Efficient and Accurate Computation of Eye Diagrams and Bit Error Rates in a Single-channel CRZ System

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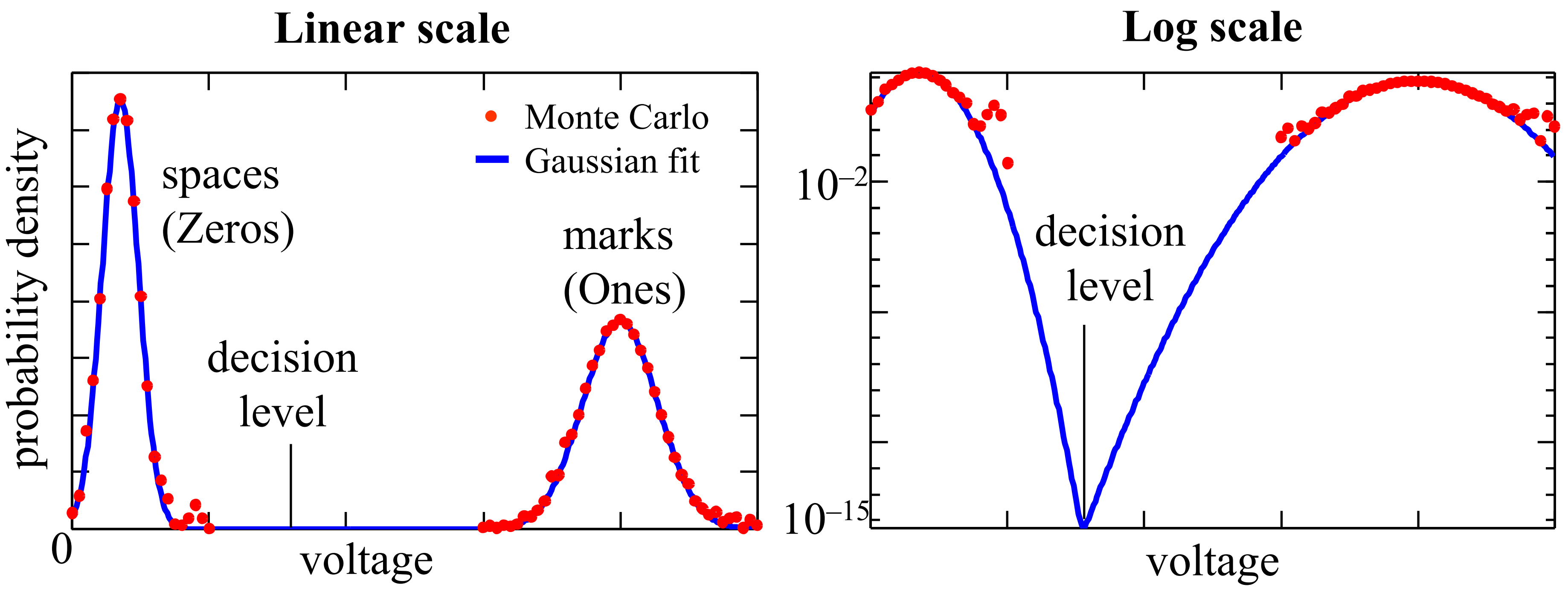
Calculating Bit Error Rates

BERs are commonly calculated from Monte Carlo simulation results using Gaussian extrapolation



But: voltage probability densities are not Gaussian [Marcuse 1990]

Outline

Goal

Accurate calculation of BER vs. decision level

1. Linearize noise propagation
2. Include signal-noise beating and data modulation
3. Model a realistic electrical receiver filter

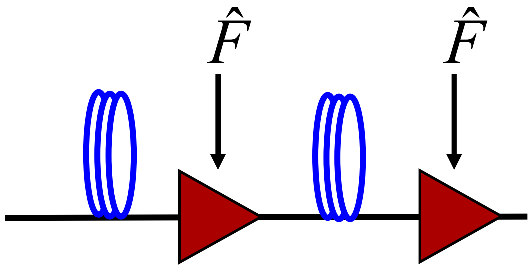
Approach

Calculate the multivariate Gaussian noise pdf of the optical field
Justification: Noise-noise interaction in the fiber is small

Linearizing the NLS

Nonlinear Schrödinger equation with ASE noise

$$i\frac{\partial u}{\partial z} + \frac{D(z)}{2}\frac{\partial^2 u}{\partial t^2} + |u|^2 u = ig(z)u + \hat{F}$$



$\hat{F}$: added Gaussian white noise

Now set $u = u_0 + \delta u,\ u_0 = \langle u \rangle$: noise-free signal

δu : accumulated noise

$$i\frac{\partial \delta u}{\partial z} + \frac{D(z)}{2}\frac{\partial^2 \delta u}{\partial t^2} + 2|u_0|^2 \delta u + u_0^2 (\delta u)^* = ig(z)\delta u + \hat{F}$$

Doob's Theorem: δu is multivariate Gaussian distributed

Noise Covariance Matrix

$$\delta u(t) = \sum_{k=-N_{FFT}/2}^{N_{FFT}/2-1} \left[\alpha_k + i\beta_k\right] \exp\left(i\omega_k t\right), \qquad N_{FFT} = 2048$$

$$\boldsymbol{a} = \left(\alpha_{-N/2}, \ldots, \alpha_{N/2-1}, \ \beta_{-N/2}, \ldots, \beta_{N/2-1}\right)^{\mathrm{T}}, \quad N = 80$$

Covariance matrix $\mathbf{K}_{kl} = \left\langle \boldsymbol{a}_k \boldsymbol{a}_l \right\rangle, \quad \mathbf{K} = \left\langle \boldsymbol{a}\boldsymbol{a}^{\mathrm{T}} \right\rangle$

Multivariate Gaussian distribution of a:

$$f_{\boldsymbol{a}}(\boldsymbol{a}, z) = \left(2\pi\right)^{-2N} \sqrt{\det \mathbf{K}^{-1}} \ \exp\left(-\tfrac{1}{2} \boldsymbol{a}^{\mathrm{T}} \mathbf{K}^{-1} \boldsymbol{a}\right)$$

