \sum_i \left(U_1 - p_i^c q_{i,p}^c\right) + \alpha_1 \sum_i \left(U_2 - p_i^{bj} q_{i,p}^{bj}\right) + (1 - \alpha_1) \sum_i \left[(p_i^c - c^c) q_{i,p}^c + (p_i^b - c^b) q_{i,p}^{bj}\right].$$

where

$$U_1 := \int\limits_{\varepsilon_{1i}} \left(\frac{\alpha}{h(\theta_i)} \left(q_{i,p}^c + \varepsilon_{1i}\right) \left[1 + \psi_{i,p}^c + \ln \theta_i - \ln s_{-i,p} - \ln \left(q_{i,p}^c + \varepsilon_{1i}\right)\right] \right) dF(\varepsilon_{1i})$$

and

$$U_2 := \int\limits_{\varepsilon_{2i}} \left(\frac{(1-\alpha)}{h(\theta_i)} \left(q_{i,p}^{bj} + \varepsilon_{2i}^j\right) \left[1 + \psi_{i,p}^b + \ln \theta_i - \ln s_{-i,p} - \ln \left(q_{i,p}^{bj} + \varepsilon_{2i}^j\right)\right] \right) dF(\varepsilon_{2i}^j)$$

Maximizing the last expression with respect to the variables $q_{i,p}^c$ and $q_{i,p}^{bj}$ we obtain the following conditions:

$$\frac{\alpha}{h(\theta_i)} [\psi_{i,p}^c + \ln \theta_i - \ln s_{-i,p} - \ln \left(q_{i,p}^c + \varepsilon_{1i}\right)] = \frac{2\alpha_1 - 1}{\alpha_1} p_i^c + \frac{1 - \alpha_1}{\alpha_1} c^c, \quad (49)$$

$$\frac{(1-\alpha)}{h(\theta_i)} [\psi_{i,p}^b + \ln \theta_i - \ln s_{-i,p} - \ln \left(q_{i,p}^{bj} + \varepsilon_{2i}^j\right)] = \frac{2\alpha_1 - 1}{\alpha_1} p_i^b + \frac{1 - \alpha_1}{\alpha_1} c^b. \quad (50)$$

Denote by W_{q^c} and W_{q^b} the respective partial derivative of W with respect to q^c and q^b. Equations (12) and (13) imply that $W_{q^c} = p_i^c - c^c$ and $W_{q^b} = p_i^b - c^b$. This implies that the left hand side of these equalities is equal respectively to p_i^c and p_i^b and that $p_i^c = c^c$ and $p_i^b = c^b$ leading to (23) and (24). Furthermore, substituting (23) and (24) into the definition of S_i^c and S_i^b and using the conditions $a_i = 0$ we obtain (30) and (31). Finally, for any positive function $h(.)$ the participation constraints (20) are satisfied. ■

Proof of Proposition 3. Since the functions S_i are differentiable and concave in $q_{i,p}^c$, $q_{i,p}^{bj}$ and a_i, condition (36) is equivalent for each i to the following first-order condition:

$$\frac{\partial q^c_{i,p}(\theta,\varepsilon_{1i})}{\partial\theta_i}\left(\frac{\alpha}{h(\theta_i)}\left[\psi^c_{i,p}+\ln\theta_i-\ln s_{-i,p}-\ln\left(q^c_{i,p}(\theta,\varepsilon_{1i})+\varepsilon_{1i}\right)\right]-p^c_i\right)$$
$$+\frac{\partial q^{bj}_{i,p}(\theta,\varepsilon^j_{2i})}{\partial\theta_i}\left(\frac{(1-\alpha)}{h(\theta_i)}\left[\psi^b_{i,p}+\ln\theta_i-\ln s_{-i,p}-\ln\left(q^{bj}_{i,p}(\theta,\varepsilon^j_{2i})+\varepsilon^j_{2i}\right)\right]-p^{bj}_i\right)$$
$$-\frac{\partial a_i(\theta,\varepsilon_{1i},\varepsilon^j_{2i})}{\partial\theta_i}=0.$$

