A Covariance Matrix Method for the Efficient and Accurate Computation of Eye Diagrams and Bit Error Rates

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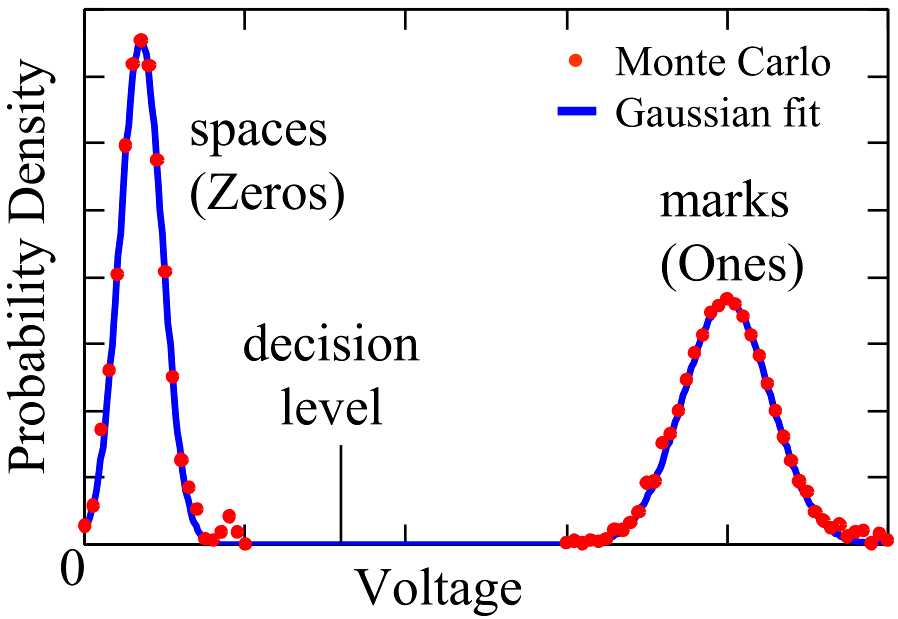
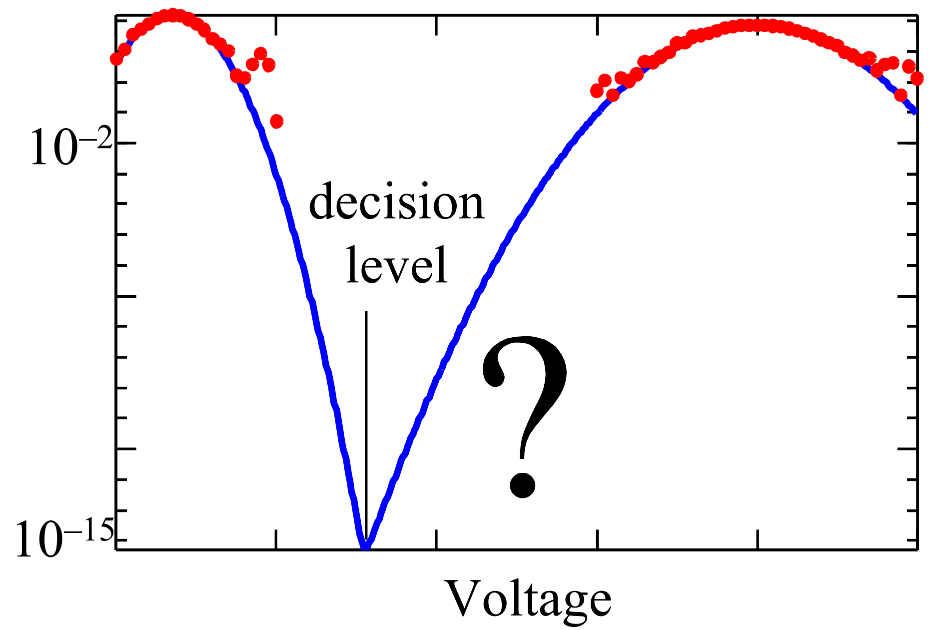
Calculating Bit Error Rates

BERs are determined by the low-probability tails of the received voltage probability density functions (pdfs)

Linear scale



Log scale



Nonlinear transmission complicates shape of the pdfs

Common Approaches

❶ Full Monte Carlo simulations + Gaussian extrapolation [Marcuse]

+ : few assumptions, simple, can be used with strong nonlinearity
– : computationally expensive, pdfs are not Gaussian

❷ CW transmission linearization + analytical pdfs [Hui, Bosco, Mazurczyk]

+ : very fast, deterministic, includes parametric gain
– : no data modulation in signal-noise interaction during transmission

❸ Optical white noise at receiver + analytical pdfs or Monte Carlo receiver simulations [Marcuse, Winzer]

+ : very fast, possibly deterministic
– : no signal-noise interaction during transmission

Validation?

Overview

Accurate calculation of BER vs. decision level

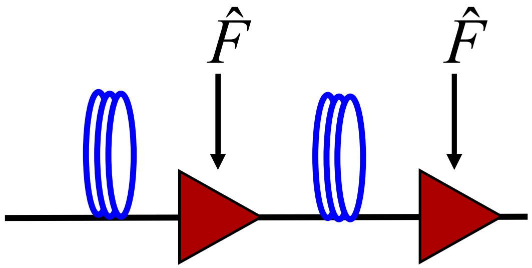
- For modulated data with multiple bits and channels
- Fiber propagation model:
  - Includes nonlinear interactions between signal and noise
  - Neglects noise-noise interactions in the fiber
  - Propagates the optical noise covariance matrix
- Receiver model:
  - Uses realistic optical and electrical filters
  - Must include noise-noise interactions in the receiver

We compute the noise covariance matrix deterministically

Linearizing the NLS

Nonlinear Schrödinger equation with ASE noise

$$i\frac{\partial u}{\partial z}+\frac{D(z)}{2}\frac{\partial^2 u}{\partial t^2}+|u|^2 u = ig(z)u+\hat{F}$$



$\hat{F}$: added Gaussian white noise

Now set $u = u_0 + \delta u$, $u_0 = \langle u \rangle$: average signal

δu : accumulated noise

$$i\frac{\partial \delta u}{\partial z}+\frac{D(z)}{2}\frac{\partial^2 \delta u}{\partial t^2}+2|u_0|^2\delta u + u_0^2(\delta u)^* = ig(z)\delta u+\hat{F}$$

Doob's Theorem: δu is multivariate Gaussian distributed

Multivariate Gaussian pdf

$$\delta u(t) = \sum_{k=1}^{N} \left[\alpha_k + i\beta_k\right] \exp\left(i\omega_k t\right)$$

$$\boldsymbol{a} = (\alpha_1, \alpha_2, \ldots, \alpha_N, \beta_1, \beta_2, \ldots, \beta_N)^T$$

Covariance matrix $\mathsf{K} = \left\langle \boldsymbol{a}\boldsymbol{a}^T \right\rangle,\ \mathsf{K}_{kl} = \left\langle \boldsymbol{a}_k \boldsymbol{a}_l \right\rangle$

Multivariate Gaussian distribution of a:

$$f_{\boldsymbol{a}}(\boldsymbol{a}, z) = \left(2\pi\right)^{-N} \sqrt{\det \mathsf{K}^{-1}} \ \exp\left(-\tfrac{1}{2}\boldsymbol{a}^T \mathsf{K}^{-1} \boldsymbol{a}\right)$$

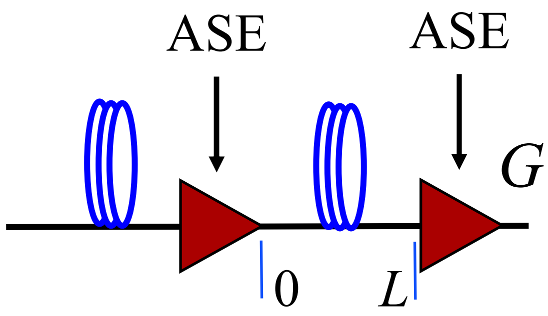
How to Compute the Covariance Matrix

Solve the linearized homogeneous propagation equation

$$\frac{\partial \boldsymbol{a}}{\partial z} = \mathsf{R}(z)\boldsymbol{a} \;\Rightarrow\; \boldsymbol{a}(L) = \boldsymbol{\Psi}\boldsymbol{a}(0)$$

$$\mathsf{K}(L) = G\,\boldsymbol{\Psi}\mathsf{K}(0)\boldsymbol{\Psi}^T + \eta\mathbf{I}$$

