Acoustic Effect and the Polarization of Adjacent Bits in Soliton Communication Lines

Ekaterina A. Golovchenko and Alexei N. Pilipetskii

***Abstract*—The excitation of radial and torsional-radial acoustic modes by solitons is analyzed depending on light polarization. The theoretical study of the timing jitter and bit error rate caused by the acoustic effect in soliton communication lines for co- and counter-polarized adjacent bits is presented. The influence of guiding filter on the acoustically induced timing jitter is calculated.**

I. Introduction

One of the main mechanisms that limits the bit rate and transmission distance in soliton communication lines [1], [2] is the so-called "long-range" interaction of solitons [3]. The physical reason of this interaction is the electrostrictional excitation of the acoustic waves by light pulses and back-influence of the acoustic wave on solitons [4], [5]. In soliton communication lines, the interaction of solitons with the acoustic wave results in the addition to the Gordon–Haus effect [6] timing jitter of solitons [7], [8]. Although the timing jitter of solitons can be significantly reduced by guiding filters [9]–[11], the detailed investigation of the acoustic effect is still important because the guiding filters cannot totally suppress the timing jitter.

In previous works [4]–[8], only the excitation of radial acoustic modes and their influence on bit error rate were discussed. But in fact, two types of acoustic modes can be excited by light pulses [12]: radial R_{om} modes for which there is only the radial component of the displacement vector, and mixed torsional-radial modes TR_{2m} with the displacement vector varying as $\cos(2\varphi)$. Spontaneous scattering of light on these groups of modes was experimentally observed in [13]. In this paper, we discuss the excitation of radial R_{om} and torsional radial TR_{2m} acoustic modes by solitons depending on their polarization state. Then we calculate the difference in values of timing jitter and bit error rates for copolarized and counterpolarized adjacent bits in soliton communication line due to the acoustic effect. The guiding filter reduction factor for the acoustically induced timing jitter of solitons is derived.

II. Theory

The main equation describing the acoustic vibrations for the components of displacement vector U_i reads as [12]:

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The authors are with the Department of Electrical Engineering, University of Maryland, 5401 Wilkens Avenue, Baltimore, MD 21228-5398 USA.

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$$\rho_0 \partial^2 U_i/\partial^2 t - C_T^2 \partial^2 U_i/\partial x_k^2 + (C_L^2 - C_T^2)\partial^2 U_k/\partial x_k \partial x_i = \partial\sigma_{ik}/\partial x_k \quad (1)$$

where ρ_0 is material density of fiber, $C_L = 5.99 \times 10^5$ cm/sec is the velocity of longitudinal acoustic wave in silica glass, $C_T = 3.74 \times 10^5$ cm/sec is the velocity of shear acoustic wave, σ_{ik} is the electrostrictional stress tensor:

$$\sigma_{ik} = -1/8\pi(2P_{11}\epsilon^2 E_i E_k + P_{12}\epsilon^2 E_l^2 \delta_{ik}) \quad (2)$$

where E_i is the electric field component. $P_{11} = 0.121$, $P_{12} = 0.27$ are the strain-optic coefficients for fused quartz and $\epsilon = 2.1$ is the dielectric permittivity, δ_{ik} is the Kroneker delta. After some calculations one can obtain the expressions for components $f_i = \partial\sigma_{ik}/\partial x_k$ of electrostrictional force in a cylindrical coordinate system

$$f_r = -\epsilon^2/8\pi \partial F/\partial r\{I(P_{11} + P_{12}) + P_{11}(I_x - I_y)\cos(2\varphi)\}$$

$$f_\varphi = \epsilon^2/8\pi \partial F/\partial r P_{11}(I_x - I_y)\sin(2\varphi) \quad (3)$$

here $I_x \equiv |E_x|^2/2$, $I_y \equiv |E_y|^2/2$, $I = I_x + I_y$, and x, y axes are parallel to light polarization ellipse axes. Function $F(r)$ describes the intensity of electric field mode distribution. Deriving (3), we dropped the terms containing double light frequency 2ω for the components f_r and f_φ.