Hence

$$\frac{\partial S_i}{\partial\theta_i}(q_{i,p}(\theta,\varepsilon_{1i},\varepsilon^j_{2i}),s_{-i,p},a_i(\theta,\varepsilon_{1i},\varepsilon^j_{2i}),\theta_i)=V_i(q_{i,p}(\theta,\varepsilon_{1i},\varepsilon^j_{2i}),\theta_i,\varepsilon_{1i},\varepsilon^j_{2i})$$

and therefore

$$S_i(q_{i,p}(\theta,\varepsilon_{1i},\varepsilon^j_{2i}),s_{-i,p},a_i,\theta_i,\varepsilon_{1i},\varepsilon^j_{2i}) = S_i(q_{i,p}(\theta_{-i},\underline{\theta_i}),s_{-i,p},a_i(\theta_{-i},\underline{\theta_i}),\underline{\theta_i},\varepsilon_{1i},\varepsilon^j_{2i}) + \int_{\underline{\theta_i}}^{\theta_i}V_i(q_{i,p}(\theta_{-i},\tilde\theta_i,\varepsilon_{1i},\varepsilon^j_{2i}),\tilde\theta_i)d\tilde\theta_i. \tag{51}$$

Substituting this expression into the definition of U we obtain the following equality:

$$\int_\Theta U(q(\theta),a(\theta),\theta)f(\theta)\,d\theta$$
$$=\int_\Theta\left(\begin{array}{c}\alpha_1\left(\sum_{i=1}^I\int_{\varepsilon_{1i}}\int_{\varepsilon^j_{2i}}S_i(q_{i,p}(\theta,\varepsilon_{1i},\varepsilon^j_{2i}),s_{-i,p},a_i(\theta,\varepsilon_{1i},\varepsilon^j_{2i}),\theta_i)dF(\varepsilon_{1i})dF(\varepsilon^j_{2i})\right)\\+(1-\alpha_1)\pi(q(\theta),a(\theta))\end{array}\right)f(\theta)\,d\theta$$
$$=\int_\Theta\left(\begin{array}{c}(2\alpha_1-1)\left(\sum_{i=1}^I\int_{\varepsilon_{1i}}\int_{\varepsilon^j_{2i}}S_i(q_{i,p}(\theta,\varepsilon_{1i},\varepsilon^j_{2i}),s_{-i,p},a_i(\theta,\varepsilon_{1i},\varepsilon^j_{2i}),\theta_i)dF(\varepsilon_{1i})dF(\varepsilon^j_{2i})\right)\\+(1-\alpha_1)W(q(\theta),\theta)\end{array}\right)f(\theta)\,d\theta$$
$$=\int_\Theta\Big((2\alpha_1-1)\Big(\sum_{i=1}^I\int_{\varepsilon_{1i}}\int_{\varepsilon^j_{2i}}S_i(q_{i,p}(\theta_{-i},\underline{\theta_i},\varepsilon_{1i},\varepsilon^j_{2i}),s_{-i,p},a_i(\theta_{-i},\underline{\theta_i},\varepsilon_{1i},\varepsilon^j_{2i}),\underline{\theta_i})dF(\varepsilon_{1i})dF(\varepsilon^j_{2i})$$
$$+\int_{\underline{\theta_i}}^{\theta_i}V_i(q_{i,p}(\theta_{-i},\tilde\theta_i,\varepsilon_{1i},\varepsilon^j_{2i}),\tilde\theta_i)\,d\tilde\theta_i\Big)+(1-\alpha_1)W(q(\theta),\theta)\Big)f(\theta)\,d\theta.$$

