Beam steering by $\chi^{(2)}$ trapping

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Received April 24, 1995

We show numerically the mutual dragging and locking together of intense fundamental and second-harmonic light beams propagating in second-order nonlinear media, and we discuss the potential applications of such effects for the steering of beams by spatial phase modulation of the input signals. © 1995 Optical Society of America

All-optical, parametric interactions of intense light signals in materials with second-order nonlinearities offer a rich variety of new opportunities for controlling light by light. One important example is the formation of solitons (more properly, solitonlike waves).[1–6] Both (1 + 1) solitons (i.e., one transverse dimension and one propagation dimension) and higher-dimensional confinement exist in bulk crystals and in planar waveguides made of $\chi^{(2)}$ media. Temporal solitons appear to be more difficult to form with currently available experimental conditions, but (2 + 1) and (1 + 1) bright spatial solitons haves already been observed by Torruellas *et al.*[7] and by Schiek *et al.*[8] in second-harmonic generation configurations in bulk KTP crystal and in a planar $LiNbO_3$ waveguide, respectively. The $\chi^{(2)}$ solitons form through the mutual trapping of the parametrically interacting waves, and the dynamics of their formation exhibits the unique features of highly nonlinear waves. One fascinating example is the mutual dragging of two intense fundamental and second-harmonic beams. In such a process, two beams whose low-power counterparts would propagate in different directions interact through the $\chi^{(2)}$ medium. Consequently the beams stick to each other and are mutually trapped, so they propagate in a common direction with the beams stuck (or locked) together. This is in some sense similar to the so-called soliton trapping that occurs in $\chi^{(3)}$ media,[9] even though both the fundamental and the second-harmonic waves are required for formation of a single soliton in $\chi^{(2)}$ media, in contrast to $\chi^{(3)}$ soliton trapping in which case two solitons can trap each other and form a single bound state. To differentiate between the two cases we propose the term beam locking for the mutual dragging and trapping of beams in $\chi^{(2)}$ media. Our goals here are to investigate the dynamics of such a process and to show that it has potential applications for the steering of beams by spatial phase modulation of the input waves.[10]

We consider continuous-wave light beams traveling in a medium with a large $\chi^{(2)}$ nonlinearity, and we study both (1 + 1) and (2 + 1) trapping. In the slowly varying envelope approximation the beam evolution can be described by the normalized equations[3,4]

$$i\,\frac{\partial a_1}{\partial \xi} - \frac{r}{2}\,\nabla_\perp{}^2 a_1 + a_1{}^* a_2 \exp(-i\beta\xi) = 0\,,$$
$$i\,\frac{\partial a_2}{\partial \xi} - \frac{\alpha}{2}\,\nabla_\perp{}^2 a_2 - i\delta\hat{\delta}\cdot\nabla_\perp a_2 + a_1{}^2 \exp(i\beta\xi) = 0\,, \tag{1}$$

where a_1 and a_2 are the amplitudes of the fundamental and the second-harmonic waves, $r = -1$, and $\hat{\delta}$ is a unitary vector along the walk-off axis. The parameters α, β, and δ are given by the ratios of the coherence length ($l_c = \pi/|\Delta k|$), the diffraction length ($l_d = k\eta^2/2$), and the walk-off length ($l_w = \eta/\rho$). Here k is the wave vector at both frequencies, $\Delta k = 2k_1 - k_2$ is the wave-vector mismatch, ρ is the walk-off angle, and η is the beam width. One has $\alpha = -l_{d1}/l_{d2}$, $\delta = \pm 2l_{d1}/l_w$, and $\beta = \mathrm{sgn}(\Delta k)2\pi l_{d1}/l_c$. The transverse coordinates are given in units of η, and we set for the propagation coordinate $z/l_{d1} = 2\xi$. For relevant experimental conditions, say, $l_c \sim 2.5$ mm, $\rho \sim 1°$, and $\eta \sim 15\ \mu$m, which yield a diffraction length $l_d \sim 1$ mm, one obtains $\alpha \simeq -0.5$, $\delta \sim \pm 1$, and $\beta \sim \pm 3$. Here we set $\alpha = -0.5$, which is representative of most situations.

