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GEOMETRIC DESIGN OF SYMMETRIC 3-RRS CONSTRAINED PARALLEL PLATFORMS

Eric Wolbrecht, Hai-Jun Su, Alba Perez and J. Michael McCarthy*
Robotics and Automation Laboratory
Department of Mechanical and Aerospace Engineering
University of California
Irvine, California 92697
Email: ewolbrec@uci.ed, suh@eng.uci.edu, maperez@uci.edu, jmmccart@uci.edu

ABSTRACT

The paper presents the kinematic synthesis of a symmetric parallel platform supported by three RRS serial chains. The dimensional synthesis of this three degree-of-freedom system is obtained using design equations for each of three RRS chains obtained by requiring that they reach a specified set of task positions. The result is 10 polynomial equations in 10 unknowns, which is solved using polynomial homotopy continuation. An example is provided in which the direction of the first revolute joint (2 parameters) and the z component of the base and platform are specified as well as the two task positions. The system of polynomials has a total degree of 4096 which means that in theory it can have as many solutions. Our example has 70 real solutions that define 70 different symmetric platforms that can reach the specified positions.

1 Introduction

This paper examines the geometric design of a three degree-of-freedom symmetric platform constructed from three RRS serial chains—R denotes a revolute, or hinge, joint, and S denotes spherical, or ball, joint. In our case, the axes of the RR chain are required to be perpendicular which means the center of the spherical joint is constrained to lie on a right circular torus. The chain has 10 design parameters, which can be determined by specifying 10 goal positions the end-effector. The result is a polynomial system of total degree over 2 million, see Su et al. 2004(9).

Chen and Roth (1967)(1) introduced this problem in their application of kinematic synthesis principles (Hartenberg and Denavit 1964(2)) to the design robot manipulators. The elementary principles of the geometric design of linkages can be found in McCarthy (2000)(3). Current research is focussed on generalizing these ideas to achieve task-based design for serial chains with two to five degrees of freedom. Su et al. 2004(8) present a detailed analysis of the synthesis of CS chains, which demonstrates the range in complexity of this design problem.

Here we use the constraint equations of the RRS chain to define design equations for a symmetric 3-RRS platform. Each of the three RRS chains have the same dimensions but are symmetrically attached to a triangular base and platform, see Figure 1. Thus, three constraint equations are be solved simultaneously for each goal position. If n is the number of specified goal positions, then $3n \leq 10$, which means we can specify at most three goal positions and one design parameter for this platform.

In what follows, we present the design equations and an example in which two goal positions are specified, which allows us to specify four design parameters for the 3-RRS platform.

2 The Symmetric 3-RRS Platform

Figure 1 shows a three degree-of-freedom symmetric 3-RRS parallel platform, formed by a moving platform that is connected to a fixed base by three identical limbs. Each limb is an RRS serial chain with one revolute joint (R) attached to the base, a spherical joint (S) attached to the moving platform and a float-

*Address all correspondence to this author.

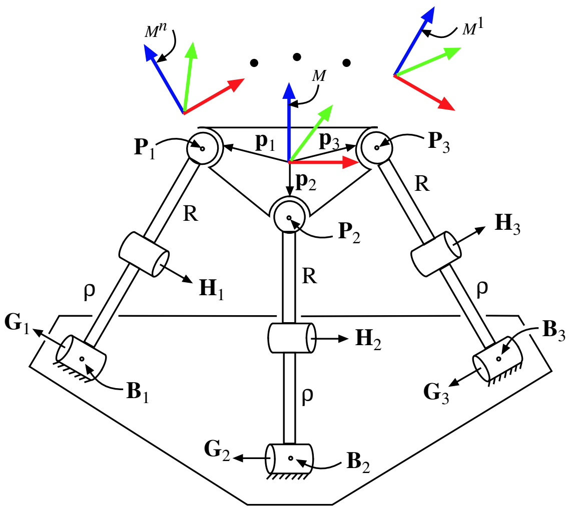

Figure 1. A Symmetric 3-RRS Parallel Platform.

ing revolute joint (R), which is perpendicular to the first R-joint, connecting them. We are interested in the symmetric case, that is, the three grounded R-joints and S-joints are located on a circle separated by 120 degrees on the base and the platform, respectively.