Integrating by parts we have

$$\int_{\underline{\theta_i}}^{\overline{\theta_i}}\int_{\underline{\theta_i}}^{\theta_i}V_i(q_{i,p}(\theta_{-i},\tilde\theta_i),\tilde\theta_i,\varepsilon_{1i},\varepsilon^j_{2i})\,d\tilde\theta_i f_i(\theta_i)\,d\theta_i=\int_{\underline{\theta_i}}^{\overline{\theta_i}}V_i(q_{i,p}(\theta,\varepsilon_{1i},\varepsilon^j_{2i}),\theta_i)(1-F_i(\theta_i))\,d\theta_i.$$

Using this relation we may rewrite the previous equality in the following form:

$$\int_\Theta U(q(\theta), a(\theta), \theta) f(\theta)\, d\theta$$

$$= \int_\Theta \int_{\varepsilon_{1i}} \int_{\varepsilon^j_{2i}} \Big((2\alpha_1 - 1) \Big(\sum_{i=1}^{I} S_i(q_{i,p}(\theta_{-i}, \underline{\theta_i}), s_{-i,p}, a_i(\theta_{-i}, \underline{\theta_i}), \underline{\theta_i})$$

$$+ V_i(q_{i,p}(\theta, \varepsilon_{1i}, \varepsilon^j_{2i}), \theta_i) \frac{1 - F_i(\theta_i)}{f_i(\theta_i)} \Big) + (1-\alpha_1) W(q(\theta), \theta) \Big) dF(\varepsilon_{1i}) dF(\varepsilon^j_{2i}) f(\theta)\, d\theta.$$

We are going to maximize pointwise the function under the integral sign. Since $\alpha_1 < 1/2$ by assumption, for any given values of $q(\theta, \varepsilon_{1i}, \varepsilon^j_{2i})$ we have to choose $a^c_i(\theta)$ and $a^b_i(\theta)$ so as to minimize the nonnegative term

$$S_i(q_{i,p}(\theta_{-i}, \underline{\theta_i}), s_{-i,p}, a_i(\theta_{-i}, \underline{\theta_i}), \underline{\theta_i}, \varepsilon_{1i}, \varepsilon^j_{2i}).$$

We choose $a_i(\theta_{-i}, \underline{\theta_i}, \varepsilon_{1i}, \varepsilon^j_{2i})$ such that

$$S_i(q_{i,p}(\theta_{-i}, \underline{\theta_i}), s_{-i,p}, a_i(\theta_{-i}, \underline{\theta_i}), \underline{\theta_i}, \varepsilon_{1i}, \varepsilon^j_{2i}) = 0. \tag{52}$$

Using the relations (52), these formulae reduces to the following:

$$\int_\Theta U(q(\theta), a(\theta), \theta) f(\theta)\, d\theta = \int_\Theta \int_{\varepsilon_{1i}} \int_{\varepsilon^j_{2i}} \Big((2\alpha_1 - 1) \Big(\sum_{i=1}^{I} V_i(q_{i,p}(\theta, \varepsilon_{1i}, \varepsilon^j_{2i}), \theta_i) \frac{1 - F_i(\theta_i)}{f_i(\theta_i)} \Big) \tag{53}$$

$$+ (1 - \alpha_1) W(q(\theta), \theta) \Big) dF(\varepsilon_{1i}) dF(\varepsilon^j_{2i}) f(\theta)\, d\theta.$$

It remains to maximize the function under the integral sign on the right-hand side. A straightforward computation yields

$$\frac{\partial V_i(q^c_{i,p}(\theta, \varepsilon_{1i}), \theta_i)}{\partial q^c_{i,p}(\theta, \varepsilon_{1i})} = \frac{\alpha}{\theta_i h(\theta_i)} - \frac{\alpha h'(\theta_i)}{h^2(\theta_i)} [\psi^c_{i,p} + \ln \theta_i - \ln s_{-i,p} - \ln \big(q^c_{i,p}(\theta, \varepsilon_{1i}) + \varepsilon_{1i} \big)],$$