We make use of three conserved quantities of the beam evolution: the total power flow, the total transverse beam momentum, and the Hamiltonian, which are given by

$$I = I_1 + I_2 = \int \{|A_1|^2 + |A_2|^2\}\mathrm{d}r_\perp\,, \tag{2}$$

$$\boldsymbol{J} = \boldsymbol{J}_1 + \boldsymbol{J}_2 = \frac{1}{4i}\int \{2(A_1{}^*\nabla_\perp A_1 - A_1\nabla_\perp A_1{}^*) + (A_2{}^*\nabla_\perp A_2 - A_2\nabla_\perp A_2{}^*)\}\mathrm{d}r_\perp\,, \tag{3}$$

$$\mathcal{H} = \frac{-1}{2}\int\Big\{r|\nabla_\perp A_1|^2 + \frac{\alpha}{2}\,|\nabla_\perp A_2|^2 - \beta|A_2|^2 + i\,\frac{\delta}{2}\,\hat{\delta}\cdot(A_2\nabla_\perp A_2{}^* - A_2{}^*\nabla_\perp A_2) + (A_1{}^{*2}A_2 + A_1{}^2A_2{}^*)\Big\}\mathrm{d}r_\perp\,, \tag{4}$$

0146-9592/95/191952-03$6.00/0

with $A_1 = a_1$ and $A_2 = a_2 \exp(-i\beta\xi)$. Solitonlike wave propagation described by Eqs. (1) occurs for a variety of input conditions, with different wave-vector mismatches and linear walk-off.[4]

Analytical solutions of the $(1 + 1)$ version of Eqs. (1) with nonzero transverse velocity can be found. At $\alpha = -0.5$, solutions with the form $a_j(\xi, r_\perp) = U_j(\xi - vr_\perp)\exp[i(\kappa_j\xi - \mu_j r_\perp)]$ exist when $\delta = 0$, with $v = r\mu_1$, $\kappa_2 = 2\kappa_1 + \beta$, and $\mu_2 = 2\mu_1$. Those solutions are a two-parameter family because μ_1 is arbitrary and the wave intensity defines a one-parameter family through κ_1.[5,6] Away from $\alpha = -0.5$ the existence of a two-parameter family is suggested by Menyuk's robustness hypothesis of solitons.[4] We emphasize that for those solutions one has $\mu_2 = 2\mu_1$ at the input face of the $\chi^{(2)}$ medium. However, we show that nonzero-velocity solitons can be excited by different conditions, in particular, by the mutual dragging of the interacting beams.

To elucidate how the wave interaction yields beam dragging, we examine the evolution of the energy centroid $\boldsymbol{\sigma}(\xi) = \int \mathbf{r}_\perp\{|a_1|^2 + |a_2|^2\}\mathrm{d}r_\perp$. One can readily show that

$$\frac{\mathrm{d}\boldsymbol{\sigma}(\xi)}{\mathrm{d}\xi} = \boldsymbol{J} - \delta I_2(\xi)\hat{\delta} - \epsilon \boldsymbol{J}_2(\xi)\,, \tag{5}$$