↑ ASE input



But: ODE is stiff due to dispersion term. Solution: perturbative approach

$$\boldsymbol{a}(0) = \boldsymbol{a}^{(k)}(0) = \varepsilon\,\hat{\boldsymbol{e}}_k \rightarrow \boldsymbol{a}^{(k)}(L) \;\Rightarrow\; \boldsymbol{\Psi}_{jk} = \frac{\boldsymbol{a}_j^{(k)}(L)}{\varepsilon}$$

Compute Ψ by perturbing each of the N frequency modes separately

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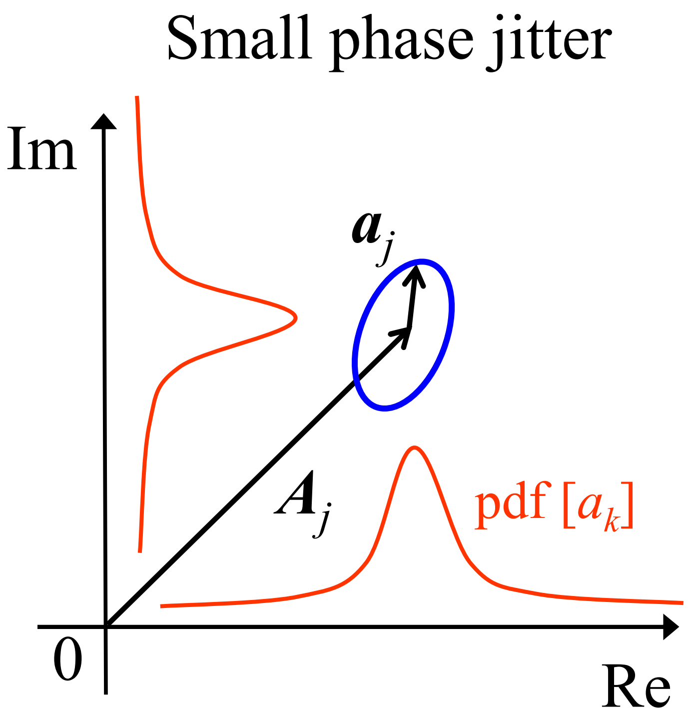
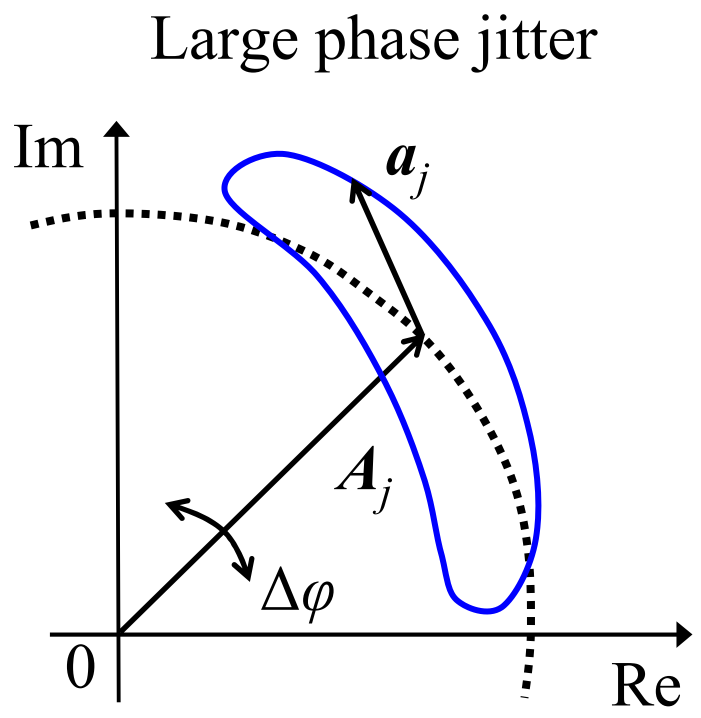
Strong Phase Jitter Requires Different Basis





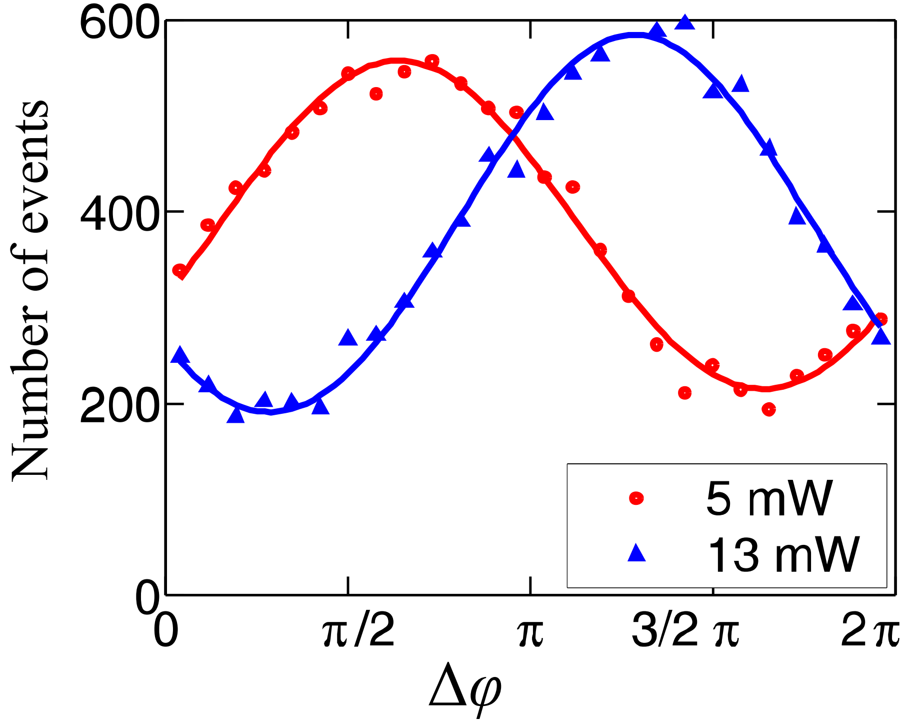
Phase jitter rotates the signal around the origin, distorting the Gaussian pdf

