

# Modeling Noise in Optical Fiber Communications Systems

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**Abstract:** We describe two different methods for calculating the probability distribution of the voltage at the decision point in the receiver, taking into account transmission nonlinearity. These two methods are in complete agreement.

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## 1. Introduction

Perhaps the most important and, at the same time, most difficult theoretical problem in optical fiber communications is finding methods that will allow the user to accurately calculate bit error ratios with reasonable efficiency. There are a number of reasons for this difficulty: The transmission system and the receiver are nonlinear systems—invalidating simple analytical approaches. At the same time, the desired bit error ratios (BERs) are low, on the order of  $10^{-12}$  or less—making standard Monte Carlo simulations impractical. Moreover, one must be able to solve the problem end-to-end, taking into account the combined effects of transmission nonlinearity, receiver nonlinearity, and error correction and/or signal processing at the decision point in the receiver.

Important progress has been made in solving portions of this problem. The most progress has been made on square law receivers, which are appropriate for on-off keyed systems. We will review this work briefly. Considerable progress has been made on the impact of transmission nonlinearity on the noise distribution prior to the receiver. Both deterministic and statistical methods have been developed to the point where they agree completely with one another for realistic systems and are mutually self-validating. The main purpose of this talk is to review this progress. Finally, important progress has been made in the past year in determining the impact of pattern dependences due to inter-channel interactions (cross-phase modulation) in a wavelength-division multiplexed system on the distribution of marks and spaces prior to the receiver and in determining the impact of iterative decoders in error correcting codes on the BER. This work was described at the European Conference on Optical Communications [1] and will not be repeated here.

## 2. Receiver model

The basic receiver structure that we will consider is shown in Fig. 1. It consists of an optical filter/demultiplexer, followed by a photodetector that squares the input optical amplitude, followed by an electrical filter, a clock recovery circuit, and a threshold detector at the decision point. While commercial receivers can be considerably more complicated, the details are often proprietary, and this model is often adequate in realistic settings. Work by Lee and Shim [2], Bosco, *et al.* [3], and Forestieri [4] has demonstrated that if the optical and electrical filters are linear and if the optical noise entering the receiver is white and Gaussian-distributed, then regardless of the signal format, the voltage distributions for the marks and for the spaces at the detection point will obey a generalized chi-square distribution. Holzlöhner, *et al.* [5] extended this earlier work slightly to show that any input multivariate Gaussian noise distribution leads to a generalized chi-square distribution.

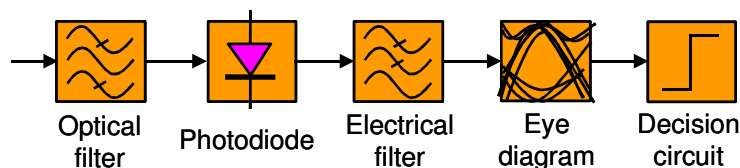


Fig. 1. Schematic illustration of the receiver model.

### 3. Noise-signal interaction

We now turn to the main subject of this presentation, which is calculating the distribution of the voltage of the marks and spaces at the detection point in the receiver, taking into account nonlinear transmission effects. Historically, two principal approaches have been used to tackle this problem. The first is complete Monte Carlo simulations. While this approach fully accounts for the nonlinearity in principle, it is not possible to calculate enough realizations to determine the full distributions of the marks and the spaces. This approach can be used to calculate the lowest moments of the distributions with reasonable accuracy in some cases; however, in some other cases, even this computation is too computationally time-consuming to be practical. The second approach is to simply neglect the nonlinear signal-noise and noise-noise interactions during transmission, in which case the noise at the entry to the receiver is well-approximated as Gaussian and white. While this approach is simple and fast, it is known by comparison to the moments of complete Monte Carlo simulations that it often produces incorrect results. As of this writing, it is not known when this method produces reliable results. Another approach, described in a review article by Golovchenko, *et al.* [6], takes into account signal-noise interactions in the approximation that the signal is treated as a continuous wave for the purpose of calculating its effect on the noise.

We present here two alternative approaches. The first approach, *the covariance matrix method*, is a deterministic approach. It is based on the assumption that once phase jitter has been separated from the noise, noise-noise beating in transmission can be neglected. Separating the phase jitter is a crucial step; Monte Carlo simulations show that this approach fails otherwise. The noise modes with the phase contribution removed are multivariate-Gaussian distributed and one can calculate the full distribution of the marks and spaces prior to the receiver for the covariance matrix [7]. Once that is done, the voltage distribution of the marks and spaces obey a generalized chi-square distribution, which can also be calculated.