with $\epsilon = 2\alpha + 1$. At phase matching one has $\alpha = -0.5$, and the last term vanishes. Otherwise it is very small (for the above-mentioned experimental conditions, $\epsilon \sim 5 \times 10^{-5}$). Hence the first two terms capture the main physics of the beam evolution. The second term shows that, for a fixed value of the walk-off parameter δ, the location of the energy centroid is governed by the second-harmonic power flow, which is not a conserved quantity. That shows how the evolution of the energy centroid is sensitive to the dynamics of the soliton excitation in terms of the input signals, such as intensities and beam shapes at both frequencies and relative phases between both waves, and also in terms of the magnitude and the sign of the wave-vector mismatch. The first term shows that one may modify the location of the energy centroid by controlling the transverse momentum of the input beams. Writing the fields in the form $a = U\exp(i\phi)$, with U and ϕ being real quantities, one obtains $\boldsymbol{J} = (1/2)\int\{2U_1{}^2\nabla_\perp\phi_1 + U_2{}^2\nabla_\perp\phi_2\}\mathrm{d}r_\perp$. Thus different values of $\boldsymbol{J}$ may be obtained by spatial phase modulation of the input beams. That can be accomplished by a spatial light modulator placed at the entrance face of the $\chi^{(2)}$ medium. Here we restrict ourselves to the simplest case, given by a phase-front tilt, which corresponds to input beams entering the $\chi^{(2)}$ medium at slightly different directions.

To investigate the dynamics of the beam dragging and to show that mutual trapping exists with different conditions and phase-front tilts, we conducted a series of numerical experiments by solving Eqs. (1) with a split-step Fourier approach. We chose as the input signal $a_1 = AU(r_\perp)\exp(i\mu\hat{x}\cdot\mathbf{r}_\perp)$ and $a_2 = BU(r_\perp)\exp(i\nu\hat{x}\cdot\mathbf{r}_\perp)$, where $U(r_\perp)$ is the transverse profile of the input beams, A and B are their amplitudes, and μ and ν are the angles of the phase-front tilt. Notice that beam dragging as predicted by Eq. (5) is useful to our present purposes only when a solitonlike wave emerges carrying most of the input energy, so that the phase tilts of the input beams amount to moderate values. We begin by analyzing $(1 + 1)$ trapping, and we set $U(r_\perp) = \mathrm{sech}^2(r_\perp)$. For $\beta = -3$, with $A = 3/\sqrt{2}$, $B = 3$, and $\nu = 2\mu$, such an input excites an exact solution of Eqs. (1). First we illustrate the effects predicted by the second term of Eq. (5) by analyzing the soliton formation with linear walk-off and input beams with no phase-front tilt. Figure 1 shows the fundamental beam at $\xi = 20$ for different values of the phase mismatch with fixed input intensities and linear walk-off. In agreement with Eq. (5), the plot shows that mutual beam dragging depends sensitively on the phase mismatch because so do the excited solitons.[4–6]

In a second set of numerical experiments we investigate the beam evolution with different values of the angles μ and ν in a configuration with negligible walk-off (i.e., $\delta \simeq 0$). Figure 2 shows the outcome at both

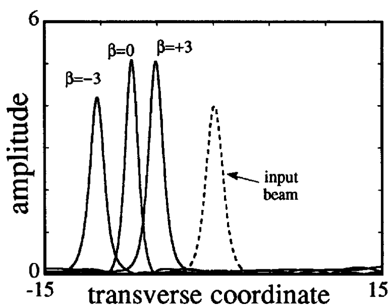

Fig. 1. Steering by $(1 + 1)$ trapping: excitation of a spatial soliton with fixed linear walk-off and input intensities, for different values of the phase mismatch. The plot shows the fundamental beam at $\xi = 20$. The second harmonic propagates stuck together with the fundamental, and it has not been plotted. In all cases $\delta = 1$, $A = 4$, and $B = 4$.