From (3), one can conclude that only two types of acoustic modes can be excited by light pulses: R_{om} are the modes with displacement vector depending only on radial coordinate r and TR_{2m} are the modes with displacement vector depending also on angle coordinate φ as $\cos(2\varphi)$ and $\sin(2\varphi)$. Analytical forms for radial dependencies of the displacement vectors for R_{om} and TR_{2m} modes can be found in [13]. The excitation of radial R_{om}-modes does not depend on polarization state, but the excitation of torsional-radial TR_{2m}-modes depends on polarization state of light. This excitation is most efficient for linear state of polarization (I_x or I_y is equal to zero) and is absent for circular state of polarization ($I_x = I_y$).

To solve (1), we first rewrite it in spectral domain, and then decompose the spectral component of the displacement vector $U_{i\Omega}$ on the eigenfunctions that coincide with the acoustic modes of the fiber as follows:

$$U_{i\Omega} = \sum_m A_m^{(j)}(\Omega) U_{mi}^{(j)} \quad (4)$$

where $j = 0, 2$ and $U_{mi}^{(j)}$ are the components of the displacement vectors of R_{om} and TR_{2m} modes correspondingly. The sum in (4) is taken over radial number of acoustic modes m. One can obtain the coefficients $A_m^{(j)}(\Omega)$ in (4) in the following form:

$$A_m^{(j)} = B_m^{(j)}/(\Omega_m^{(j)2} + 2i\Gamma_m^{(j)}\Omega - \Omega^2) \tag{5}$$

where

$$B_m^{(j)} = \int_0^R \int_0^{2\pi} (f_{i\Omega} U_{mi}^{(j)}) r\, dr\, d\varphi \Big/ \int_0^R \int_0^{2\pi} (U_{mi}^{(j)} U_{mi}^{(j)}) r\, dr\, d\varphi. \tag{6}$$

Deriving (5) we used the orthogonality of the acoustic modes. Also, we introduced here the acoustic mode linewidths $\Gamma_m^{(j)}$, where $j = 0, 2$ for R_{om} and TR_{2m} modes correspondingly. The widths Γ_m are mainly determined by the reflection of acoustic vibrations from the boundary between cladding and external polymeric cladding [13]. By taking the inverse Fourier transform of (4), one can obtain the solution of (1) in temporal domain.

We can now calculate the back influence of acoustic excitations on the light pulses. Excitation of R_{om} acoustic modes leads to isotopic perturbation of the dielectric permittivity as follows:

$$\delta\epsilon_{xx} = \delta\epsilon_{yy} = \epsilon^2(P_{11} + P_{12})(S_{rr} + S_{\varphi\varphi})$$

$$\delta\epsilon_{xy} = \delta\epsilon_{yx} = 0 \tag{7}$$

where $S_{rr} = \partial U_r/\partial r$ and $S_{\varphi\varphi} = U_r/r$ are the components of the strain tensor in cylindrical coordinates. TR_{2m} acoustic modes cause birefringent perturbation of the dielectric permittivity of the fiber. The principal axes of the tensor of this perturbation coincide with ellipse polarization axes. Dropping the terms containing $\cos(2\varphi)$ and $\cos(4\varphi)$, we get

$$\delta\epsilon_{xx} = \epsilon^2 P_{11}/4(\partial U_r^{(2)}/\partial r - \partial U_\varphi^{(2)}/\partial r + U_r^{(2)}/r - U_\varphi^{(2)}/r)$$

$$\delta\epsilon_{yy} = -\delta\epsilon_{xx}, \quad \delta\epsilon_{xy} = \delta\epsilon_{yx} = 0 \tag{8}$$

where $U_r^{(2)}$ and $U_\varphi^{(2)}$ are the radial dependences of the displacement vector for TR_{2m} modes. Using this theory, one can calculate the electrostrictional excitation of acoustic waves by light pulses in single-mode fibers and calculate the interaction of solitons with acoustic wave. In this interaction [7]–[10], light pulses excite acoustic waves resulting in changes of the effective refractive index for the following pulses. The small changes of the effective refractive index of the fiber δn can be calculated with the help of perturbation theory

$$\delta n = (1/2n)\int \delta\epsilon F^2(r)\, ds \Big/ \int F^2(r)\, ds. \tag{9}$$