Solution: Separate phase jitter from $\boldsymbol{a}^{(k)}(L)$

Phase and Timing Jitter

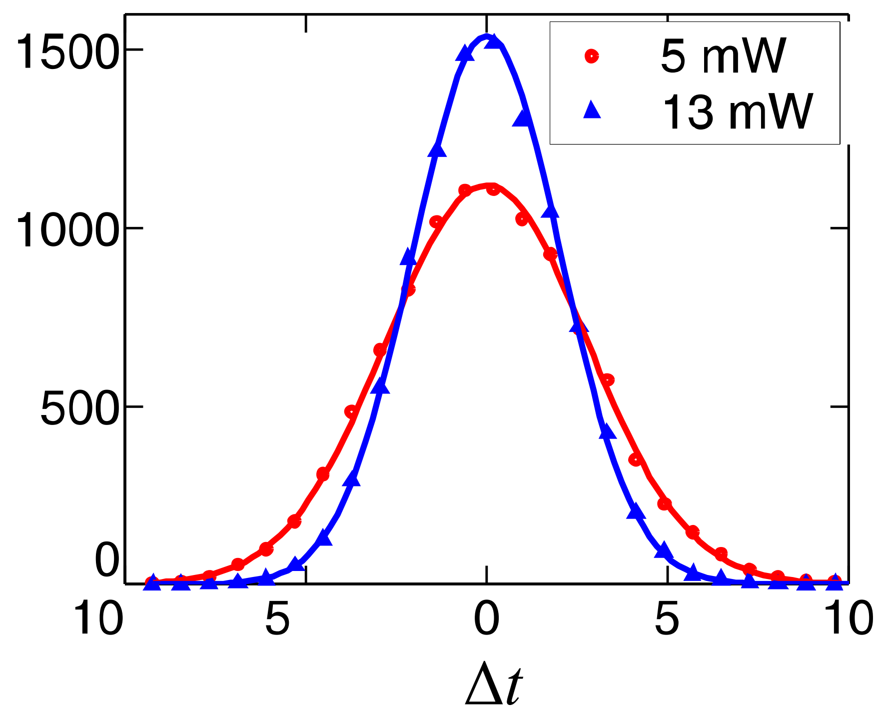
DMS system, 24,000 km

Phase jitter histogram



fit: Jacobi θ-function

Timing jitter histogram



fit: Gaussian

Timing and phase jitter are Gaussian distributed

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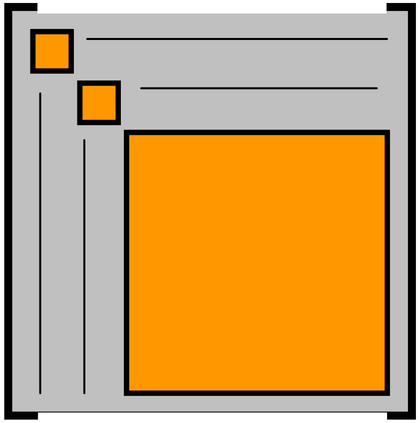
Phase and Timing Jitter Separation

Separate phase and timing jitter by projecting out their noise modes using a 2-step Gram-Schmidt orthogonalization procedure:

$$\boldsymbol{v} = \mathrm{FT}\{ i u_0(t) \}, \quad \boldsymbol{w} = \mathrm{FT}\{ \partial_t u_0(t) \}$$

$$\tilde{\boldsymbol{w}} = \boldsymbol{w} - \frac{(\boldsymbol{w}, \boldsymbol{v})}{(\boldsymbol{v}, \boldsymbol{v})} \boldsymbol{v}$$

$$\tilde{\boldsymbol{a}}^{(k)} = \boldsymbol{a}^{(k)} - \frac{(\boldsymbol{a}^{(k)}, \boldsymbol{v})}{(\boldsymbol{v}, \boldsymbol{v})} \boldsymbol{v} - \frac{(\boldsymbol{a}^{(k)}, \tilde{\boldsymbol{w}})}{(\tilde{\boldsymbol{w}}, \tilde{\boldsymbol{w}})} \tilde{\boldsymbol{w}}$$

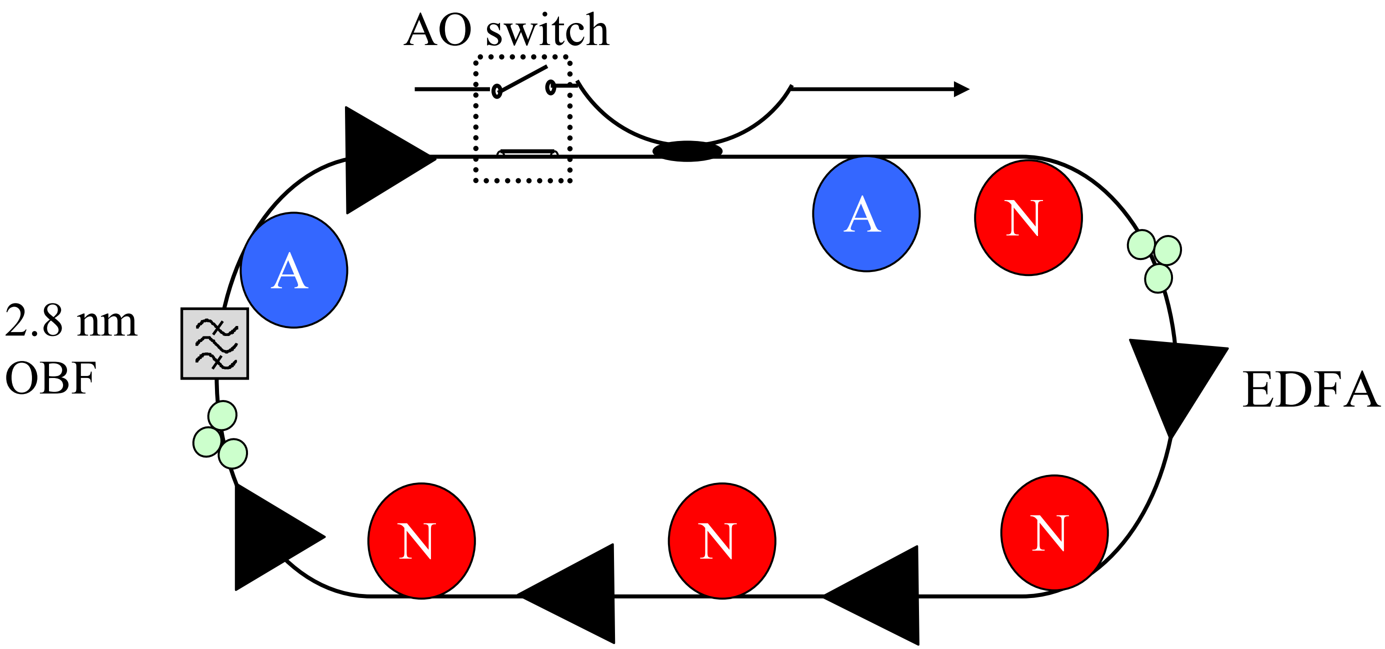


$\tilde{K}$

Jitter separation must be applied to each pulse individually

Test System 1: 10 Gb/s DMS over 24,000 km

R.-M. Mu *et al*., IEEE J. Sel. Topics Quant. Electronics **6**, 248–257 (2000)