Let $\mathbf{G}_i$ and $\mathbf{H}_i$ represent the two R-joint axes of the ith limb. Let $\mathbf{v}_i$ be their common normal with ρ_i as the normal distance. The intersection of $\mathbf{v}_i$ with $\mathbf{G}_i$ and $\mathbf{H}_i$ are denoted by the points $\mathbf{B}_i$ and $\mathbf{Q}_i$ respectively. Let the position of the S-joint be the vector $\mathbf{P}_i$ and R_i be the normal distance from $\mathbf{P}_i$ to the axis $\mathbf{H}_i$. The location of the S joint of the ith limb is defined in the moving frame by the vector $\mathbf{p}_i$. Its coordinates in the fixed frame are given by $\mathbf{P}_i = [M]\mathbf{p}_i$, where $[M]$ is the 4×4 homogeneous transform that defines the location of the moving platform.

In this paper, we assume $\mathbf{G}_i$ and $\mathbf{H}_i$ are perpendicular to each other and all three limbs have the same dimensions that is

$$\rho = \rho_1 = \rho_2 = \rho_3, \quad \text{and} \quad R = R_1 = R_2 = R_3. \tag{1}$$

By symmetry, the normal points $\mathbf{B}_2$ and $\mathbf{B}_3$ can be determined by rotating $\mathbf{B}_1$ around the z-axis by $2\pi/3$ and $4\pi/3$ radians, respectively. The vectors $\mathbf{G}_2$, $\mathbf{G}_3$ and $\mathbf{p}_2$, $\mathbf{p}_3$ can be determined in the same way. We have

$$\begin{aligned} \mathbf{B}_2 &= [Z(2\pi/3)]\mathbf{B}_1, \quad \mathbf{B}_3 = [Z(4\pi/3)]\mathbf{B}_1, \\ \mathbf{G}_2 &= [Z(2\pi/3)]\mathbf{G}_1, \quad \mathbf{G}_3 = [Z(4\pi/3)]\mathbf{G}_1, \text{ and} \\ \mathbf{p}_2 &= [Z(2\pi/3)]\mathbf{p}_1, \quad \mathbf{p}_3 = [Z(4\pi/3)]\mathbf{p}_1, \end{aligned} \tag{2}$$

where $[Z(\theta)]$ is the 4×4 transform that defines a rotation by θ around z-axis.

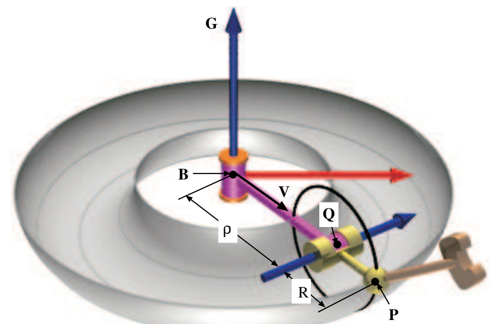

Figure 2. The circular torus traced by the wrist center of a "right" RRS serial chain.

3 Circular Torus Constraint Equation

The center of the S-joint of an RRS chain in which the axes of the R-joints are perpendicular traces a right circular torus (Fig. 2), and, we refer to this chain as a "right" RRS chain. The equation of the right circular torus can be used to define the dimensions of this chain such that it reaches a specified set of goal positions. This equation for the torus is obtained as follows.

First, recall that $\mathbf{G}$ is the axis of the torus. Introduce a unit vector $\mathbf{v}$ perpendicular to this axis so the center of the generating circle is given by $\mathbf{Q} - \mathbf{B} = \rho\mathbf{v}$. Next define $\mathbf{u}$ to be the unit vector in the direction $\mathbf{G}$, then a point $\mathbf{P}$ on the torus is defined by the vector equation,

$$\mathbf{P} - \mathbf{B} = \rho\mathbf{v} + R(\cos\phi\mathbf{v} + \sin\phi\mathbf{u}), \tag{3}$$

where ϕ is the angle measured from $\mathbf{v}$ to the radius vector of the generating circle.