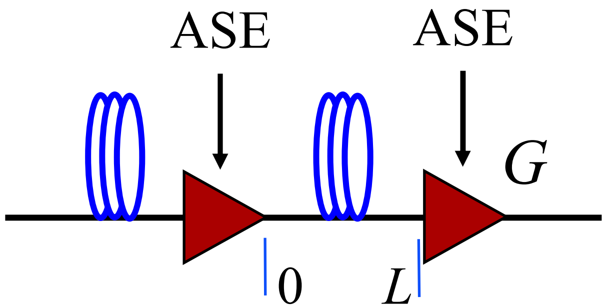
How to Compute the Covariance Matrix

Solve the linearized homogeneous propagation equation

$$\frac{\partial \boldsymbol{a}}{\partial z} = \mathbf{R}(z)\boldsymbol{a} \quad \Rightarrow \quad \boldsymbol{a}(L) = \boldsymbol{\Psi}\,\boldsymbol{a}(0)$$

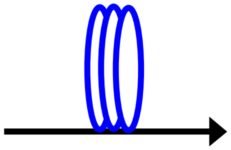
$$\mathbf{K}(L) = G\,\boldsymbol{\Psi}\mathbf{K}(0)\boldsymbol{\Psi}^{\mathrm{T}} + \eta\,\mathbf{I}$$

↑ ASE input



But: ODE is stiff due to dispersion term. Solution: perturbative approach

$$\boldsymbol{a}^{(k)}(0) = \varepsilon \hat{e}_k$$

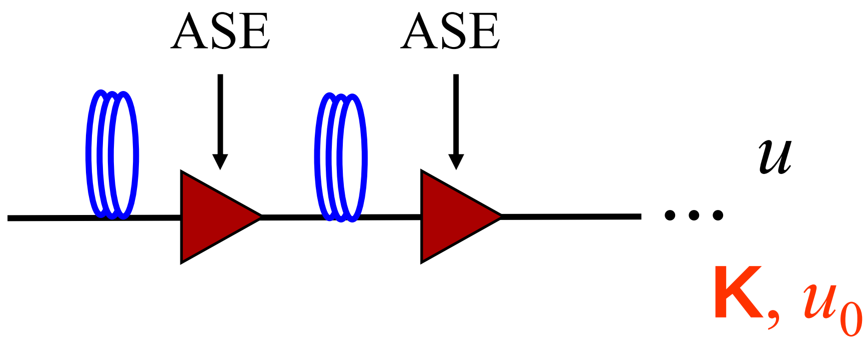


$$\boldsymbol{a}^{(k)}(L) \Rightarrow \boldsymbol{\Psi}_{jk} = \frac{\boldsymbol{a}_j^{(k)}(L)}{\varepsilon}$$

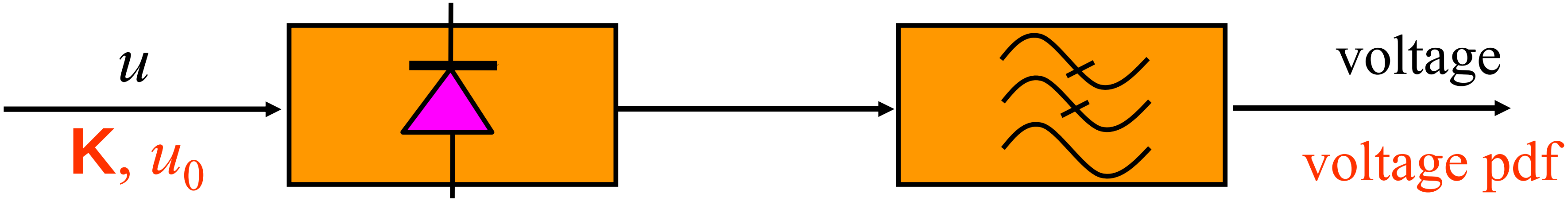
Compute Ψ by perturbing each of the N frequency modes separately

Simulation Setup

Propagation:



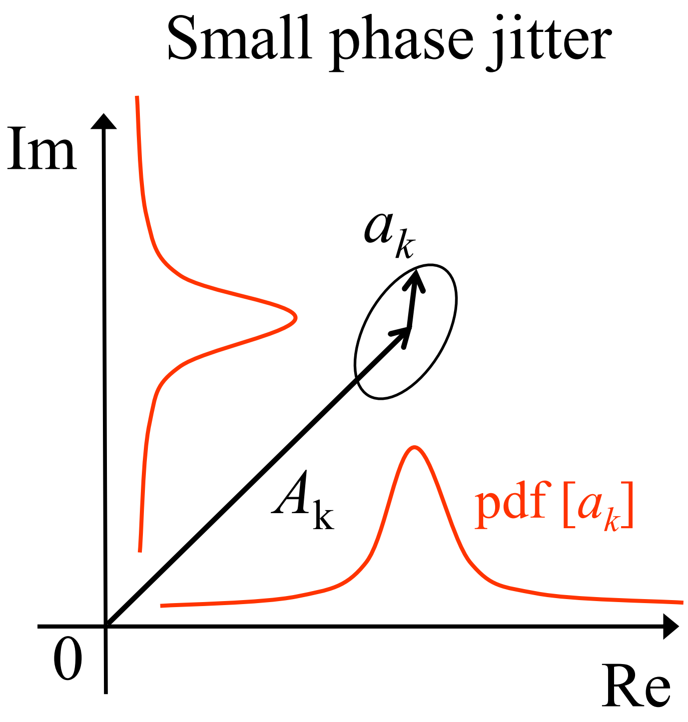
Receiver model:

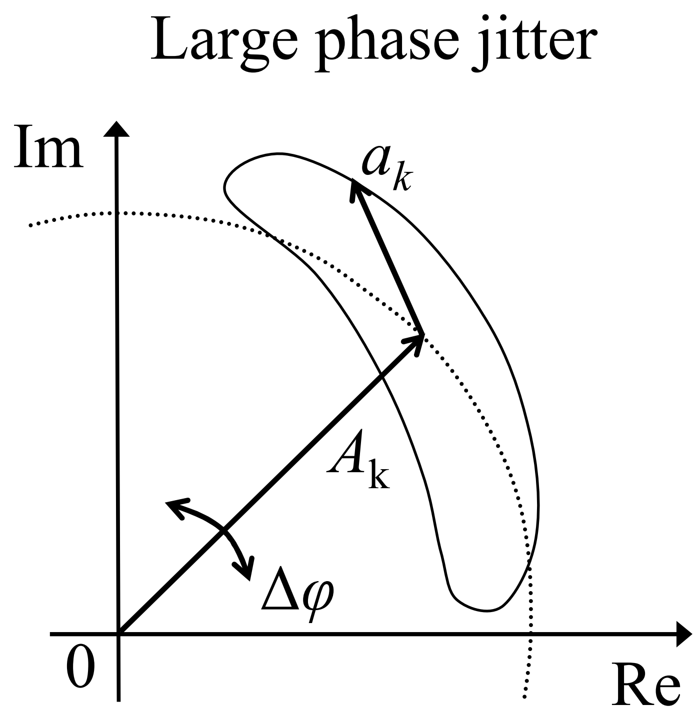


Square-law detector   Narrow-band filter (Bessel)

Quadratic noise-noise terms in the receiver cannot be neglected !

Strong Jitter Distorts the Gaussian pdf





Separate phase jitter from $\boldsymbol{a}^{(k)}(L)$

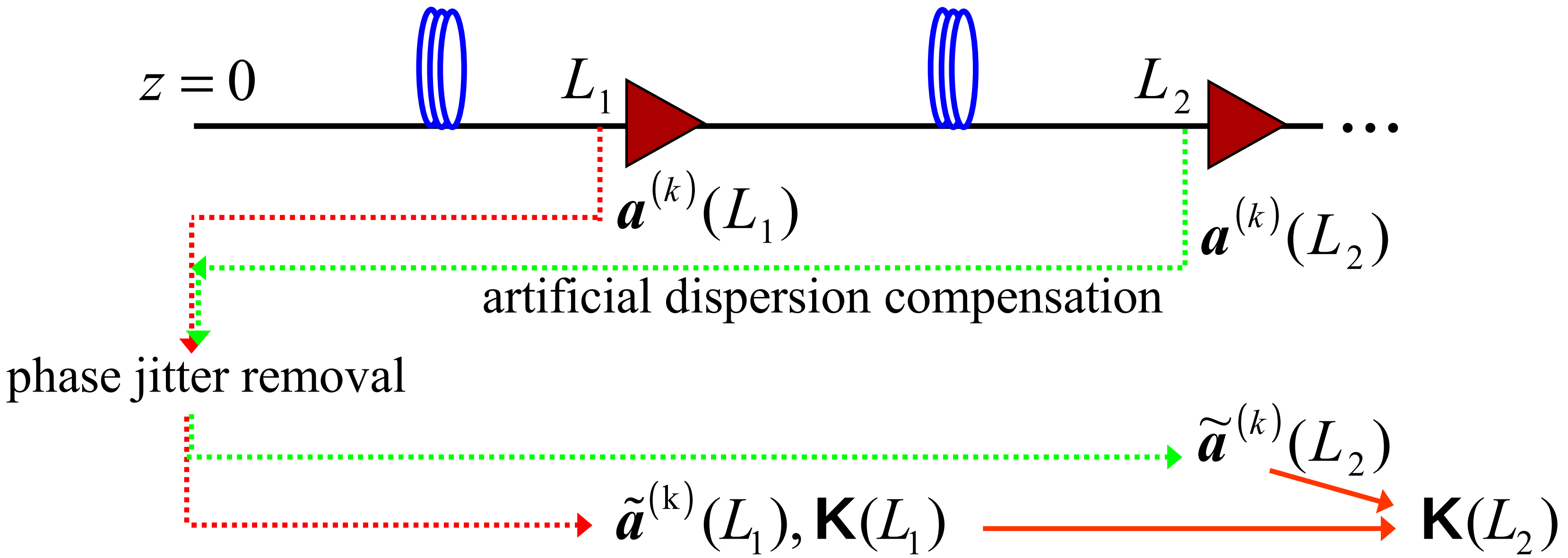
Phase jitter rotates signal around origin, distorting the Gaussian pdf