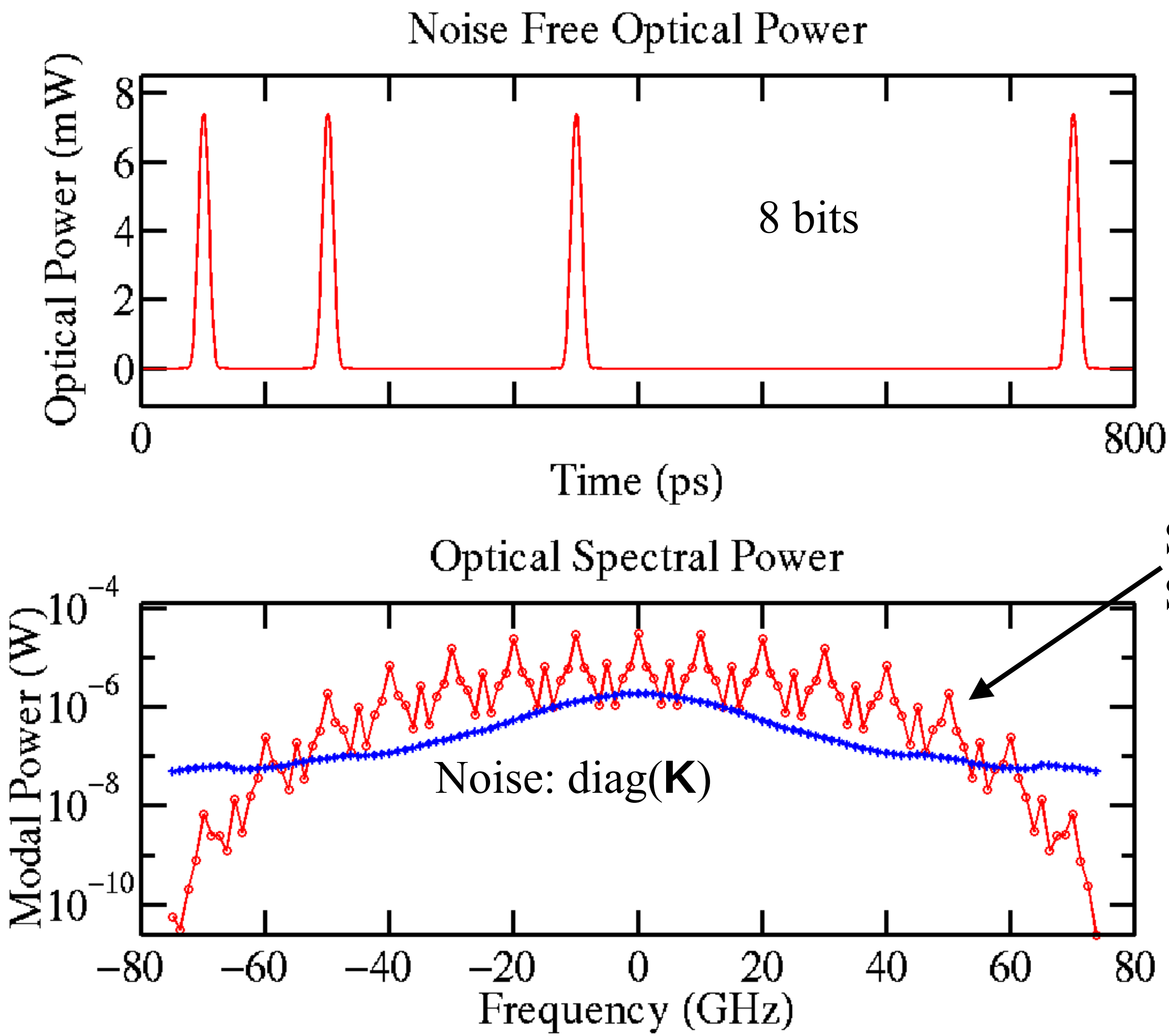


N: 4 × 25 = 100 km Normal dispersion fiber, *D* = – 1.1 ps/nm-km

A: 2 × 3.5 km Anomalous dispersion fiber, *D* = 16 ps/nm-km

Highly nonlinear system, hence stringent test of our approach

Noise-free Optical Signal at Receiver

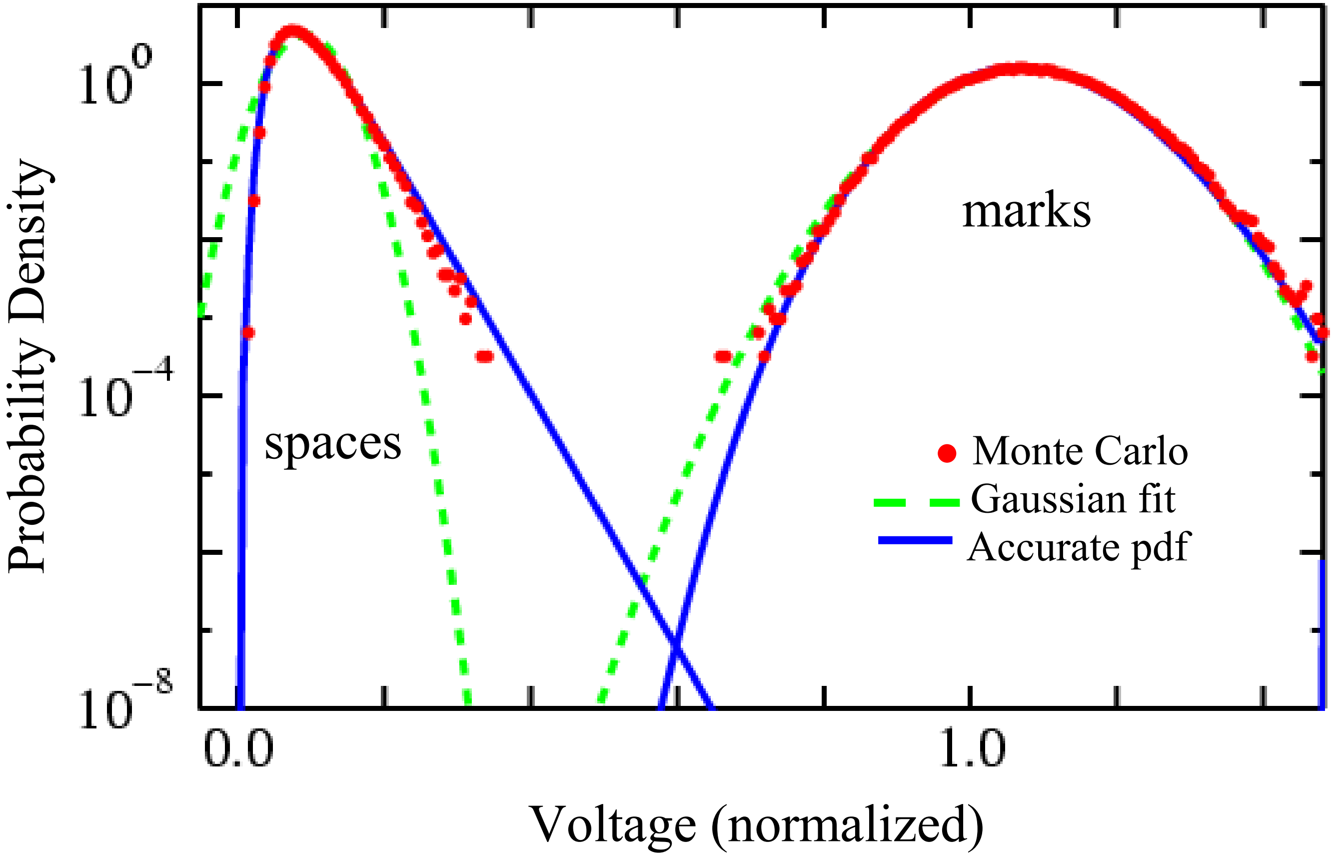


Signal Spectrum