An algebraic equation of the torus is obtained from Eq. (3) by computing the magnitude

$$(\mathbf{P} - \mathbf{B})^2 = \rho^2 + R^2 + 2\rho R\cos\phi. \tag{4}$$

Next compute the dot product of $\mathbf{u}$ with (3) yields

$$(\mathbf{P} - \mathbf{B}) \cdot \mathbf{u} = R\sin\phi. \tag{5}$$

Eliminate $\cos\phi$ and $\sin\phi$ from these equations, in order to obtain

$$((\mathbf{P} - \mathbf{B})^2 - \rho^2 - R^2)^2 + 4\rho^2((\mathbf{P} - \mathbf{B}) \cdot \mathbf{G})^2 - 4\mathbf{G}^2\rho^2R^2 = 0. \tag{6}$$

Note we have used $\mathbf{u} = \mathbf{G}/|\mathbf{G}|$. This is the equation of a right circular torus. It has 10 independent parameters, the two scalars ρ and R, three each for $\mathbf{P}$ and $\mathbf{B}$, and two for $\mathbf{G}$. See Su et al. (2004) (9) for solutions to the constraint equation for the general torus.

Platform	Task Positions	Free Parameters
3-RRS	2	4
	3	1
4-RRS	2	2
5-RRS	2	0

Table 1. The number of task positions and free parameters for symmetric 3, 4 and 5-RRS platforms.

4 Platform Design Equations

The design equations for the 3-RRS platform are defined by the torus constraint equations for each of the three chains,

$$((\mathbf{P}_i^k - \mathbf{B}_i)^2 - \rho^2 - R^2)^2 + 4\rho^2((\mathbf{P}_i^k - \mathbf{B}_i)\cdot\mathbf{G}_i)^2 - 4\rho^2 R^2 \mathbf{G}_i^2 = 0, \quad i = 1,2,3. \tag{7}$$

The point $\mathbf{P}_i^k = [M^k]\mathbf{p}_i$ is the position of S-joint of chain i when the end effector is located at the kth task position. We obtain three equations (7) for each specified task position.

The symmetric 3-RRS platform has the 10 independent design parameters $\mathbf{r} = (\rho, R, \mathbf{B}_1, \mathbf{p}_1, \mathbf{G}_1)$–note $\mathbf{G}_1$ is a unit vector so it has only two independent parameter. Equations (1) and (2) define the remaining dimensions of the limbs supporting this platform in terms of the elements of the design vector $\mathbf{r}$.

Because we can have no more than 10 design equations, we find that the number n of task positions must satisfy the condition $3n \leq 10$. The result is two basic design problems (i) two task positions ($n = 2$), which allows the values of four dimensional parameters to be selected by the designer, and (ii) three task positions ($n = 3$), which allows one dimensional parameter to be selected.

It is interesting to note that design equations for symmetric 4-RRS and 5-RRS platforms can be defined in the same way. The 4-RRS platform has two degrees of freedom, and each task position requires four equations similar to (7). This means it can be defined to reach two task positions with two free parameters that can be specified by the designer. The 5-RRS platform has one degree of freedom and can be designed to reach two task positions with no extra free parameters. These results are summarized in Table 1.

In what follows, we seek the all of the solutions to the design equations using the polynomial homotopy method.

5 The Polynomial Homotopy Method

Polynomial homotopy method is a globally convergent numerical technique for finding all the isolated solutions to systems of polynomial equations. The method is based on the fact that small changes in the coefficients of a polynomial system result in small changes to the system of roots.

A homotopy is a function that smoothly transforms one system of polynomials $Q(\mathbf{z}) = 0$ called the start system into a second set $P(\mathbf{z}) = 0$ called the target system. While there are many ways to define the homotopy, perhaps the simplest is given by

$$H(\lambda, \mathbf{z}) = (1-\lambda)Q(\mathbf{z}) + \lambda P(\mathbf{z}), \tag{8}$$

where $\lambda \in [0,1)$ is the real-valued homotopy parameter.