$$\frac{\partial V_i(q^{bj}_{i,p}(\theta, \varepsilon^j_{2i}), \theta_i)}{\partial q^{bj}_{i,p}(\theta, \varepsilon^j_{2i})} = \frac{(1-\alpha)}{\theta_i h(\theta_i)} - \frac{(1-\alpha) h'(\theta_i)}{h^2(\theta_i)} [\psi^b_{i,p} + \ln \theta_i - \ln s_{-i,p} - \ln \Big(q^{bj}_{i,p}(\theta, \varepsilon^j_{2i}) + \varepsilon^j_{2i} \Big)],$$

$$\frac{\partial W(q(\theta, \varepsilon_{1i}, \varepsilon^j_{2i}), \theta)}{\partial q^c_{i,p}(\theta, \varepsilon_{1i})} = \frac{\alpha}{h(\theta_i)} [\psi^c_{i,p} + \ln \theta_i - \ln s_{-i,p} - \ln \big(q^c_{i,p}(\theta, \varepsilon_{1i}) + \varepsilon_{1i} \big)] - c^c,$$

$$\frac{\partial W(q(\theta, \varepsilon_{1i}, \varepsilon^j_{2i}), \theta)}{\partial q^{bj}_{i,p}(\theta, \varepsilon^j_{2i})} = \frac{(1-\alpha)}{h(\theta_i)} [\psi^b_{i,p} + \ln \theta_i - \ln s_{-i,p} - \ln \Big(q^{bj}_{i,p}(\theta, \varepsilon^j_{2i}) + \varepsilon^j_{2i} \Big)] - c^b$$

and therefore the following first-order conditions (solving the problem pointwise):

$$\frac{\alpha}{h(\theta_i)} [\psi^c_{i,p} + \ln \theta_i - \ln s_{-i,p} - \ln \big(q^c_{i,p}(\theta, \varepsilon_{1i}) + \varepsilon_{1i} \big)] = c^c_{i1}(\theta_i, \varepsilon_{1i}), \tag{54}$$

$$\frac{(1-\alpha)}{h(\theta_i)} [\psi^b_{i,p} + \ln \theta_i - \ln s_{-i,p} - \ln \Big(q^{bj}_{i,p}(\theta, \varepsilon^j_{2i}) + \varepsilon^j_{2i} \Big)] = c^b_{i1}(\theta_i, \varepsilon^j_{2i}). \tag{55}$$

Solving this system we obtain (38)–(39). Using these results, we deduce from (12), (13), (**??**) and (52) that

$$a_i(\theta,\varepsilon_{1i},\varepsilon_{2i}^j)=\frac{\alpha}{h(\theta_i)}\left(q_{i,p}^c+\varepsilon_{1i}\right)[1+\psi_{i,p}^c+\ln\theta_i-\ln s_{-i,p}-\ln\left(q_{i,p}^c+\varepsilon_{1i}\right)]-p_i^c q_{i,p}^c$$
$$+\frac{(1-\alpha)}{h(\theta_i)}\left(q_{i,p}^{bj}+\varepsilon_{2i}^j\right)[1+\psi_{i,p}^b+\ln\theta_i-\ln s_{-i,p}-\ln\left(q_{i,p}^{bj}+\varepsilon_{2i}^j\right)]-p_i^{bj}q_{i,p}^{bj}$$
$$-\int_{\underline{\theta_i}}^{\theta_i}V_i(q_{i,p}(\theta_{-i},\tilde{\theta}_i),\tilde{\theta}_i,\varepsilon_{1i},\varepsilon_{2i}^j)\;d\tilde{\theta}_i.$$