In due turn, this induced time-dependent refractive index δn leads to the changes in the mean frequencies of the light pulses

$$\delta\omega \approx \frac{\omega}{c}\frac{\partial(\delta n)}{\partial t} Z \tag{10}$$

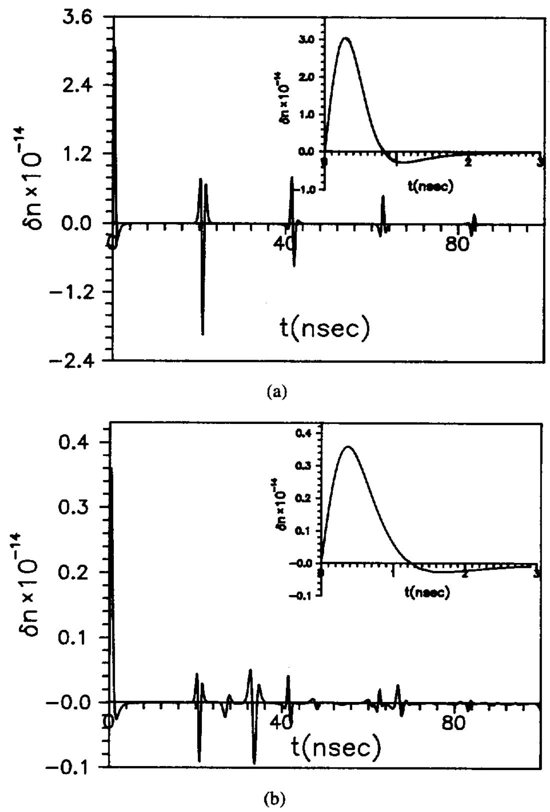

Fig. 1. Acoustic responses to the soliton with duration 50 psec (D = 1 psec/(nm*km) and the effective cross-section of the fiber $A_{\rm eff}$ = 35 μm^2) for (a) R_{om} modes and (b) TR_{2m} modes. The value of the acoustic modes linewidths $2\Gamma_m = 6 \times 10^7$ sec^{-1} was estimated from the experimental data obtained in [13]. The inserted graphs show the first spikes of corresponding acoustic responses.

where Z is the propagation distance. Consequently, this results in the temporal shifts (timing jitter) of light pulses relative to each other [7]–[10].

Acoustic responses $\delta n_R(t)$ and $\delta n_{TR}(t)$ (perturbations of the effective refractive index of light mode) due to excitation of R_{om} and TR_{2m} modes by short light pulse with linear polarization are shown in Fig. 1(a) and (b), correspondingly. The magnitude of acoustic response function is proportional to the energy of short light pulse [8]–[10], and the main addition to the effective refractive index is caused by excitation of radial acoustic modes.

The excitation of TR_{2m} modes depends on the polarization state of light pulses. Solitons in real communication fiber experience all polarization states with equal probabilities due to random linear birefringence [14]. Perturbations in birefringence caused by TR_{2m} acoustic modes are much smaller than random linear birefringence, and they cannot change the polarization state of the soliton. Thus, the acoustic response of TR_{2m} modes should be averaged on polarization states. The influence of the acoustic wave on the soliton results in additional phase shifts (and frequency shifts). This means that to calculate the averaged acoustic response of TR_{2m} modes, one should calculate the averaged on all polarization states phase shift of the light. It can be shown that the averaged acoustic response of the TR_{2m} modes equals, with the coefficient $2/\pi$, the acoustic response for pulse with linear polarization.