Phase Jitter Removal

Remove phase jitter by projecting out the noise proportional to iu_0

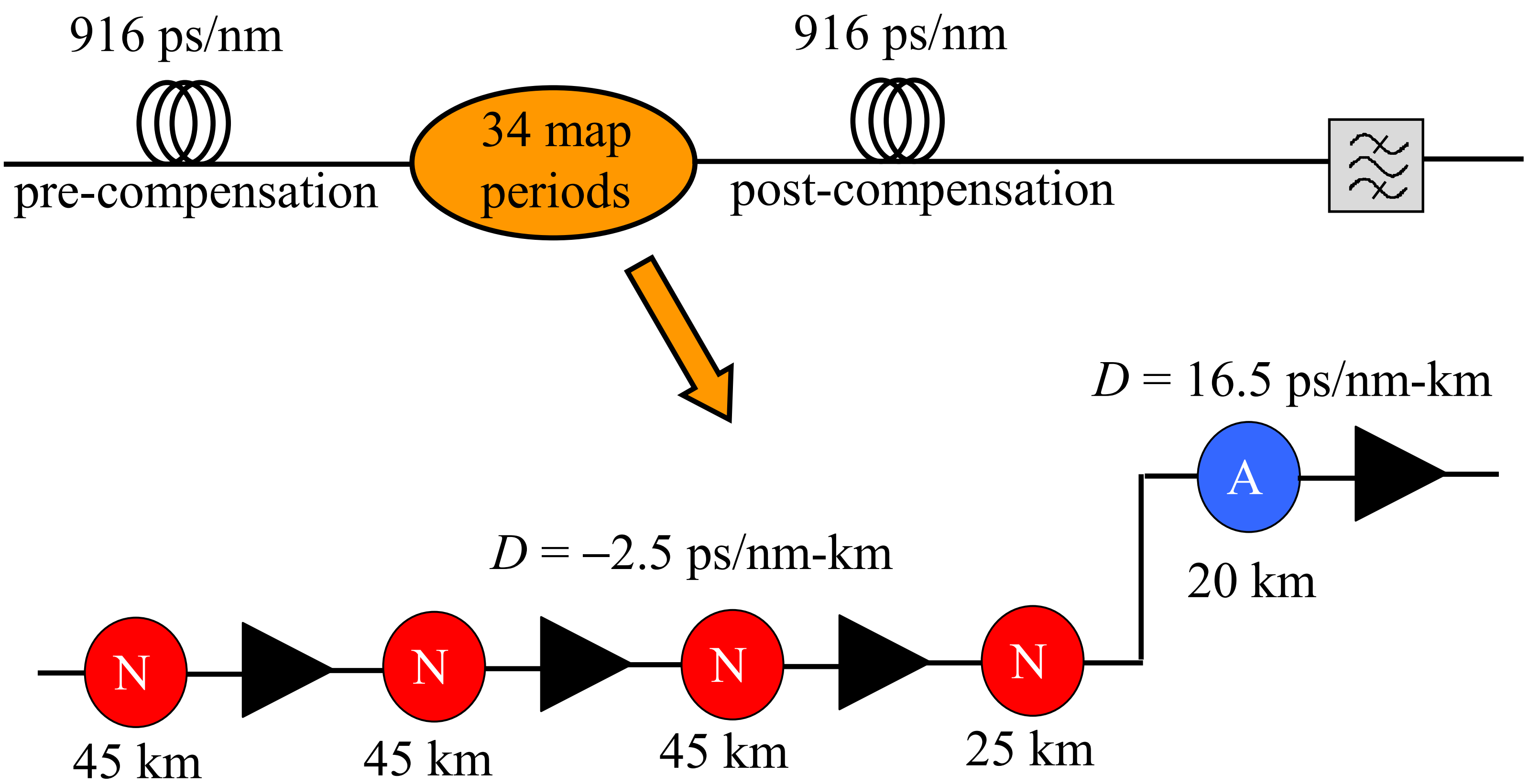
$$\tilde{\boldsymbol{a}}^{(k)} = \boldsymbol{a}^{(k)} - \frac{\left(\boldsymbol{a}^{(k)}, \boldsymbol{v}\right)}{\left(\boldsymbol{v}, \boldsymbol{v}\right)} \boldsymbol{v}, \qquad \boldsymbol{v} = \mathrm{FT}\left\{ iu_0(t) \right\}$$

Jitter removal requires artificial dispersion compensation in CRZ:



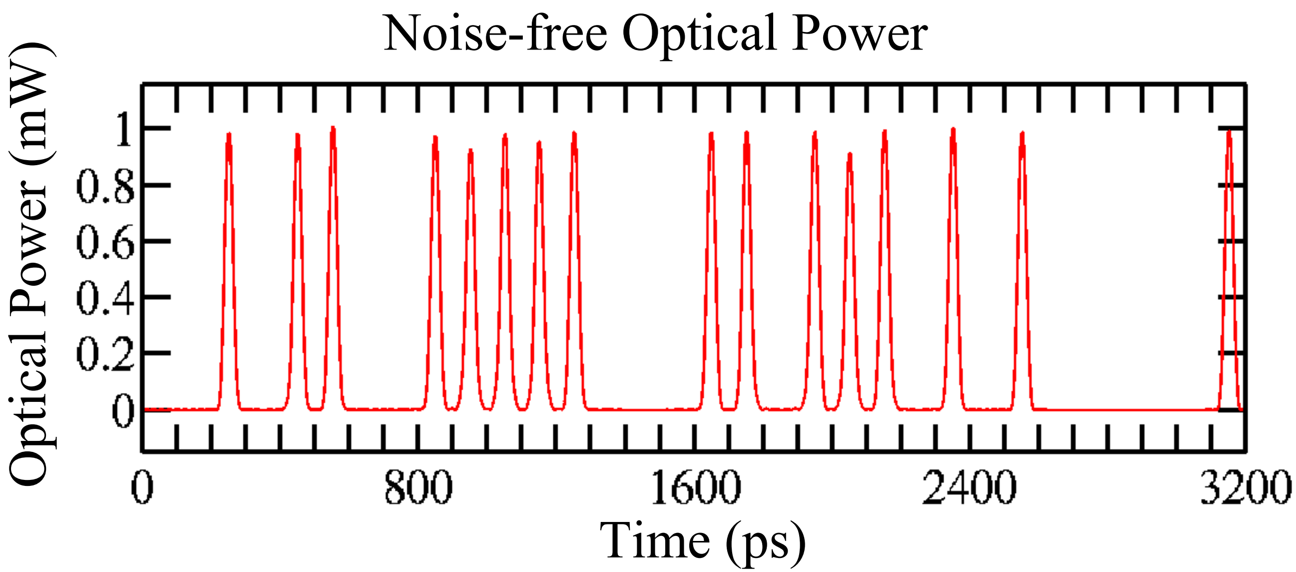
Test System: Submarine CRZ, 6100 km

Modeled on a transatlantic communications system by Tyco Communications, Inc.