It follows from (51), (54) and (55) that

$$V_i(q_{i,p}(\theta,\varepsilon_{1i},\varepsilon_{2i}^j),\theta_i,\varepsilon_{1i},\varepsilon_{2i}^j) = q_{i,p}^c(\theta,\varepsilon_{1i})\left(\frac{\alpha}{\theta_i h(\theta_i)}-\frac{\alpha h'(\theta_i)}{h^2(\theta_i)}-\frac{h'(\theta_i)}{h(\theta_i)}c_{i1}^c(\theta_i,\varepsilon_{1i})\right) \tag{56}$$
$$+q_{i,p}^{bj}(\theta,\varepsilon_{2i}^j)\left(\frac{(1-\alpha)}{\theta_i h(\theta_i)}-\frac{(1-\alpha)h'(\theta_i)}{h^2(\theta_i)}-\frac{h'(\theta_i)}{h(\theta_i)}c_{i1}^b(\theta_i,\varepsilon_{2i}^j)\right)$$

Since all factors in this expression are nonnegative by (36)–(37), the participation constraints (33) follow from (36) and (37). ■

Proof of Proposition 4. By repeating the first part of the proof of the preceding proposition, we are led again to the maximization of the integral (53). If $\theta\in\Theta$ satisfies $\theta_i>\underline{\theta_i}$ for all i, then our former reasoning still leads to the first-order conditions (54) and (55). Since almost all points $\theta\in\Theta$ have this property, we can still solve this system and we obtain (41) and (42). Furthermore, since each S_i is a decreasing function of a_i, it remains to show that by choosing $a_i(\theta,\varepsilon_{1i},\varepsilon_{2i}^j)=0$ the participation constraints are satisfied. Finally, the nonnegativity of S_i follows from its explicit formulae

$$S(q_{i,p}(\theta),s_{-i,p},a_i(\theta,\varepsilon_{1i},\varepsilon_{2i}^j),\theta_i,\varepsilon_{1i},\varepsilon_{2i}^j)=\frac{\alpha}{h(\theta_i)}\left(q_{i,p}^c+\varepsilon_{1i}\right)+\frac{(1-\alpha)}{h(\theta_i)}\left(q_{i,p}^{bj}+\varepsilon_{2i}^j\right).$$

■

8 Appendix 2. Estimation results

Table 1: Estimation of structural parameters (Constant MU of income and unobservables ε_{1i}, ε^j_{2i} Gumbel distributed)

		EER(07)	Model 1	EER(07)	Model 1	EER(07)	Model 1
Variable	Par.	Peak	Peak	Off-Peak	Off-Peak	t-test	t-test
MU. Transport	α	0.895 (0.012)	0.819 (0.015)	0.894 (0.010)	0.884 (0.012)	0.06	-3.41
Distr. θ	μ	0.300 (0.002)	0.589 (0.005)	0.386 (0.003)	0.718 (0.007)	-22.18	-14.97
MU income	h	1.941 (0.259)	2.039 (0.211)	2.286 (0.283)	2.434 (0.333)	-0.90	-1.00
Dist. ε_{1i}	μ_1	—	45.78 (0.836)	—	42.428 (0.558)	—	3.33
Dist. ε^j_{2i}	μ_2	—	207.75 (17.18)	—	325.42 (32.31)	—	-3.22
Bus	β^b_1	1.618 (0.177)	2.243 (0.164)	0.058 (0.192)	1.021 (0.180)	5.98	5.02
Bus Freq.	β^b_2	0.106 (0.017)	0.091 (0.016)	0.206 (0.024)	0.154 (0.022)	-3.47	-2.28
Distance O-D	β^b_3	-0.011 (0.015)	0.056 (0.015)	0.078 (0.019)	0.146 (0.020)	-3.62	-3.64
Time O-D	β^b_4	0.005 ($9e-4$)	0.004 ($9e-4$)	-0.001 (0.002)	-0.005 (0.002)	3.32	4.73
Stud.-Scholar	β^b_5	0.444 (0.104)	0.020 (0.089)	0.348 (0.132)	0.200 (0.142)	0.57	-1.07
Car	β^c_1	1.518 (0.179)	2.152 (0.150)	0.620 (0.204)	1.076 (0.182)	3.31	4.56
Distance O-D	β^c_2	0.082 (0.009)	0.116 (0.012)	0.125 (0.011)	0.169 (0.016)	-3.02	-2.73
Time O-D	β^c_3	$5e-4$ ($5e-4$)	$7e-4$ ($4e-4$)	-0.012 ($9e-4$)	-0.007 ($7e-4$)	11.75	10.05
Car Power	β^c_4	0.034 (0.006)	0.023 (0.005)	0.048 (0.009)	0.040 (0.007)	-1.35	-1.90
Unemployed	β^c_5	-0.200 (0.093)	-0.126 (0.080)	-0.220 (0.078)	-0.164 (0.069)	0.16	0.36
Stud.-Scholar	β^c_6	-0.490 (0.099)	-0.366 (0.086)	-0.416 (0.118)	-0.248 (0.105)	-0.48	-0.87
Age	β^c_7	0.048 (0.009)	0.035 (0.007)	0.026 (0.009)	0.024 (0.007)	1.76	1.03
Age*age	β^c_8	$-6e-4$ ($1e-4$)	$-4e-4$ ($1e-4$)	$-4e-4$ ($1e-4$)	$-3e-4$ ($1e-4$)	-1.41	-0.71
Mean Log L.		-7.663	-3.945	-6.618	-3.679		