Knowing the values of the acoustic responses, we can now discuss the contribution of the acoustic effect to the value of timing jitter of solitons in communication lines. It follows from (8) that light pulses in orthogonal polarizations generate TR_{2m} modes with opposite signs. Thus, the contribution of TR_{2m} modes to the value of timing jitter is smaller for the case of information transmission by counter-polarized adjacent bits than for the case of copolarized adjacent bits. The value of timing jitter variance in the case of copolarized adjacent bits is as follows:

$$\sigma_{\parallel}^2 \cong \left(\frac{D\lambda}{c}\right)^2 \left[K \int_0^{\infty} (\delta n_R'(\theta) + \delta n_{TR}'(\theta))^2 \, d\theta\right] \frac{z^4}{16} \quad (11)$$

and in the case of counter polarized adjacent bits the same value reads as

$$\sigma_{\perp}^2 \cong \left(\frac{D\lambda}{c}\right)^2 \left[K \int_0^{\infty} (\delta n_R'^2(\theta) + \delta n_{TR}'^2(\theta)) \, d\theta\right] \frac{z^4}{16} \quad (12)$$

where $\delta n'_{R,TR}(t)$ are the temporal derivatives of the response functions, D is fiber dispersion, K is the bit rate, z is the propagation distance. Deriving (11) and (12), we were following the approach developed in [7], [8]. Equations (11) and (12) give the values of timing jitter variances that are summing up with Gordon–Haus jitter σ_{GH}. The total resulting timing jitter σ is:

$$\sigma^2 = \sigma_{\rm GH}^2 + \sigma_{\parallel,\perp}^2 \quad (14)$$

Gordon–Haus effect can be significantly suppressed by guiding filters [9]–[11]. One can conclude that the acoustically induced timing jitter should be also suppressed by guiding filters. However the nature of Gordon–Haus and acoustic effects is completely different. That can be seen, for instance, in the different dependences of timing jitter on the propagation length z for these two effects: $\sigma_{\rm GH}^2 \propto z^3$ and $\sigma_{\parallel,\perp}^2 \propto z^4$. Thus, the functional form of guiding filter reduction factor [10] for the acoustic effect should differ from that for the Gordon–Haus effect. Following the derivation developed in [10], we get the following equation for the soliton frequency shift due to the acoustic effect and filter influence:

$$\frac{\partial \Omega}{\partial z} = -\frac{4}{3}\zeta_2 \Omega + \frac{\Omega}{c}\sum \delta n'(t) \quad (15)$$

where ζ_2 is the filter factor [11] and the sum is taken over all the responses of the previous pulses for each given pulse. Solving (15) and calculating the average values of solitons temporal shifts as described in [9] and [10], we obtain the acoustically induced timing jitter variance under the influence of guiding filter

$$\sigma^2 = f_a(x)\sigma_{\parallel,\perp}^2 \quad (16)$$

where $f_a(x) = \frac{4}{x^4}(x - 1 - e^{-x})^2$ is the filter reduction factor for the acoustic effect and $x = 2\alpha z$, where α is the excess amplifier gain. In comparison the reduction factor for Gordon–Haus effect is $f_{\rm GH}(x) = \frac{3}{2x^3}(2x - 3 + 4e^{-x} - e^{-2x})$ [10]. Fig. 2 shows both $f_a(x)$ and $f_{\rm GH}(x)$.

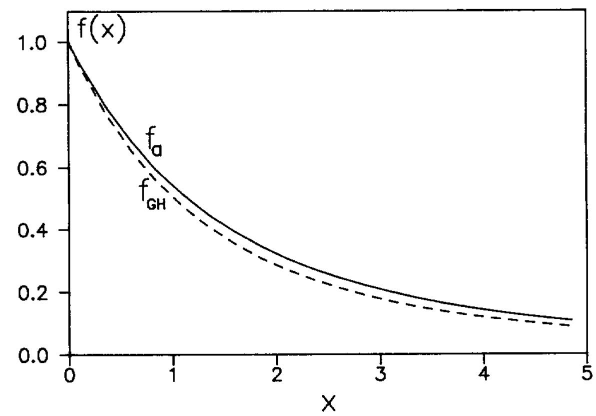

Fig. 2. Reduction factors of the variance of the timing jitter for Gordon–Haus $f_{\rm GH}(x)$ [9] (dots) and acoustic $f_a(x)$ [Eq. (16)] (solid line) effects.