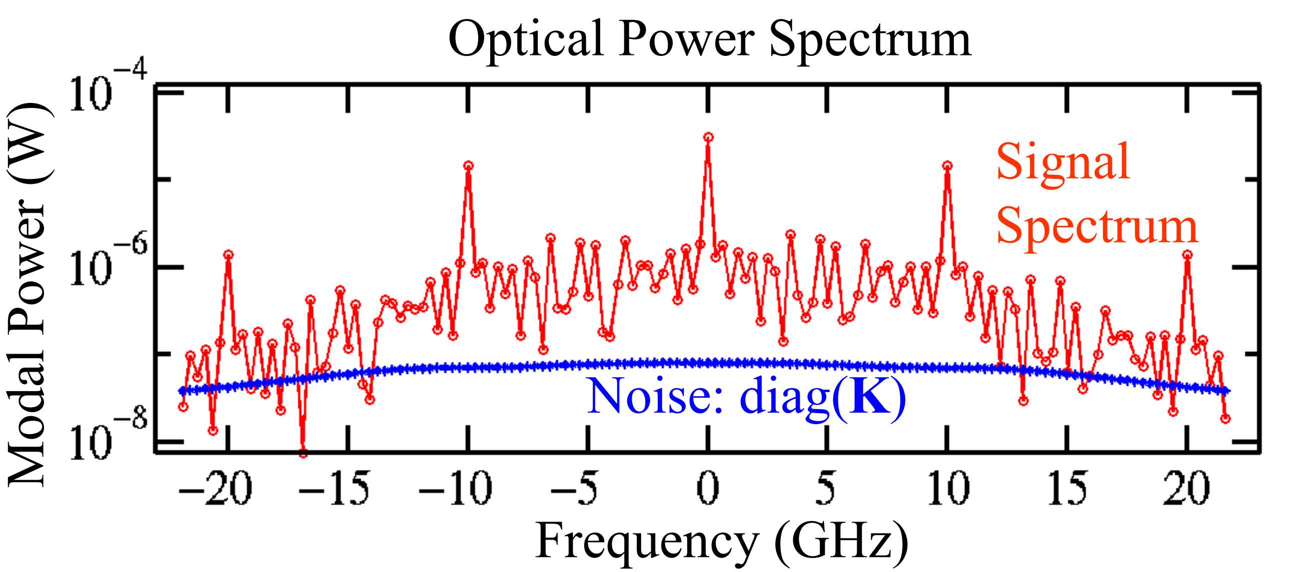


Non-periodic evolution: medium nonlinearity, but strong pulse overlap

Noise-Free Optical Signal at Receiver



32 bits PRBS



OSNR: 11.7 dB

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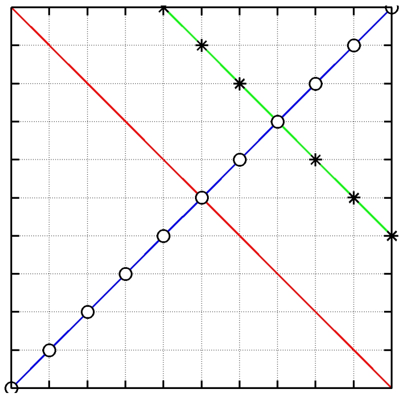
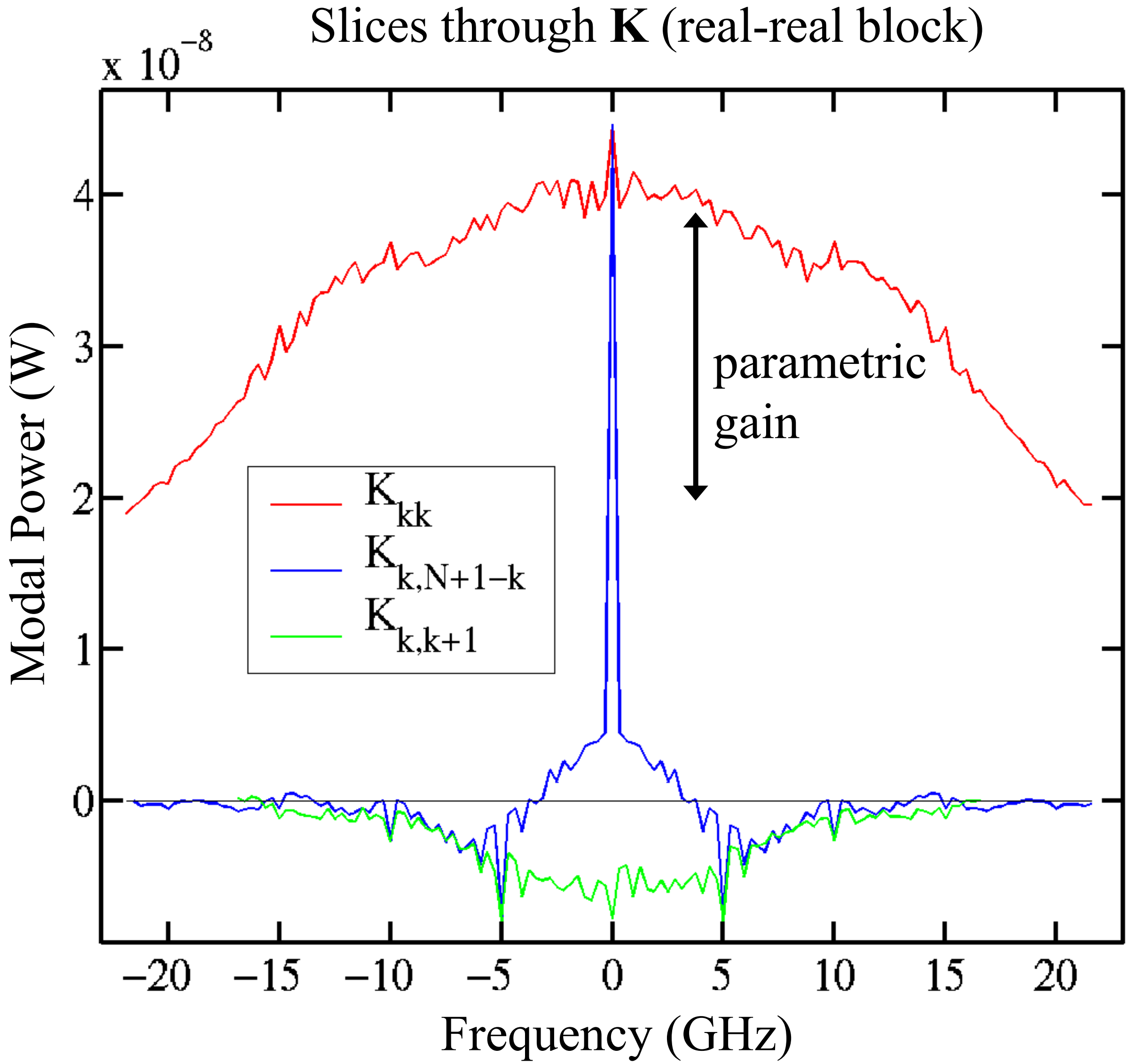
Characterizing the Covariance Matrix