For root of the start system $Q(\mathbf{z}) = 0$, denoted $\mathbf{z} = \mathbf{a}_j$, $j = 1, \ldots, N$, the homotopy equation $H(\lambda, \mathbf{z}) = 0$ has an associated zero curve γ_a, which is the connected component of $H^{-1}(0)$ containing the start point $(0, \mathbf{a}_j)$. Each zero curve leads either to a point $(1, \mathbf{z}_a)$ where $P(\mathbf{z}_a) = 0$, or it diverges to a root "at infinity."

Each zero curve of the homotopy can be parameterized by its arc length s, so γ_a has the form $(\lambda(s), \mathbf{z}(s))$. Tracking this curve involves numerical computation of points $\mathbf{y}_i \approx (\lambda(s_i), \mathbf{z}(s_i))$, where $\{s_i\}$ is an increasing sequence of arc lengths. This can be done using a predictor-corrector strategy described in Watson et al. (1997)(10) and Wise et al. (2000)(11).

Along the zero curve γ_a, we have $H(\lambda(s), \mathbf{z}(s)) = 0$, therefore we can compute

$$\frac{d}{ds}H(\lambda, \mathbf{z}) = \left[H_\lambda \ H_\mathbf{z}\right] \begin{Bmatrix} d\lambda/ds \\ d\mathbf{z}/ds \end{Bmatrix} = 0, \tag{9}$$

where $[J_H] = [H_\lambda, H_\mathbf{z}]$ is the $n \times (n+1)$ matrix of partial derivatives of the homotopy $H(\lambda, \mathbf{z})$. Notice that the vector $\mathbf{v} = (d\lambda/ds, d\mathbf{z}/ds)^T$ tangent to the zero curve γ_a is in the null space of the Jacobian matrix $[J_H]$. Integration of this equation yields tracks the zero curve from the known initial roots of the start system to the unknown roots of the target system.

To solve this problem, we use our POLSYS-GLP software which is a generalized version of the homotopy solver POLSYS-PLP developed by Watson et al. (1997)(10). It is a Fortran routine that runs in a parallel computing environment.

6 Example

As an example of the design of a symmetric 3-RRS chain, we solve the two position case with $\mathbf{G}$, p_z and B_z chosen. This requires the solution of six quartic equations which form a polynomial system of total degree $4^6 = 4,096$. For comparison, the three position case requires the solution of nine equations of degree five with a total degree of $5^9 = 1,953,125$ where we have treated ρ^2 and R^2 as independent variables.

The 4×4 matrices defining the task positions of the end-effector are given by

$$M^1 = \begin{bmatrix} -0.7314 & 0.5348 & 0.4232 & -0.4350 \\ -0.5909 & -0.8068 & -0.0017 & 0.4918 \\ 0.3405 & -0.2513 & 0.9060 & 1.1661 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (10)$$

$$M^2 = \begin{bmatrix} 0.7960 & 0.5967 & 0.1011 & -0.0318 \\ -0.6052 & 0.7869 & 0.1208 & -0.2604 \\ -0.0075 & -0.1573 & 0.9875 & 1.0613 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (11)$$

The two position problem provides the designer the freedom to specify four of the dimensional parameters of the 3-RRS platform. In this example we chose the axis $\mathbf{G}_1$ to be along the z-axis, that is $\mathbf{G}_1 = (1,0,0)$, as well as the parameter values $B_z = 0$ and $p_z = -0.25$. This places the base in the x-y coordinate plane of the fixed frame, and the platform slightly below the x-y plane of the end-effector frame.

Our execution of POLSYS-GLP for this set of design equations yielded 70 real solutions, which are listed in Tables 4 and 5. As an example we present the table of Denavit-Hartenberg parameters (Table 2) for the 4th solution in Table 4—the three RRS chains are identical and have the same set of DH parameters. Table 3 defines the joint angles of each RRS chain that positions the platform in the two task position. Figures 3 and 4 illustrate the movement of our 3-RRS platform between the two task positions—the dark sequence of end-effectors is the actual trajectory.