OSNR: 8.44 dB

Accurate Probability Density Functions

39,000 Monte Carlo noise realizations



Optimum Accurate BER: 3.2×10^{-9}

Optimum Gaussian BER: 2.3×10^{-14}

$Q = 7.54$

Accurate pdfs deviate strongly from Gaussians in tails

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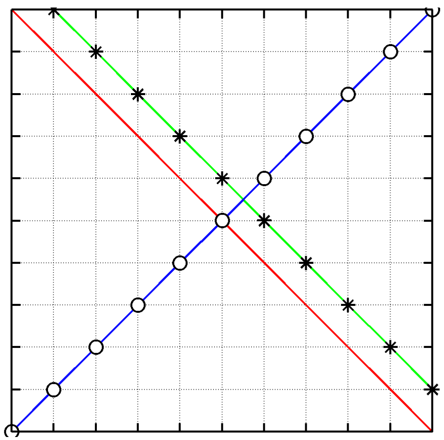
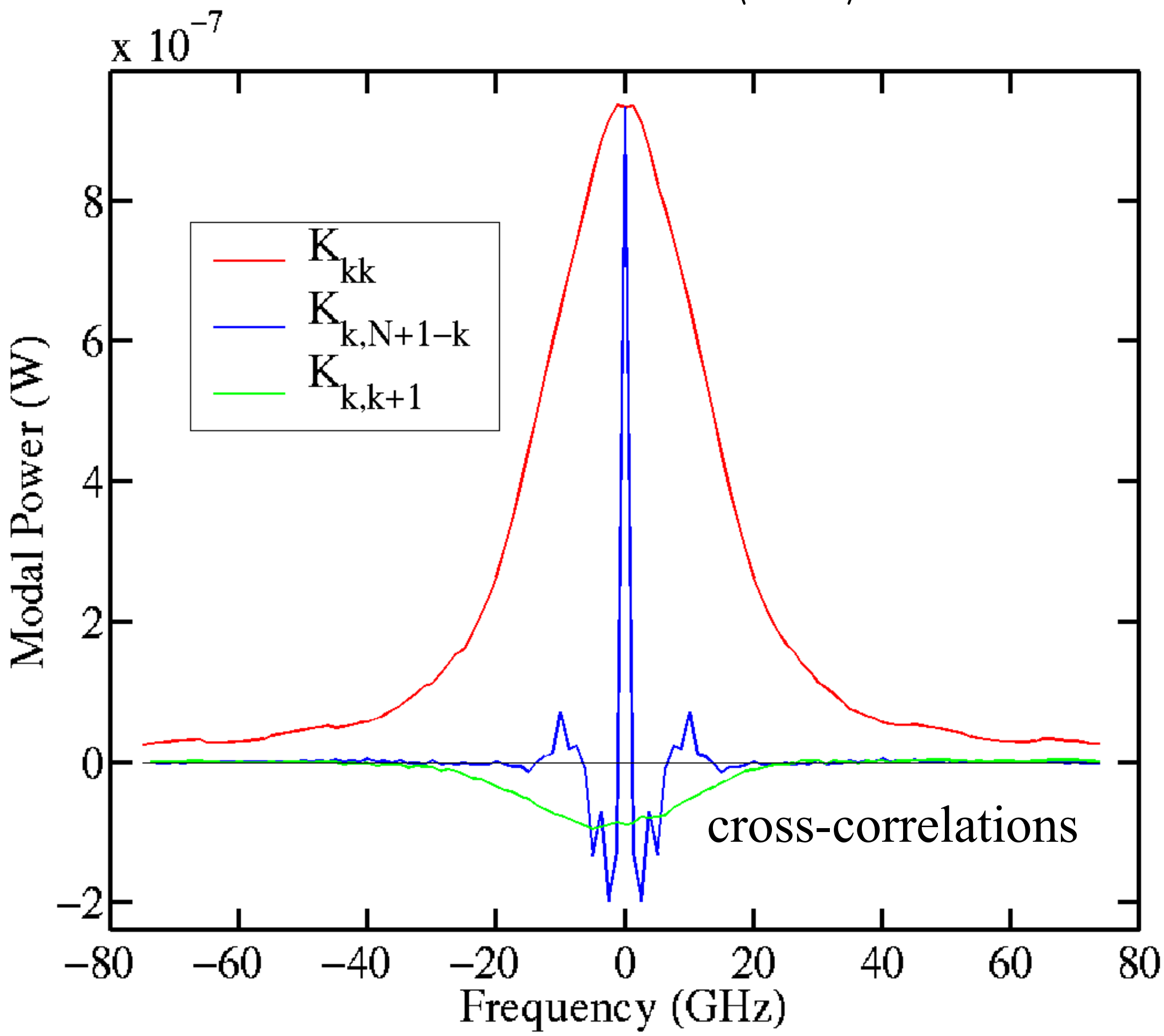
Cross-correlations in the Covariance Matrix



Slices through **K** $\langle \alpha_k \alpha_l \rangle$



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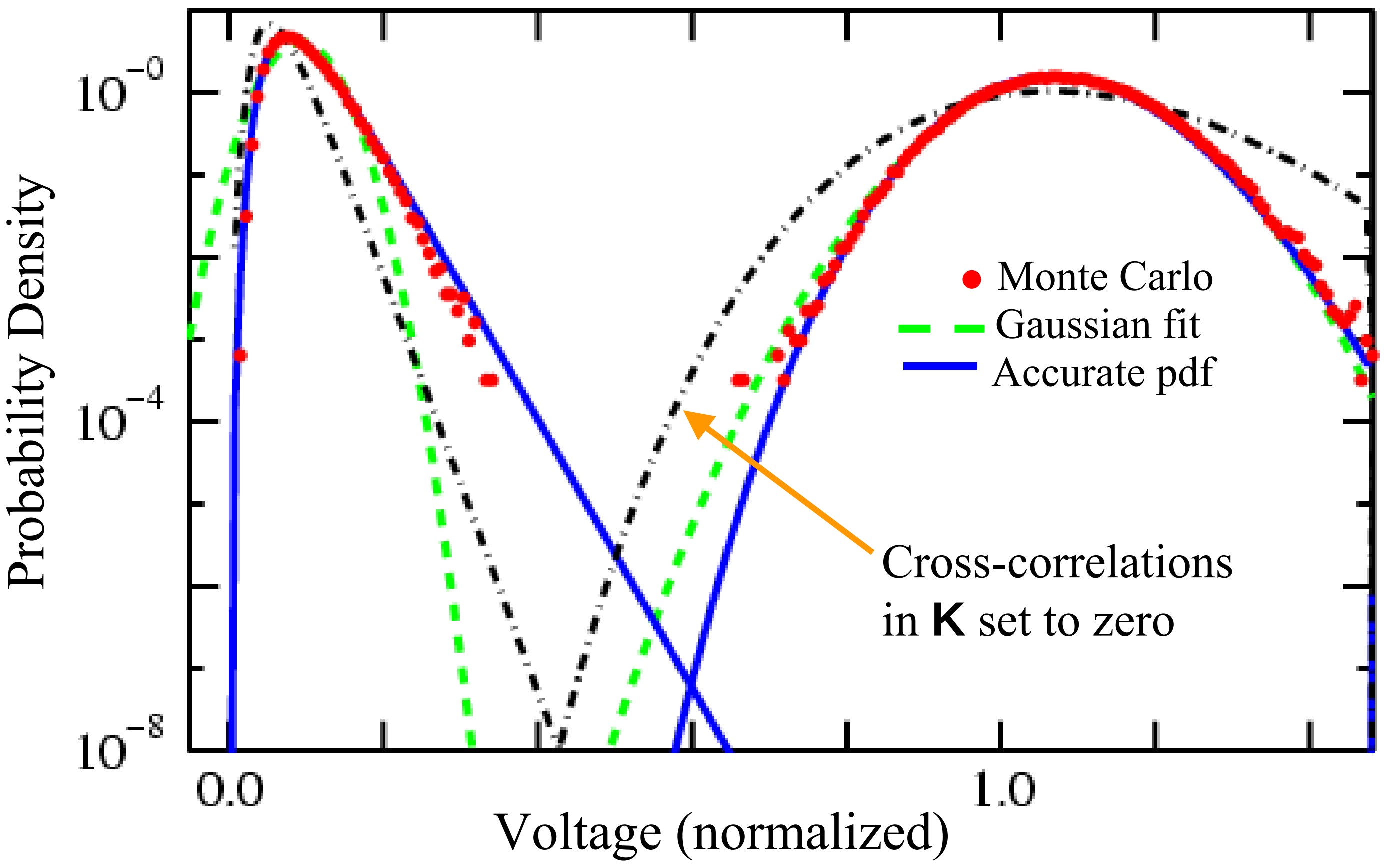
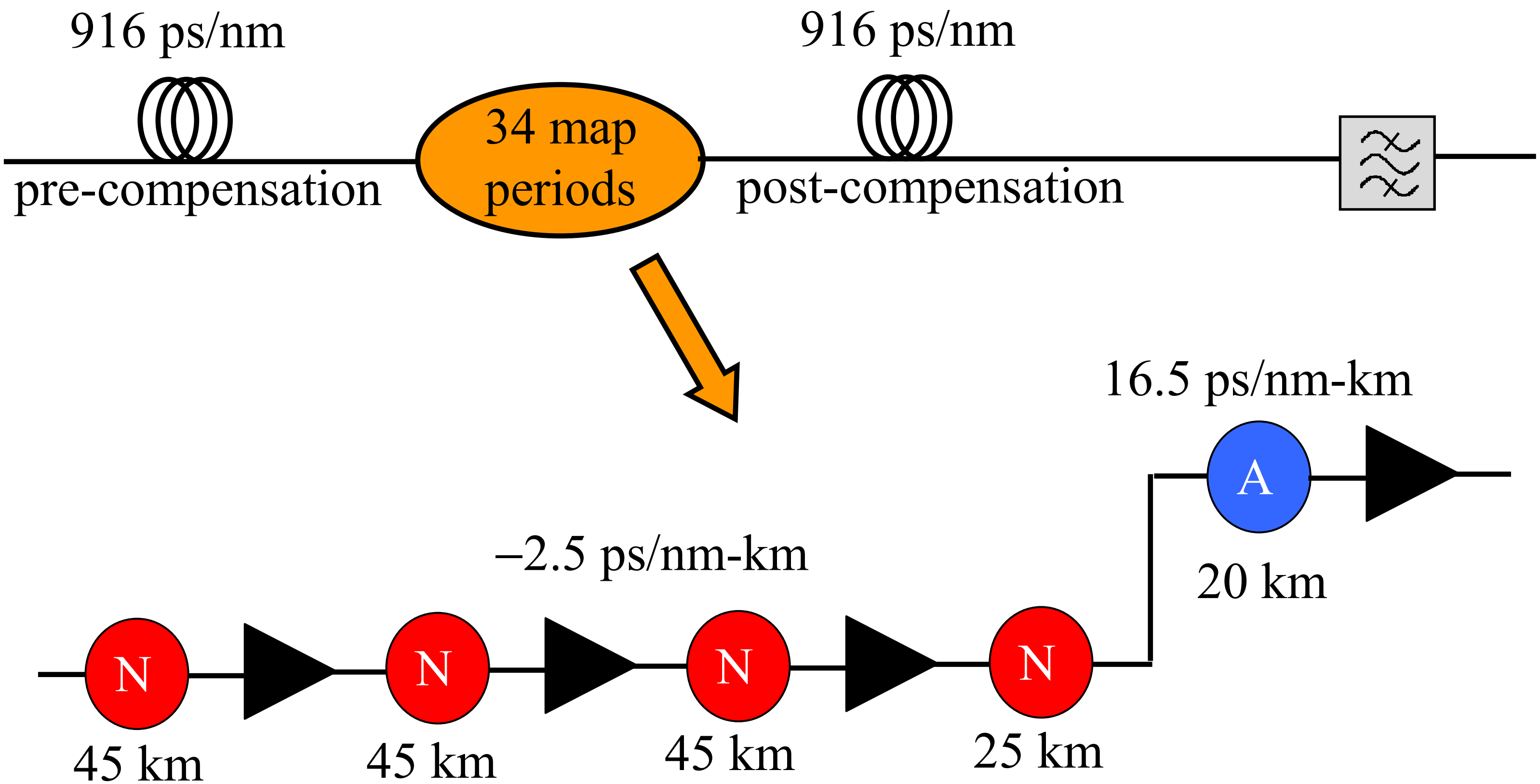
Are the Cross-correlations Relevant?