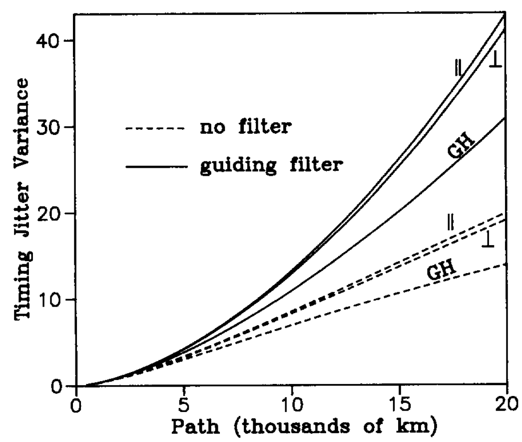

Fig. 3. Timing jitter variance vs distance with guiding filters (solid lines) and with no filtering at all (dashed lines) for three cases: pure Gordon–Haus effect, and total resulting timing jitter variance [Eq. (14)] for Gordon–Haus and acoustic effects for copolarized adjacent bits (∥) and counterpolarized bits (⊥).

III. Results and Discussion

Now we have all the necessary information to calculate the dependence of the timing jitter variance and bit error rate on propagation distance z. To compare our results with the experimental situation, we used the parameters from the experiments of Mollenauer *et al.* [2]: $K = 5$ GBit/s, $D = 0.7$ ps/nm/km, $\tau = 41$ ps, and the effective fiber cross-section $A_{\rm eff} = 50\ \mu{\rm m}^2$. We first calculate $\sigma_{\rm GH}(z)$ for the pure Gordon–Haus effect, and then we find the value of the total timing jitter variance for the cases of the adjacent bits transmitted in co- and counter-polarizations $\sigma_{\parallel,\perp}(z)$. The dependences $\sigma_{\rm GH,\parallel,\perp}(z)$ are shown in Fig. 3. Calculating $\sigma_{\parallel,\perp}(z)$ we took into account only the influence of the first spike of the response function. Such an assumption is justified by the action of the variations of fiber cladding diameter along

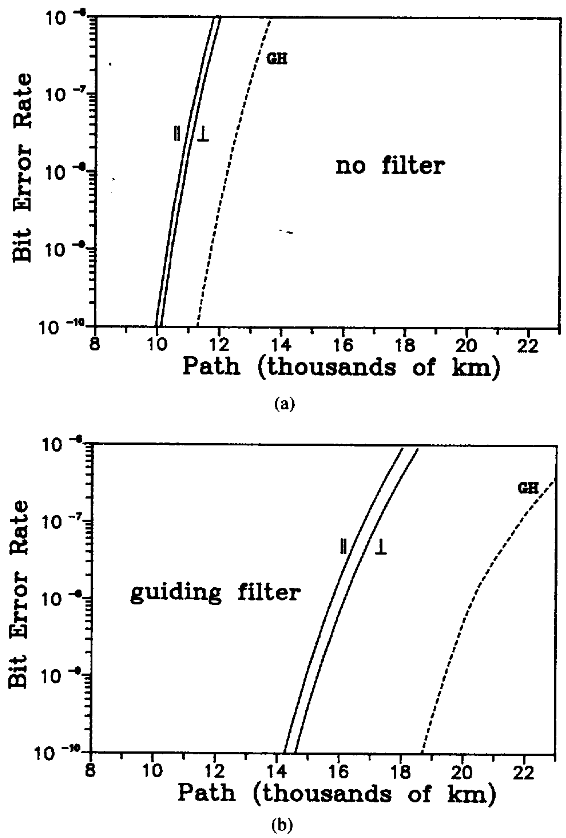

Fig. 4. (a) Dependence of bit error rates on propagation distance: dashed line—pure Gordon–Haus effect, solid lines total BER from Gordon–Haus and acoustic effects (copolarized adjacent bits—|| and counter-polarized bits—⊥). (b) The same, but under the influence of guiding filter.