It is important to notice that the workspace of the 3-RRS platform is three dimensional, which means that an it cannot generate arbitrary trajectories between the two task positions. To plan this trajectory, we use our SYNTHETICA software to modify a given trajectory, the light set of frames, into a trajectory that lies in the workspace, the dark set of frames (Su et al. 2003(7)).

joint	α	a	θ	d
1	-	-	θ_1	0
2	90°	1.5813	θ_2	0
3	0°	0.7529	θ_3	0
4	90°	0	θ_4	0
5	-90°	0	θ_5	0

Table 2. The Denavit-Hartenberg parameters solution 4 in Table 4

chain	$\theta_1(^\circ)$	$\theta_2(^\circ)$	$\theta_3(^\circ)$	$\theta_4(^\circ)$	$\theta_5(^\circ)$
1	-132.823	229.953	-15.512	48.441	101.711
	-147.046	189.092	-1.543	50.317	-131.693
2	-128.437	108.140	57.784	61.158	15.492
	177.960	201.637	-18.484	100.546	113.137
3	-122.309	-13.916	143.511	15.825	-66.863
	-175.881	162.975	8.054	84.375	-6.446

Table 3. Joint parameters for task positions 1 and 2

7 Conclusion

This paper presents the geometric design theory for symmetric 3-RRS parallel platforms. It is shown that these systems have 10 design parameters and that they can be designed to reach three arbitrary task positions. An example of the two position synthesis of this platform is provided, which requires the solution of a polynomial system of total degree 4,096. We obtain 70 real solutions, any one of which can be used as a platform to guide the end-effector between the goal positions.

The primary benefit of parallel platforms that have less than the full six degrees-of-freedom of a typical robotic system lies in the ability to leverage the structural constraint to amplify the performance of the actuators. Further research will use this mechanical advantage to evaluate and classify designs for these constrained robotics systems.

ACKNOWLEDGMENT

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REFERENCES

Chen, P. and Roth, B., 1967, "Design Equations for Finitely and Infinitesimally Separated Position Synthesis of Binary Link and Combined Link Chains," *ASME J. Engineering for Industry* 91:209-219.

Hartenberg, R., S., and Denavit, J., 1964, *Kinematic Synthesis of Linkages*, McGraw-Hill, NY.

McCarthy, J. M., 2000, *Geometric Design of Linkages*. Springer-Verlag, New York.

Morgan, A. P., 1987, *Solving polynomial systems using continuation for engineering and scientific problems*, Prentice-Hall, Englewood Cliffs, NJ..

Tsai, L. W., and Roth, B., 1972, "Design of Dyads with

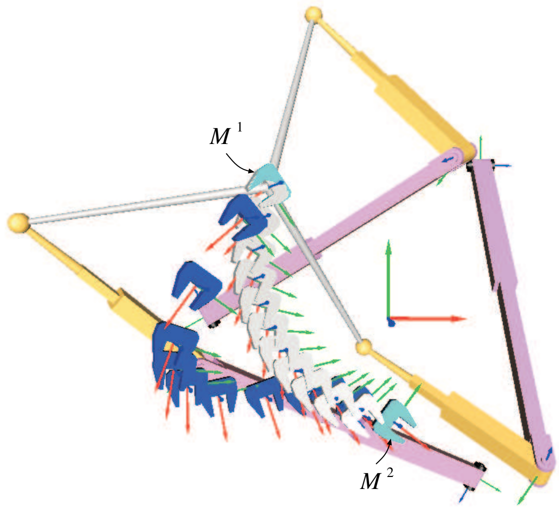

Figure 3. An Symmetric 3-RRS platform reaching M^1

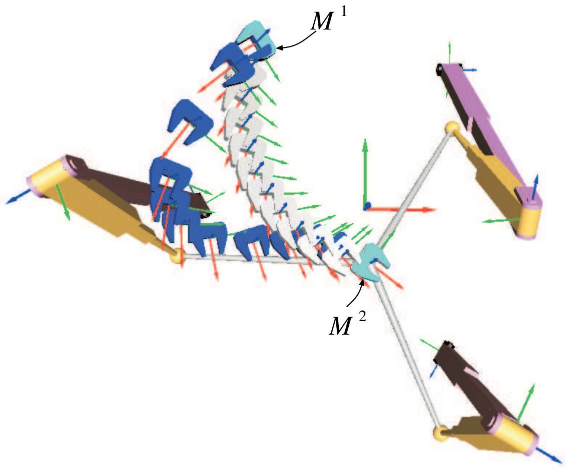

Figure 4. An Symmetric 3-RRS platform reaching M^2

Helical, Cylindrical, Spherical, Revolute and Prismatic Joints," *Mechanism and Machine Theory*, 7:591-598.