Table 2: Estimation of structural parameters ($\theta \rightarrow$Beta(1,μ) and unobservables ε_{1i}, ε_{2i}^j Gumbel distributed)

Model 2		Peak	Peak	Off-Peak	Off-Peak	t-test	t-test
Variable	Par.	C. Inf.	Inc. Inf.	C. Inf.	Inc. Inf.	C. Inf.	Inc. Inf.
MU. Transport	α	0.747 (0.017)	0.800 (0.017)	0.873 (0.011)	0.865 (0.010)	-6.143	-3.254
Dist. θ	μ	—	0.454 (0.052)	—	0.380 (0.003)	—	12.429
MU income	k	3.587 (0.314)	3.096 (0.347)	3.788 (0.351)	4.379 (0.422)	-0.429	-2.346
Dist. ε_{1i}	μ_1	50.24 (1.163)	46.875 (0.988)	43.013 (0.537)	43.389 (0.518)	5.637	3.125
Dist. ε_{2i}^j	μ_2	149.28 (10.208)	188.036 (15.857)	297.92 (25.40)	279.518 (21.092)	-5.428	-3.467
Bus	β_1^b	2.735 (0.205)	2.109 (0.173)	1.525 (0.231)	0.818 (0.208)	3.916	4.775
Bus Freq.	β_2^b	0.0595 (0.023)	0.094 (0.017)	0.136 (0.027)	0.160 (0.024)	-2.158	-2.257
Distance O-D	β_3^b	0.012 (0.023)	0.053 (0.015)	0.180 (0.025)	0.143 (0.027)	-4.865	-2.946
Time O-D	β_4^b	0.004 (0.001)	0.005 ($9e-4$)	-0.015 (0.002)	-0.005 (0.002)	7.980	5.019
Stud.-Scholar	β_5^b	-0.321 (0.136)	0.024 (0.093)	0.206 (0.199)	0.184 (0.151)	-2.183	-0.906
Car	β_1^c	2.507 (0.265)	2.014 (0.163)	2.043 (0.320)	0.839 (0.219)	1.117	4.309
Distance O-D	β_2^c	0.116 (0.013)	0.132 (0.013)	0.205 (0.013)	0.200 (0.015)	-4.793	-3.382
Time O-D	β_3^c	0.002 ($9e-4$)	$7e-4$ ($5e-4$)	-0.019 (0.0015)	-0.008 ($8e-4$)	11.777	9.434
Car power	β_4^c	0.025 (0.009)	0.024 (0.006)	0.043 (0.016)	0.0455 (0.009)	-0.938	-2.005
Unemployed	β_5^c	-0.166 (0.141)	-0.131 (0.090)	-0.315 (0.136)	-0.188 (0.079)	0.764	0.480
Stud.-Scholar	β_6^c	-0.752 (0.160)	-0.387 (0.093)	-0.475 (0.207)	-0.297 (0.128)	-1.060	-0.569
Age	β_7^c	0.055 (0.012)	0.040 (0.008)	0.031 (0.013)	0.0300 (0.009)	1.339	0.875
Age*age	β_8^c	$-6e-4$ ($1e-4$)	$-5e-4$ ($1e-4$)	$-4e-4$ ($1e-4$)	$-4e-4$ ($1e-4$)	-1.414	-0.707
Mean Log L.		-3.733	-3.990	-2.766	-3.493		