Matrix sketch



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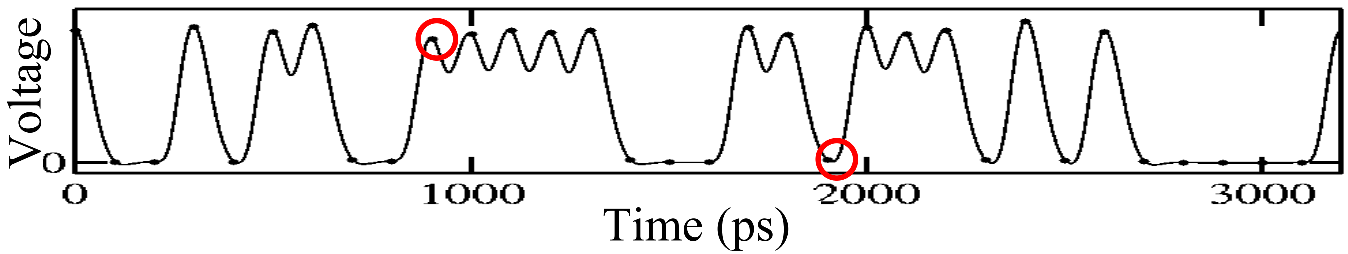
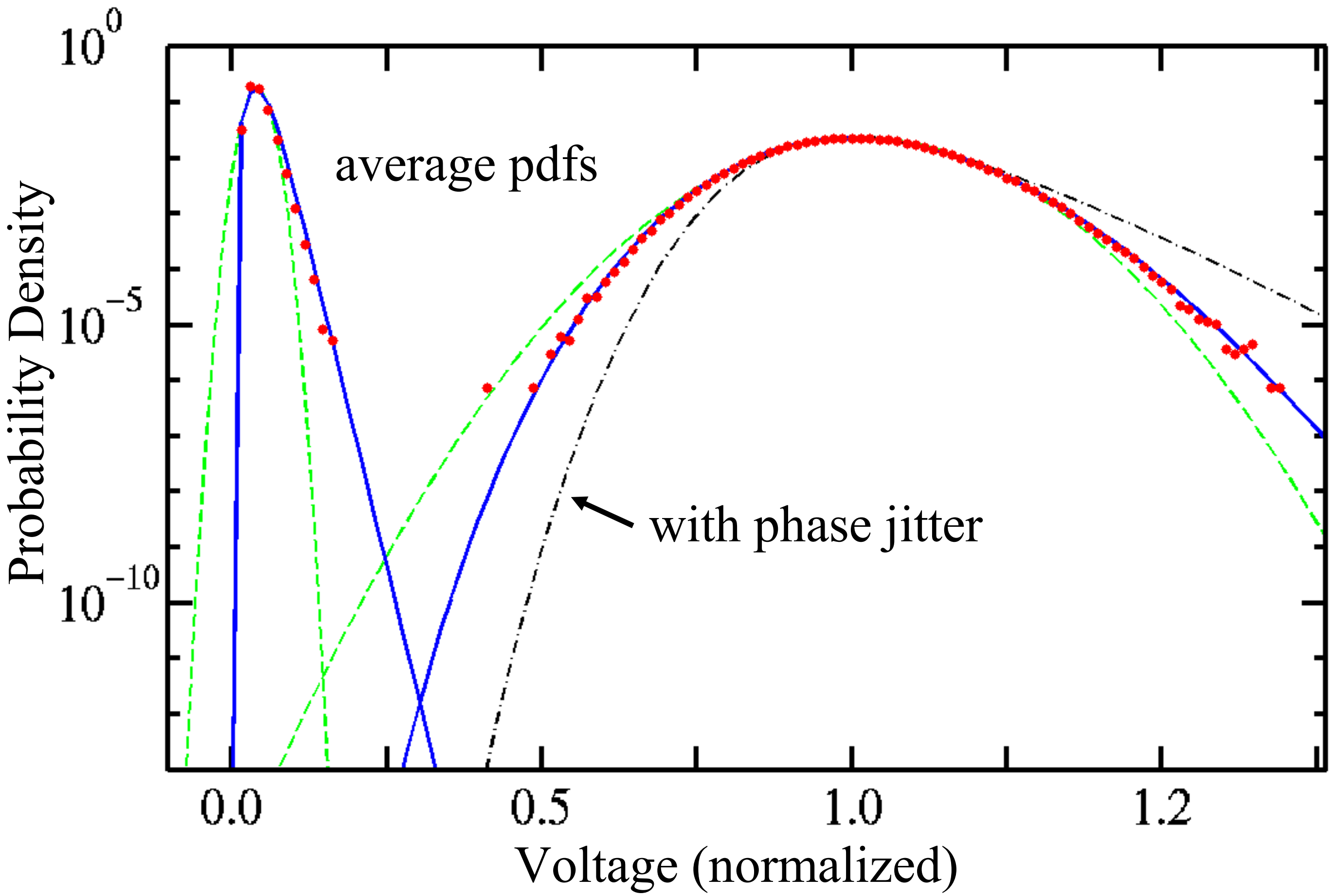
Pdfs of the Electrical Signal