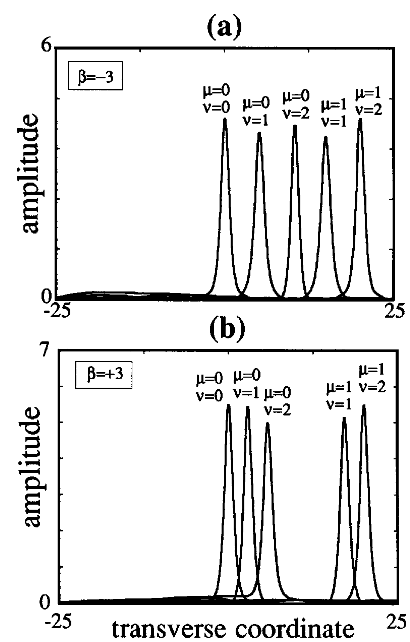

Fig. 2. Steering by $(1 + 1)$ trapping: fundamental beam at $\xi = 20$ of a spatial soliton excited by input beams with transverse phase modulation. (a) $\beta = -3$, $A = 4$, $B = 4$; (b) $\beta = 3$, $A = 5$, $B = 3$.

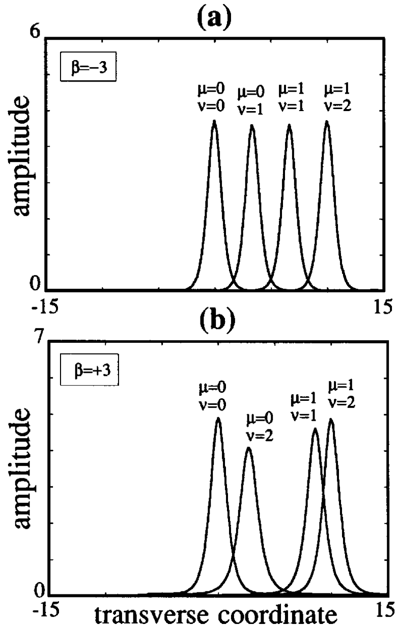

Fig. 3. Steering by (2 + 1) trapping: fundamental beam at $\xi = 10$ of a trapped state excited with Gaussian beams with transverse phase modulation. (a) $\beta = -3$, $A = 3$, $B = 5$; (b) $\beta = 3$, $A = 5$, $B = 3$.

signs of the phase mismatch. The plots show how the beams drag and trap each other, so that for moderate initial angular deviations they stick together and a soliton with nonzero transverse velocity is excited. Large values of ν with $\mu = 0$ prevent the formation of a soliton by making the interaction length between the beams too short. The excitation conditions have been chosen in such a way that they fall close to a minimum of the Hamiltonian. Because the stationary, zero-velocity solutions occur at the minimum of $\mathcal{H}$ for a given I, this choice minimizes the amount of radiation generated. The same procedure can be followed with any shapes of the input beams.

In a third set of numerical experiments we analyze (2 + 1) trapping. We examine Gaussian input beams, so that $U(r_\perp) = \exp(-r_\perp{}^2)$. We chose the beam amplitudes in such a way that the Hamiltonian is close to its minimum value for Gaussian beams. Figure 3 is representative of the beam evolution with different values of μ and ν. It shows a slice of the fundamental beam after propagating 20 diffraction lengths. The plot shows results similar to those for (1 + 1) trapping, but the dynamics of the beam evolution is different in both cases. In particular, for $\mu = 0$, mutual trapping is possible with larger values of ν in (1 + 1) than in (2 + 1). Regarding the experimental implications of our results, we notice that different propagation directions in the $\chi^{(2)}$ medium might correspond also to significantly different values of the linear phase mismatch between the interacting waves, except when either noncritical or quasi-phase matching occurs.

In conclusion, we have investigated the trapping of fundamental and second-harmonic beams propagating in second-order nonlinear media. We have observed numerically the dragging and locking together of intense beams whose low-power counterparts would propagate in different directions, and we have discussed briefly the implications of such effects for the steering of beams by spatial phase modulation of the input signals.

L. Torner acknowledges support from the Polytechnic University of Catalonia funded through grant PR9414. Support by Fundacio Catalana per la Recerca and the Centre de Supercomputacio de Catalunya through the allocation of supercomputer resources is also gratefully acknowledged. We thank Ewan M. Wright for many valuable discussions.

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