Standard Deviations in parentheses.

Table 3: Welfare (Euros) – Peak Period, constant MU of income (with unobservables)

		Current pricing	Opt. pricing	Opt. pricing	Opt. pricing
α_1		—–	1/8	1/4	1/2
$q^c_{i,p}$	mean	2.2	2.16	2.19	2.41
	s. dev	0.9	1.12	1.14	1.25
$q^{bj}_{i,p}$	mean	1.09	1.09	1.09	1.10
	s. dev.	0.49	0.27	0.27	0.27
$a^c_i + a^b_i$	mean	0.15	6.26	6.30	6.23
	s.dev	0.47	2.95	2.97	2.95
p^c_i	mean	2.01	3.20	3.01	1.04
	s.dev	1.78	—–	—–	—–
p^{bj}_i	mean	0.94	1.00	0.98	0.75
	s.dev	0.32	—–	—–	—–
Profit	mean	3,820	23,753	23,287	18,488
	s. dev.	2.42	3.81	3.73	2.95
Surplus	mean	71,021	41,190	42,213	51,843
	s. dev	8.67	8.23	8.40	10.02
Welfare	mean	74,841	64,943	65,500	70,331
	s. dev	9.32	12.04	12.14	12.97

1. Welfare weight $\alpha_1 \in (0,1)$ placed on travelers' surplus by the regulator

Table 4: Welfare (Euros) – Off-Peak Period, constant MU of income (with unobservables)

		Current pricing	Opt. pricing	Opt. pricing	Opt. pricing
α_1			1/8	1/4	1/2
$q^c_{i,p}$	mean	0.93	0.93	0.94	1.04
	s. dev	1.13	1.15	1.17	1.28
$q^{bj}_{i,p}$	mean	0.36	0.45	0.44	0.39
	s. dev.	0.50	0.42	0.41	0.37
$a^c_i + a^b_i$	mean	0.05	2.40	2.42	2.44
	s.dev	0.30	2.85	2.88	2.91
p^c_i	mean	2.18	3.05	2.88	1.06
	s.dev	1.50	—–	—–	—–
p^{bj}_i	mean	0.55	0.74	0.74	0.76
	s.dev	0.53	—–	—–	—–
Profit	mean	1,603	9,448	9,275	7,582
	s. dev.	1.69	3.67	3.60	2.91
Surplus	mean	27,294	12,915	13,291	16,546
	s. dev	10.45	8.65	8.80	10.10
Welfare	mean	28,897	22,364	24,109	24,128
	s. dev	10.45	12.32	13.02	13.01

Table 5: Welfare (Euros)– Peak Period, heterogenous MU of income, Nonlinear Heterogenous Pricing (Het. P.)