Su, H.-J., Collins, C., McCarthy, J. M., 2002,"An Extensible Java Applet for Spatial Linkage Synthesis," DETC2002/MECH-34371, *ASME Design Engineering Technical Conference, Montreal, Canada, Sept.29-Oct.02, 2002*.

Su, H., Dietmaier, P., and McCarthy, J. M., 2003 Trajectory Planning for Constrained Parallel Manipulators, ASME Journal of Mechanical Design, 125(4):709-716, December.

Su, H. , Wampler, C. W. , McCarthy, J. M. , 2004, Geometric Design of Cylindric PRS Serial Chains, *ASME Journal of Mechanical Design*, (in press).

Su, H. , McCarthy, J. M. , and Watson, L. , 2004, Generalized Linear Product Homotopy Algorithms and the Computation of Reachable Surfaces, *ASME Journal of Computers and Information Science and Engineering*, (in press).

Watson, L. T., Sosonkina, M., Melville, R. C., Morgan, A. P., and Walker, H. F. 1997, "Algorithm 777: HOMPACK90: A suite of Fortran 90 codes for globally convergent homotopy algorithms," *ACM Trans. Math. Software* 23(4):514-549.

Wise, S. M., Sommese, A. J., and Watson, L. T., 2000, "Algorithm 801: POLSYS_PLP: A Partitioned Linear Product Homotopy Code for Solving Polynomial Systems of Equations." *ACM Trans. Math. Software* 26(1):176-200.

Sol.	ρ	R	B_x	B_y	p_x	p_y
1	0.1997	1.0318	0.2863	0.3472	-0.2781	-0.1239
2	0.1932	1.0368	-0.3926	0.1697	0.2457	-0.2119
3	0.1842	1.0439	0.1513	-0.4494	0.0491	0.3458
4	1.5813	0.7529	0.4488	0.7514	-0.0460	0.7096
5	0.1594	1.0573	-0.2252	-0.2324	0.2510	0.0973
6	0.1594	1.0573	-0.2252	-0.2324	0.2510	0.0973
7	1.6217	0.7265	-0.2897	-0.8382	-0.0674	-0.5905
8	0.7925	1.7082	1.0366	0.1694	0.4643	0.4225
9	0.1550	1.0629	0.3546	-0.0626	-0.2125	0.1613
10	0.6260	1.5247	-0.1381	0.8031	-0.3017	0.5176
11	0.2539	1.1646	0.3126	0.7065	-0.4989	-0.0485
12	0.3160	1.1853	-0.2042	-0.3028	-0.0152	-0.2423
13	0.1443	1.0746	-0.0806	0.3747	-0.0401	-0.2646
14	0.3307	1.1975	-0.0692	0.3376	-0.1434	0.2135
15	0.3323	1.1984	0.3586	-0.0039	0.2070	0.1246
16	0.6546	1.5813	-0.5982	-0.8458	-0.0459	-0.6881
17	0.2085	1.2050	-0.4619	-0.5138	0.3180	-0.0508
18	0.2530	1.1618	-0.7149	0.0105	0.2962	-0.5095
19	0.2156	1.1636	0.3867	-0.6151	0.1981	0.5026
20	0.2024	1.2127	0.6876	-0.0890	-0.1604	0.2716
21	0.1989	1.1368	0.4103	0.1060	-0.3832	0.3060
22	0.2061	1.1277	-0.1249	-0.3235	0.4482	0.1727
23	0.2270	1.2187	-0.3943	-0.0822	0.2334	0.2634
24	0.2379	1.1584	-0.3231	0.2053	0.4718	-0.2268
25	0.1941	1.2038	0.3427	-0.2684	-0.3182	0.0492
26	0.1649	1.2020	-0.1639	0.6459	-0.1438	-0.2571
27	0.1704	1.2067	0.0716	0.4748	0.0585	-0.3007
28	0.1772	1.1505	-0.2326	0.3276	-0.0775	-0.4971
29	0.2803	1.1528	0.1944	0.2427	-0.4197	-0.4202
30	0.2361	1.2927	-0.6509	0.5146	0.2011	-0.1683
31	0.2055	1.2714	0.0413	-0.8272	0.0384	0.2767
32	0.2242	1.1614	0.1399	-0.3854	-0.0578	0.5833
33	0.1748	1.0370	0.0153	0.0019	0.0100	0.0158
34	0.2762	1.3243	0.7212	0.5202	-0.2144	-0.1058
35	0.2469	1.3091	0.7941	0.1160	-0.3226	0.4538