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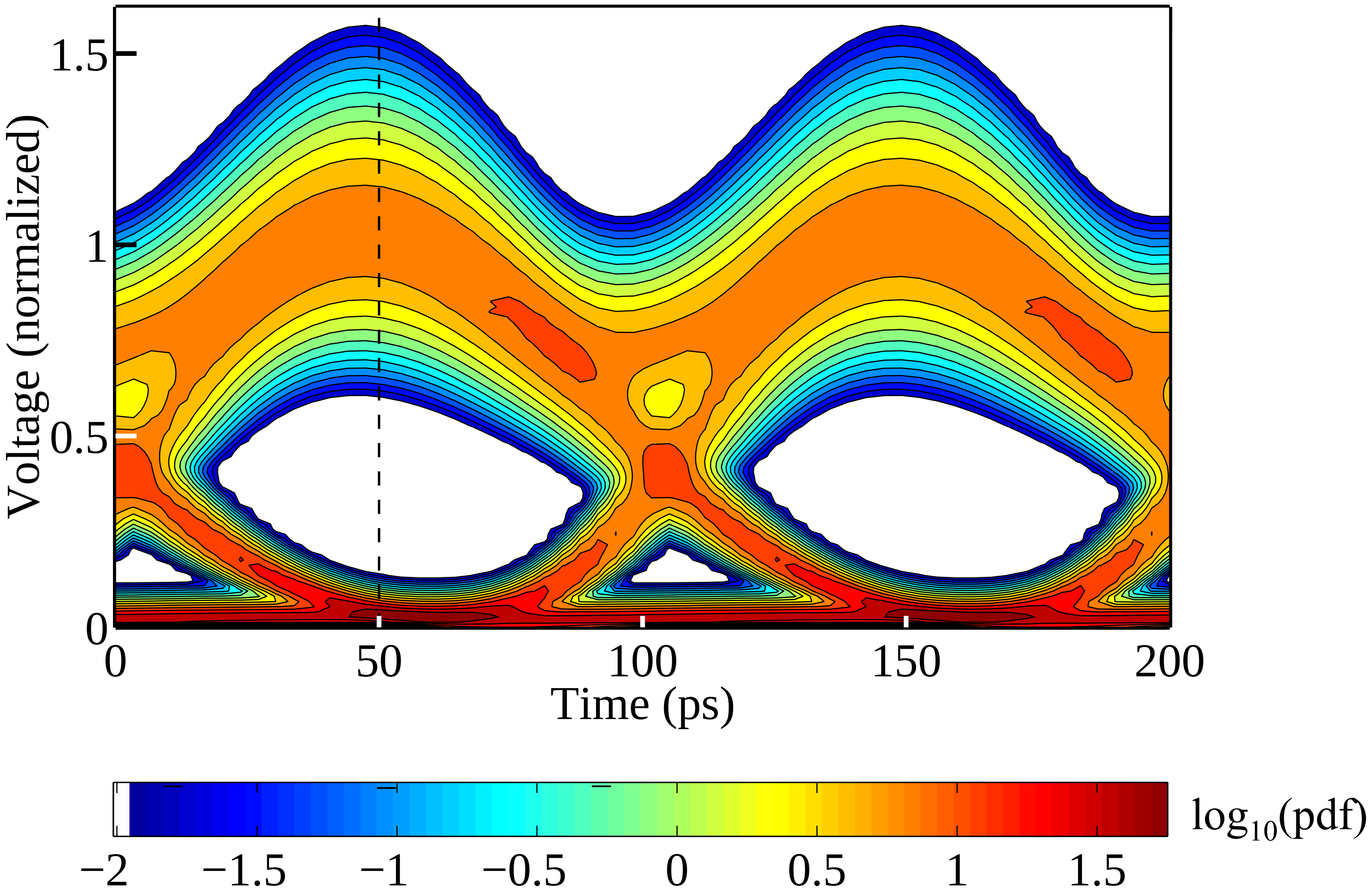
Eye Diagram from Linearization