		Current P.	Compl. Inf	Het. P.	Het. P.	Het. P.
α_1			$> 1/2$	$1/2$	$3/4$	0.99
$q^c_{i,p}$	mean	2.23	3.46	2.48	2.44	1.55
	s. dev	1.07	2.35	1.52	1.50	0.96
$q^{bj}_{i,p}$	mean	0.96	1.05	0.94	0.72	0.95
	s. dev.	0.46	0.50	0.24	0.18	0.24
$a^c_i + a^b_i$	mean	0.15	0	0	0	0
	s.dev	0.47	0	0	0	0
p^c_i	mean	2.01	1.07	1.02	15.95	14.3
	s.dev	1.78	—	—	—	—
p^{bj}_i	mean	0.94	0.76	0.72	3.82	3.50
	s.dev	0.33	—	—	—	—
Profit	mean	3,728	0	0	-5,474	17,078
	s. dev.	2.52	0	0	1.06	3.37
Surplus	mean	74,280	59,952	78,923	61,182	52,242
	s. dev	10.52	26.53	14.81	11.66	9.44
Welfare	mean	78,008	59,952	78,923	55,707	69,320
	s. dev	11.40	26.53	14.81	10.6	12.81

Table 6: Welfare (Euros) – Off-Peak Period, heterogenous MU of income, Heterogenous Nonlinear Pricing

		Current P.	Compl. Inf.	Heter. P.	Heter. P.	Heter. P.
	α_1		$> 1/2$	$1/2$	$3/4$	0.99
$q^c_{i,p}$	mean	0.92	1.34	1.06	1.20	0.70
	s. dev	1.53	1.81	1.63	1.84	1.08
$q^{bj}_{i,p}$	mean	0.26	0.37	0.27	0.25	0.63
	s. dev.	0.43	0.35	0.26	0.24	0.59
$a^c_i + a^b_i$	mean	0.05	0	0	0	0
	s. dev.	0.30	0	0	0	0
p^c_i	mean	2.18	1.07	0.98	9.00	9.58
	s. dev	1.50	—	—	—	—
p^{bj}_i	mean	0.55	0.76	0.70	1.61	1.42
	s. dev	0.53	—	—	—	—
Profit	mean	1,222	0	0	38,648	23,833
	s. dev.	1.75	0	0	73.32	46.65
Surplus	mean	22,963	23,190	25,214	22,781	17,755
	s. dev.	11.01	10.87	11.90	10.80	8.06
Welfare	mean	24,186	23,190	25,214	61,429	41,588
	s. dev.	10.87	10.87	11.90	76.18	48.72

Table 7: Welfare (Euros)– Peak Period, heterogenous MU of income, Nonlinear Homogenous Pricing (Hom. P)

α_1		1/2	3/4	0.99
$q_{i,p}^c$	mean	2.51	1.13	1.66
	s. dev	0.67	0.30	0.44
$q_{i,p}^{bj}$	mean	1.13	0.99	1.06
	s. dev	0.38	0.33	0.35
$a_i^c+a_i^b$	mean	0	0	0
	s. dev	0	0	0
p_i^c	mean	1.07	10.20	5.84
	s. dev	——	——	——
p_i^{bj}	mean	0.76	2.09	1.45
	s. dev	——	——	——
Profit	mean	0	21,808	0
	s. dev	0	1.60	0
Surplus	mean	76,829	38,260	53,034
	s. dev	5.65	2.40	3.62
Welfare	mean	76,829	60,068	69,216
	s. dev	5.65	3.99	4.87

Table 8: Welfare (Euros)–Off Peak Period, heterogenous MU of income, Nonlinear Homogenous Pricing (Hom. P)

		Homog. P.	Homog. P.	Homog. P.
	α_1	1/2	3/4	0.99
$q_{i,p}^c$	mean	0.98	0.16	0.64
	s. dev	0.65	0.14	0.43
$q_{i,p}^{bj}$	mean	0.37	3.90	0.70
	s. dev	0.20	1.85	0.36
$a_i^c+a_i^b$	mean	0	0	0
	s. dev	0	0	0
p_i^c	mean	1.07	14.92	4.62
	s. dev	——	——	——
p_i^{bj}	mean	0.76	0.31	0.65
	s. dev	——	——	——
Profit	mean	0	892	4,323
	s. dev	0	1.11	0.98
Surplus	mean	25,075	17,883	18,507
	s. dev	4.79	2.54	3.37
Welfare	mean	25,075	18,775	22,830
	s. dev	4.79	3.15	4.34

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