The second approach that we will present is based on the *multicanonical Monte Carlo method* [8,9]. This approach fully accounts for all the transmission nonlinearity. It is a method for iteratively biasing Monte Carlo simulations, so that *a priori* knowledge of how to bias the simulation is not needed, in contrast to most other importance sampling techniques.

For systems that we have studied, the covariance matrix method requires approximately as much computer time as 200 noise-free simulations. The reason is that each noise mode must be propagated separately from amplifier to amplifier. The multicanonical Monte Carlo method requires approximately  $10^5$  realizations, which costs  $10^5$  times as much computer time as a noise-free simulation, in cases that we studied. This number of realizations amounts to several hours to several days of computer time on a modern computer cluster, which is acceptable for validation studies, but is not usually acceptable for production studies.

In order to test our approaches, we applied them to a chirped return to zero system that is based on a submarine transmission system. The model system, shown in Fig. 2, consists of 34 map periods, each with five amplifiers, over a total length of 6120 km. It includes pre-compensation, post-compensation, and the receiver model shown in Fig. 1. If we define the nonlinear scale length as the length scale  $1/\gamma P$ , where  $\gamma$  is the Kerr coefficient and  $P$  is the path-averaged maximum pulse intensity, the system is approximately 3 nonlinear scale lengths long. We have studied the dynamics of this system using standard simulations, and we have shown that this nonlinear scale length is typical for a variety of submarine and terrestrial systems [10].

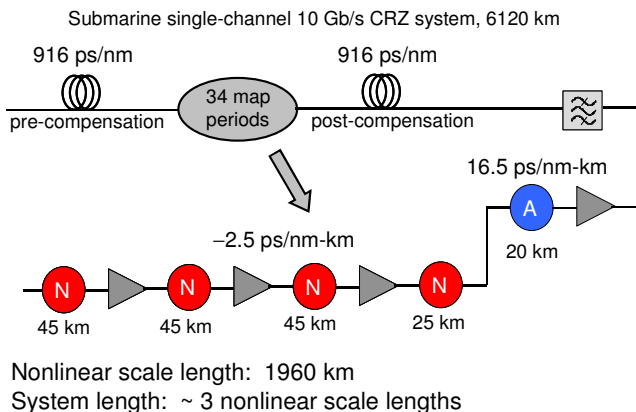


Fig. 2. Schematic illustration of the model submarine system.

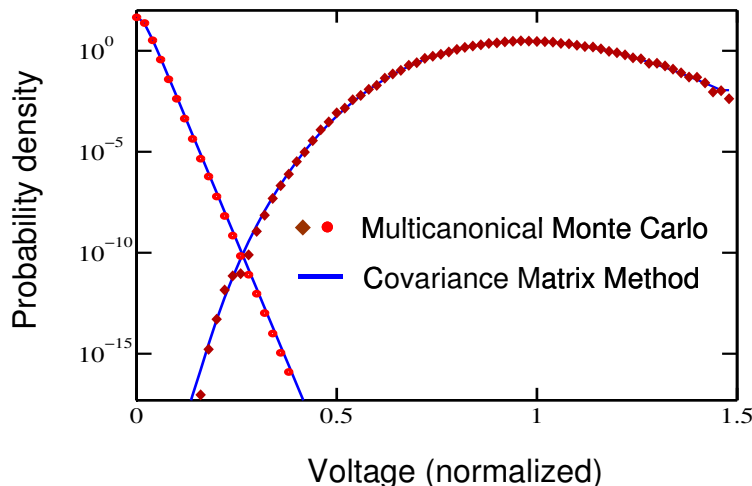


Fig. 3. Probability density functions for the voltage of the marks and spaces for the model system in Fig. 2.

We show our key result in Fig. 3 [11]. Here, we compare the covariance matrix method and multicanonical Monte Carlo method for this model system. We are showing the voltage distribution for the marks and for the spaces at the detection point in the receiver after clock recovery, which has been realistically modeled. The distributions for the marks and spaces are separately calculated using both methods. Both methods are capable of calculating the distribution function at the level of  $10^{-20}$ , and they are in perfect agreement!

#### 4. Conclusions

There is much that remains to be done. These results should be extended to other modulation formats, like non-return to zero and differential phase shift keying, as well as to other system configurations. Polarization effects should be included. Moreover, these methods can and should be used to give us information about when simple approximations, like the Gaussian white noise approximation, will yield accurate results in realistic systems.

Nonetheless, important progress has been made. Since the nonlinearity in the model system of this paper is at the high end of most commercial systems, it is our view that the covariance matrix method will apply to any commercial system. Certainly, it appears possible to use the multicanonical Monte Carlo method with any commercial system.

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