the fiber length on the response function [12]. The random cladding diameter variations result in random variations of the eigenfrequencies of the acoustic modes along the fiber. This, in turn, leads to variations in the time delay in the appearance of the subsequent spikes and to averaging of the second and subsequent spikes of the response function [12]. The solid lines correspond to a communication line without guiding filters and the dots show the influence of guiding filter. Fig. 4(a) shows the bit error rate without guiding filter and Fig. 4(b) gives the same value under the influence of guiding filter.

The acoustic effect is essentially contributing to the value of timing jitter and bit error rate. It is worth noting, that with the increase of the transmission distances and bit rates, the acoustically induced timing jitter can become larger than that due to the Gordon–Haus effect. Our calculations also reveal the difference in the timing jitter values for co- and counter-polarized adjacent bits, which can be explained by the difference in contribution of TR_{2m} acoustic modes. However, the experimentally [11] observed difference in bit error rates for co- and counter-polarized bits was somewhat larger, than produced by our theoretical calculations. Such a discrepancy results from the fact, that in our calculations, we were not taking into account the direct Kerr-like interaction of solitons. In reality, the Kerr soliton interaction is also contributing to the value of bit error rate, and the strength of this interaction is weaker for counter-polarized solitons.

IV. Conclusion

We have presented a detailed theory of the acoustic effect in soliton communication lines. Two types of the acoustic modes can be excited by solitons: R_{om} and TR_{2m} modes. The main contribution to the acoustically induced soliton timing jitter is determined by R_{om} modes. The contribution of TR_{2m} modes depends on the relative polarization of the adjacent bits, and it is weaker for the adjacent bits in counter-polarizations. Utilization of guiding filters is suppressing both Gordon–Haus and acoustic effects, but the functional form of the guiding filter reduction factor of the timing jitter for the acoustic effect, which we have calculated, is different from that for the Gordon–Haus effect.

The theory can be applied to estimations of the bit error rates in long-distance soliton communication lines. It also provides a better understanding of the nonlinear phenomena in soliton transmission systems.

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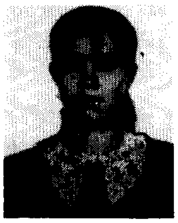
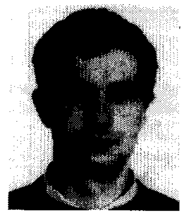


Ekaterina Golovchenko was born in Moscow, U.S.S.R. on July 30, 1962. She received the B.S. degree in physics from the Physical Department, Moscow State University, Moscow, U.S.S.R., in 1985 and the Ph.D. degree from the General Physics Institute, Academy of Sciences of the U.S.S.R., Moscow, in 1991. Her research interests include Raman stimulated scattering, solitons, modulational instability and ultrashort pulse generation in optical fibers.

In 1985, she joined the Fiber Optics Department, General Physics Institute, Academy of Sciences of Russia, as a Research Scientist. Since 1994 she has been with the University of Maryland.



Alexei Pilipetskii was born in Moscow, U.S.S.R. on March 23, 1962. He received the B.S. degree in physics from the Physical Department, Moscow State University, Moscow, U.S.S.R., in 1985 and the Ph.D. degree from the General Physics Institute, Academy of Sciences of the U.S.S.R., Moscow, in 1990. His research interests include Brillouin and Raman stimulated scatterings, solitons, and modulational instability in optical fibers.

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