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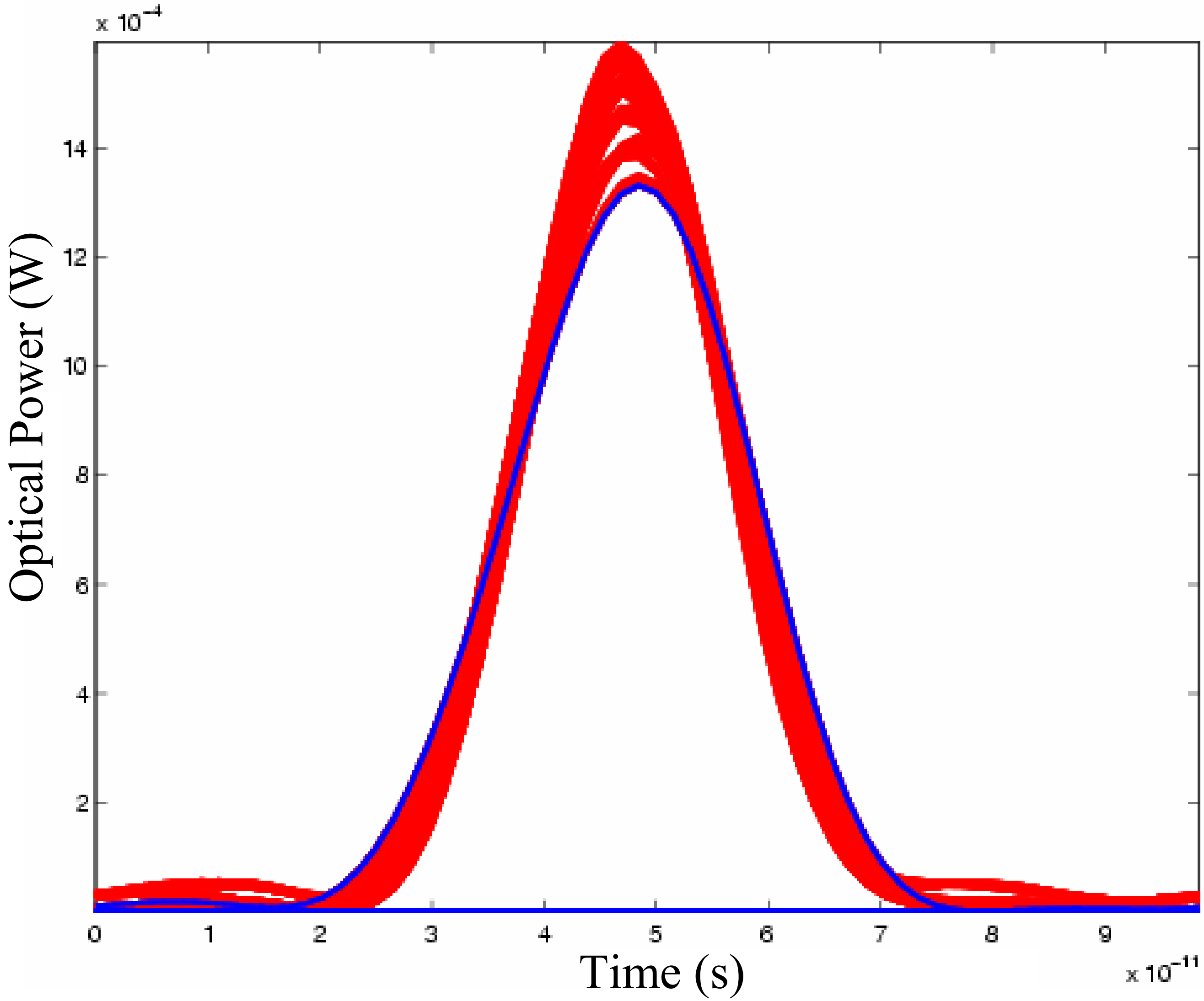
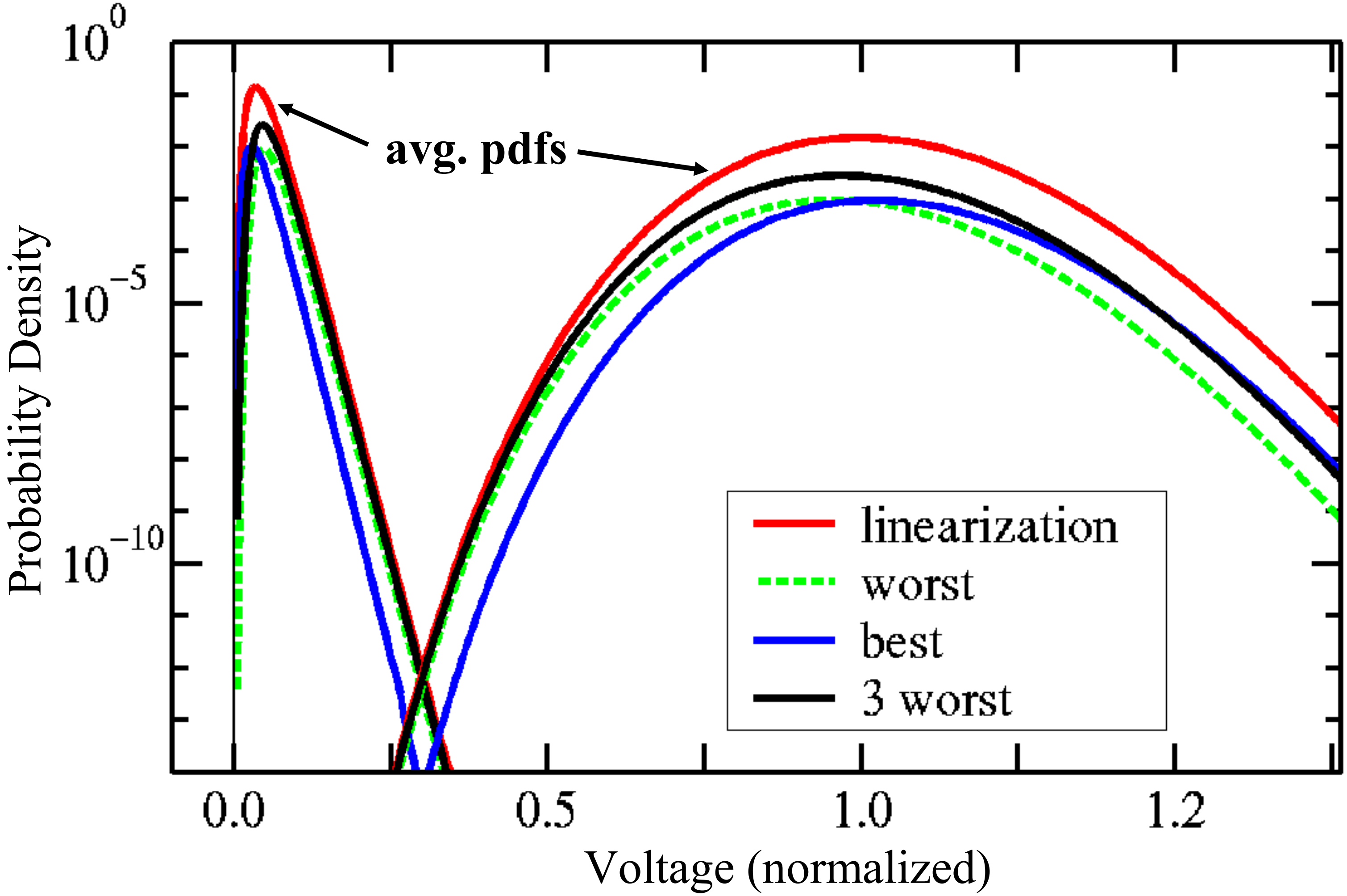
Focus on Worst Pattern

by Brian Marks



32 Bits CRZ: Focus on Worst Patterns



Conclusions

❶ Linearization method was successfully applied to CRZ system

❷ Critical steps: Phase jitter separation + dispersion compensation

❸ Bit patterns are important, focus on worst patterns

❹ Computational cost equal to *2N* Monte Carlo noise realizations

Approach might be practical in realistic systems