Table 4. Real solutions 1-35

Sol.	ρ	R	B_x	B_y	p_x	p_y
36	1.4718	0.6389	-0.0217	-0.3956	0.0269	-0.1840
37	0.2697	1.3362	-0.3456	-0.7301	0.6242	-0.0025
38	0.1934	1.3010	-0.3676	0.6213	-0.2160	-0.5100
39	0.1825	1.3503	0.0070	0.7626	-0.0401	-0.2695
40	0.2638	1.4645	-0.5559	0.6012	0.4370	-0.1338
41	0.1988	1.2970	-0.0584	0.3971	0.0469	-0.5538
42	0.2414	1.2511	0.3676	-0.4986	0.0855	0.7222
43	0.2327	1.4308	-0.0379	-0.7513	-0.1187	0.5050
44	0.2407	1.4292	0.7530	-0.3708	-0.2323	0.1403
45	0.3606	1.5641	0.7320	0.3432	-0.2836	-0.4821
46	0.3329	1.5485	-0.6404	-0.4911	0.4912	0.1612
47	0.2246	1.4271	0.3214	-1.0446	0.2363	0.4195
48	0.2698	1.4781	0.7281	-0.1466	-0.4439	0.2884
49	0.2030	1.4090	-0.1756	0.6648	-0.0867	-0.5179
50	0.2860	1.4948	-1.0953	0.3805	0.1961	-0.5033
51	0.2908	1.3389	-0.2759	-0.1157	0.4620	0.4145
52	0.2445	1.3294	0.3591	-0.0877	-0.5821	0.1955
53	0.3068	1.5213	-0.8415	-0.3973	0.2620	0.0869
54	0.3676	1.6034	0.6201	0.7064	-0.5441	-0.3458
55	0.2959	1.5345	-0.9096	0.3534	0.4087	-0.4689
56	0.2397	1.4802	0.2135	-0.8586	0.0597	0.5986
57	0.2929	1.2972	-0.6184	-0.0185	0.5809	-0.5569
58	0.3196	1.5695	0.8682	0.9844	-0.4325	0.0394
59	0.3470	1.3485	0.1264	0.6933	-0.7782	-0.3950
60	2.4395	1.7477	0.1290	-0.7945	1.0613	-0.0046
61	2.5094	2.3699	0.1405	-1.3974	2.2148	0.1147
62	0.5583	2.0769	-0.3055	0.1229	0.9897	1.0887
63	0.7932	3.1719	-1.6497	0.8659	0.4510	1.4714
64	0.4932	6.8554	5.1915	6.0287	-2.2153	1.6317
65	0.7927	4.5208	-0.2347	0.0271	-2.6058	-3.9375
66	1.0484	9.7704	-0.2351	-9.0415	11.4363	0.0131
67	1.3228	9.6956	0.6411	-7.6408	11.7093	1.1856
68	2.7151	18.6248	16.8378	16.9468	-12.4889	13.8047
69	0.3425	16.9464	-11.8791	5.1014	-7.8788	-19.2191
70	16.1257	24.8574	-4.2586	-24.9544	24.7880	-15.2163

Table 5. Real solutions 36-70