Noise cross-correlations impact the pdfs significantly in this system

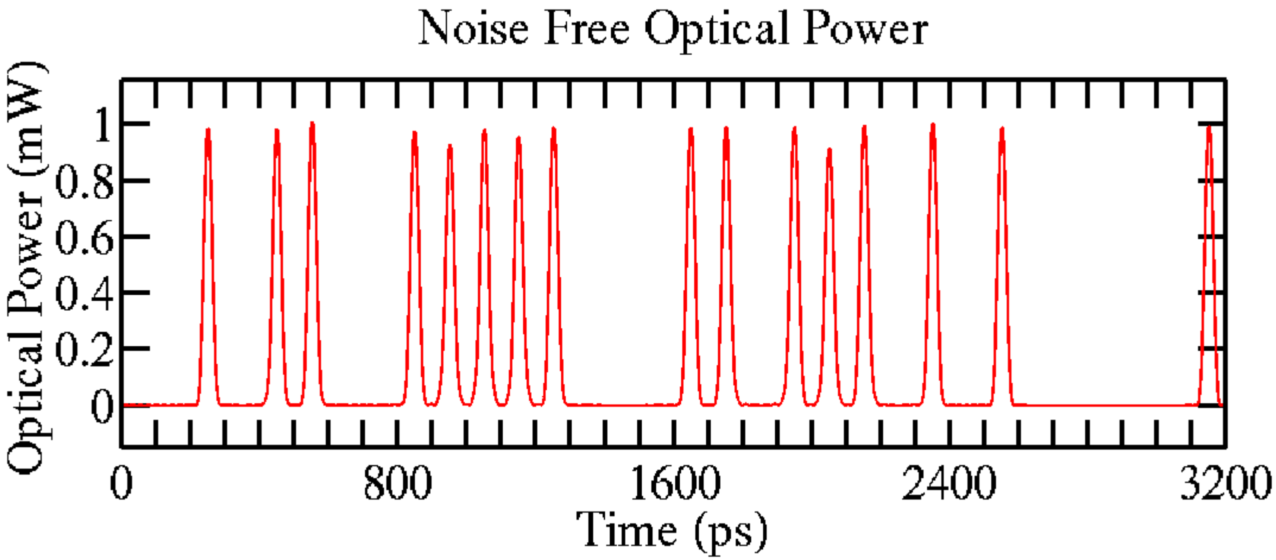
Test System 2: Submarine CRZ, 6100 km

10 Gb/s, single-channel

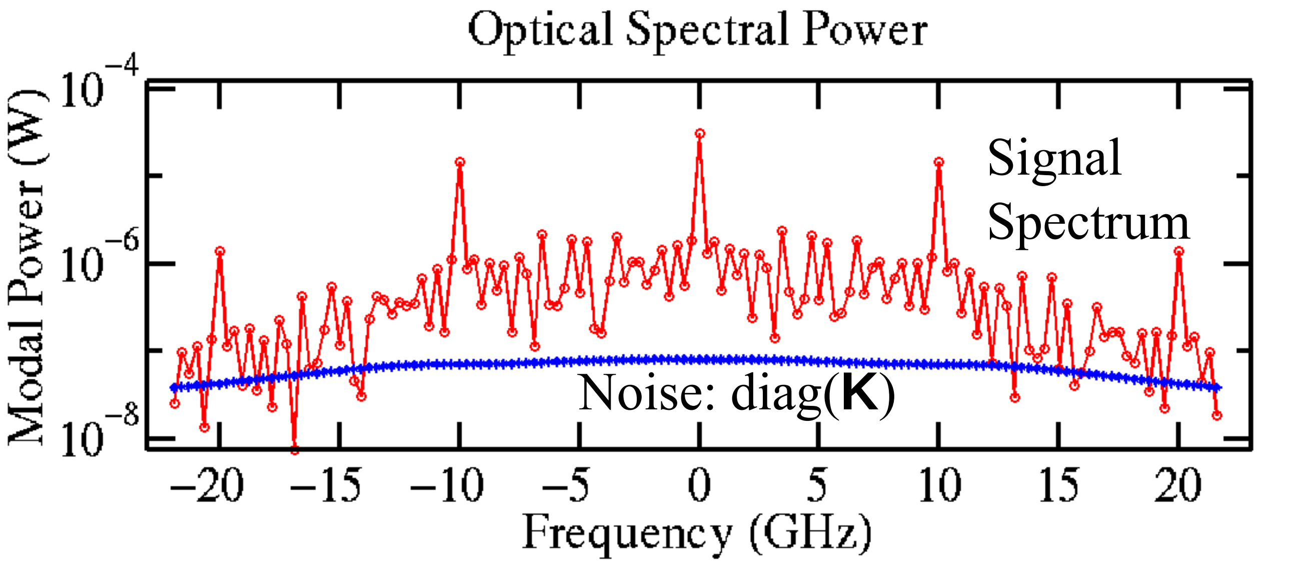


Non-periodic evolution: medium nonlinearity, but strong pulse overlap

Noise-free Optical Signal at Receiver



32 bits PRBS



OSNR: 11.7 dB

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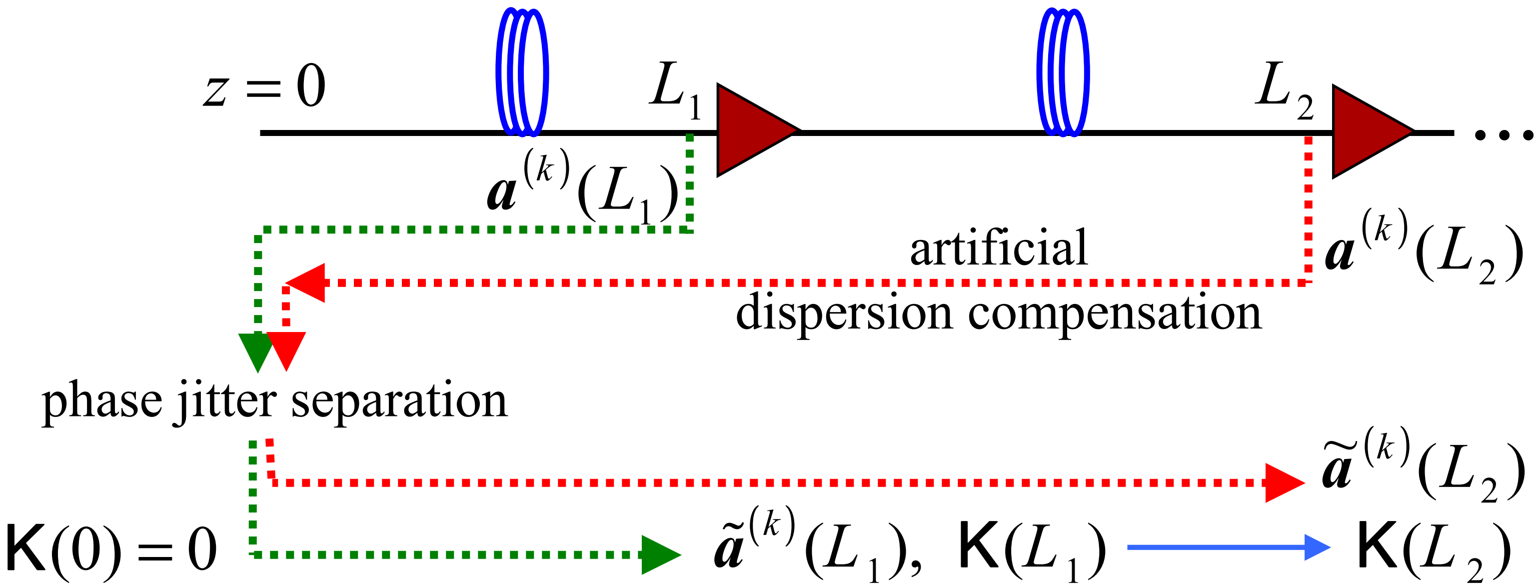
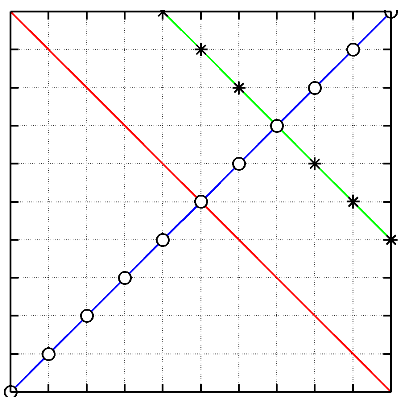
Compress Pulses, then Separate Jitter

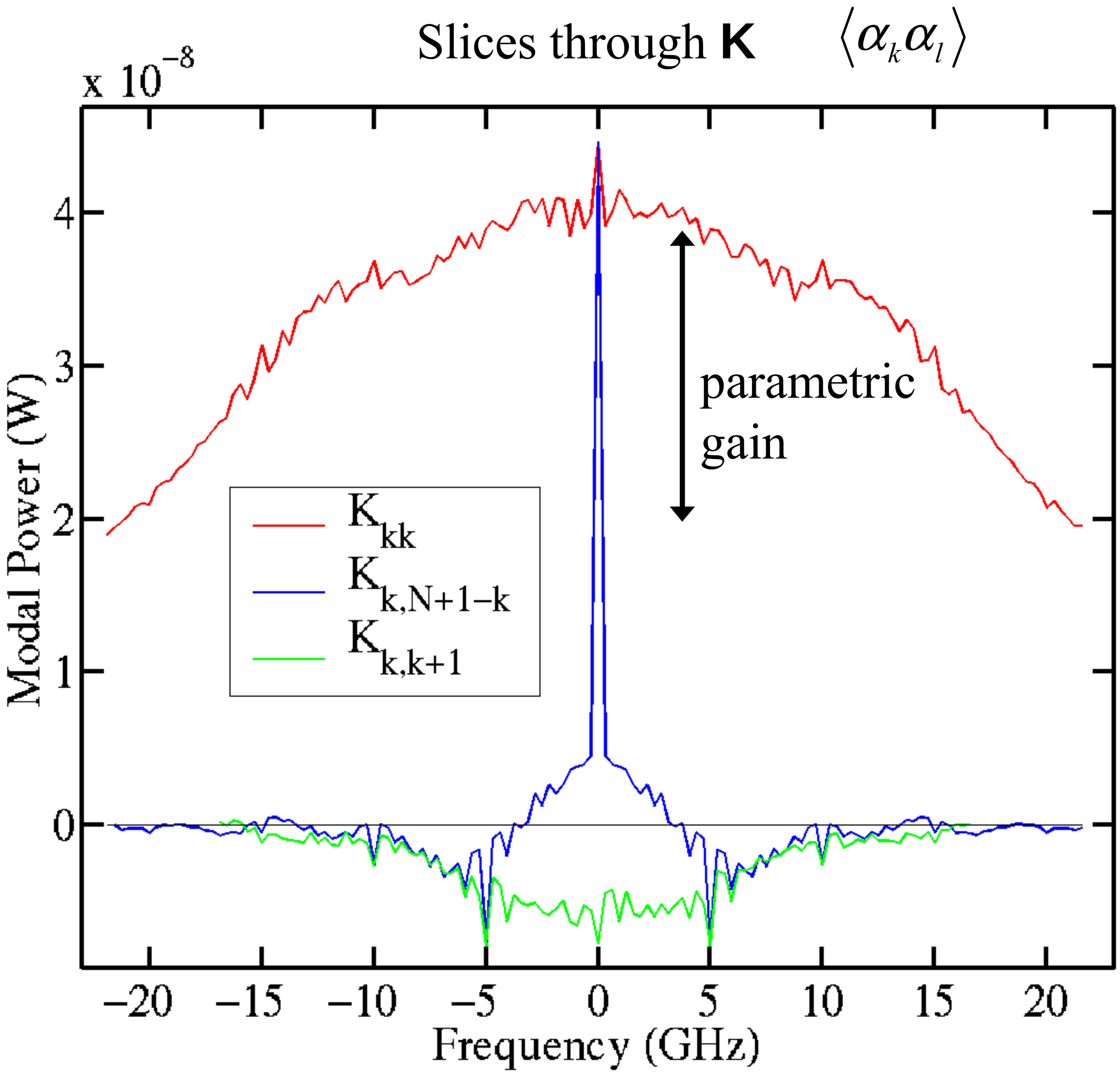
Jitter separation requires artificial dispersion compensation in CRZ:



Pulses in CRZ evolve separately, despite their strong overlap

Characterizing the Covariance Matrix





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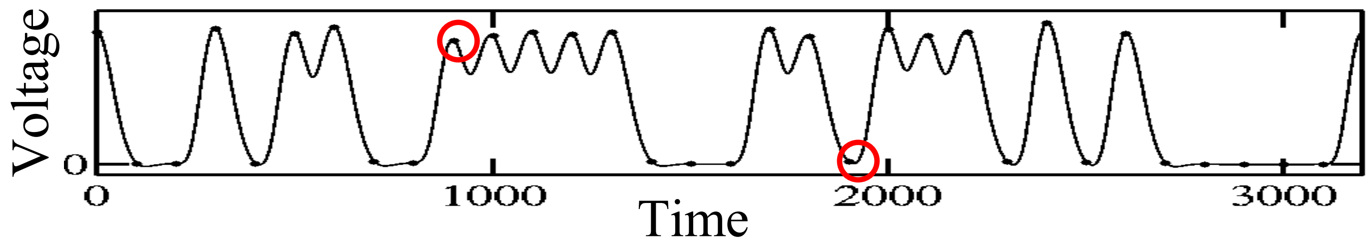
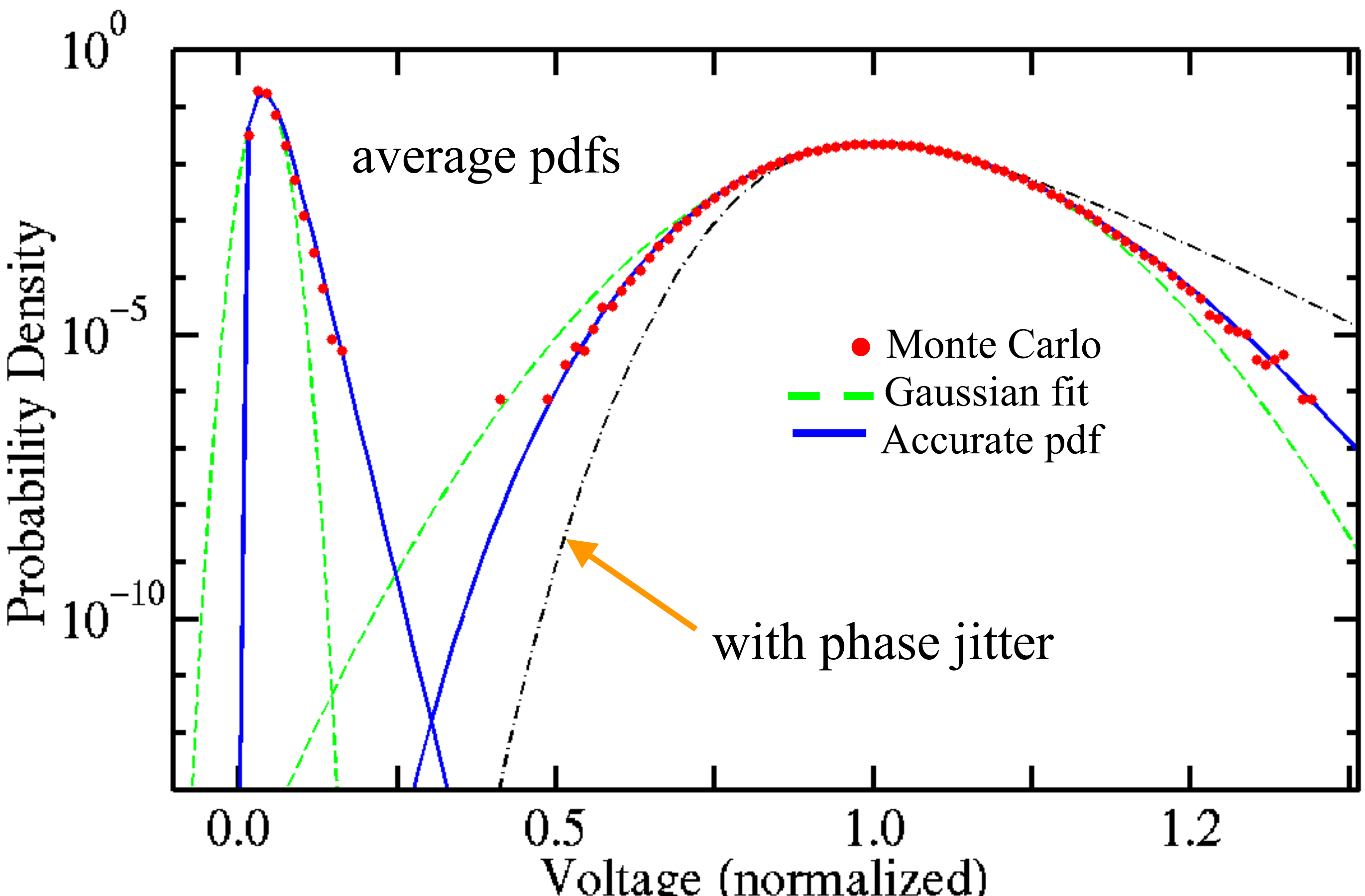
Pdfs of the Electrical Signal



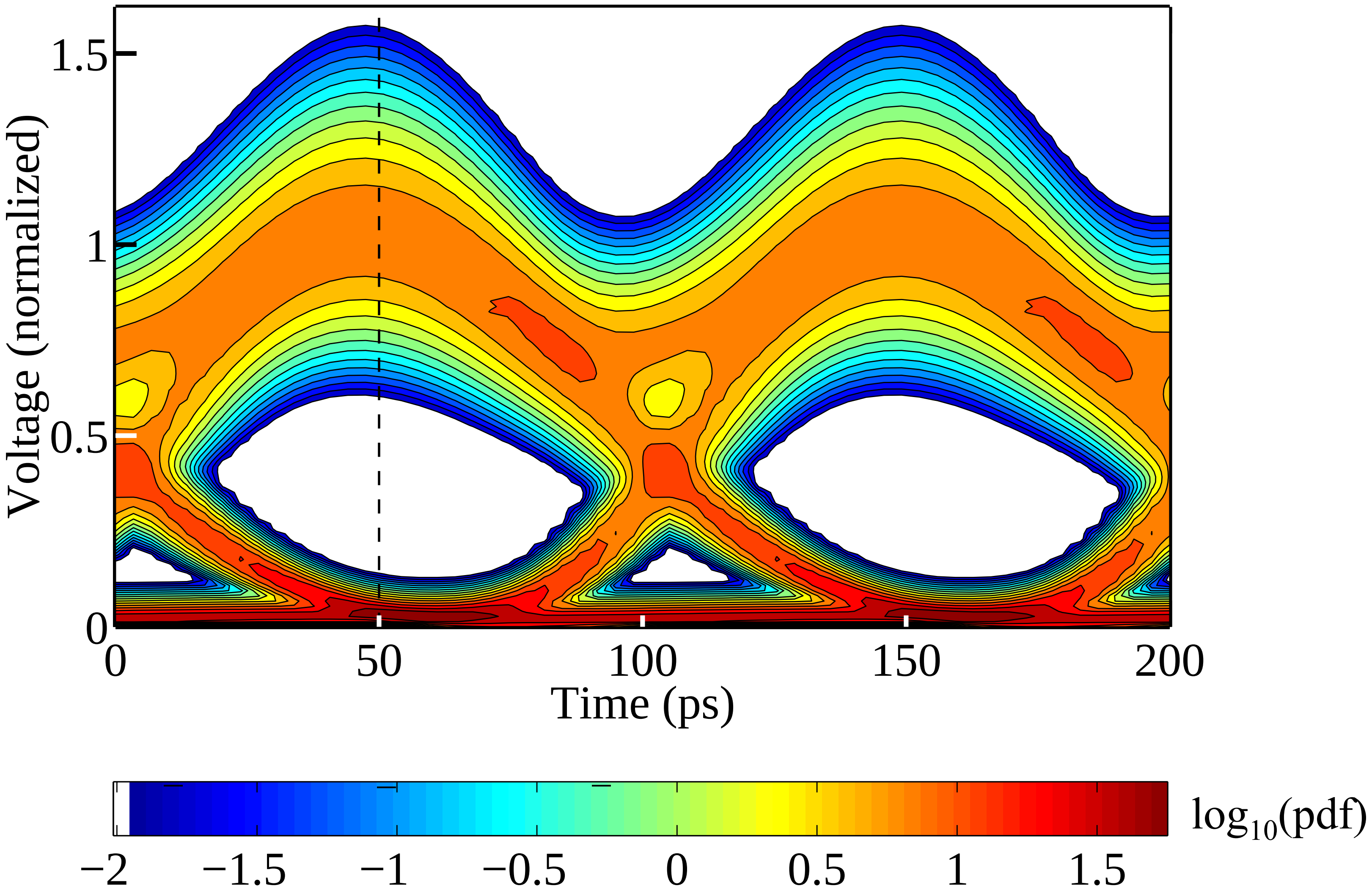


Optimum
Accurate BER:
3.1×10^{-12}

Optimum
Gaussian BER:
1.0×10^{-11}
$Q = 6.7$
(100,000 realiz.)

Accurate Contour Eye Diagram

CRZ system, 10 Gb/s, 32 bits PRBS, single-channel, 6100 km



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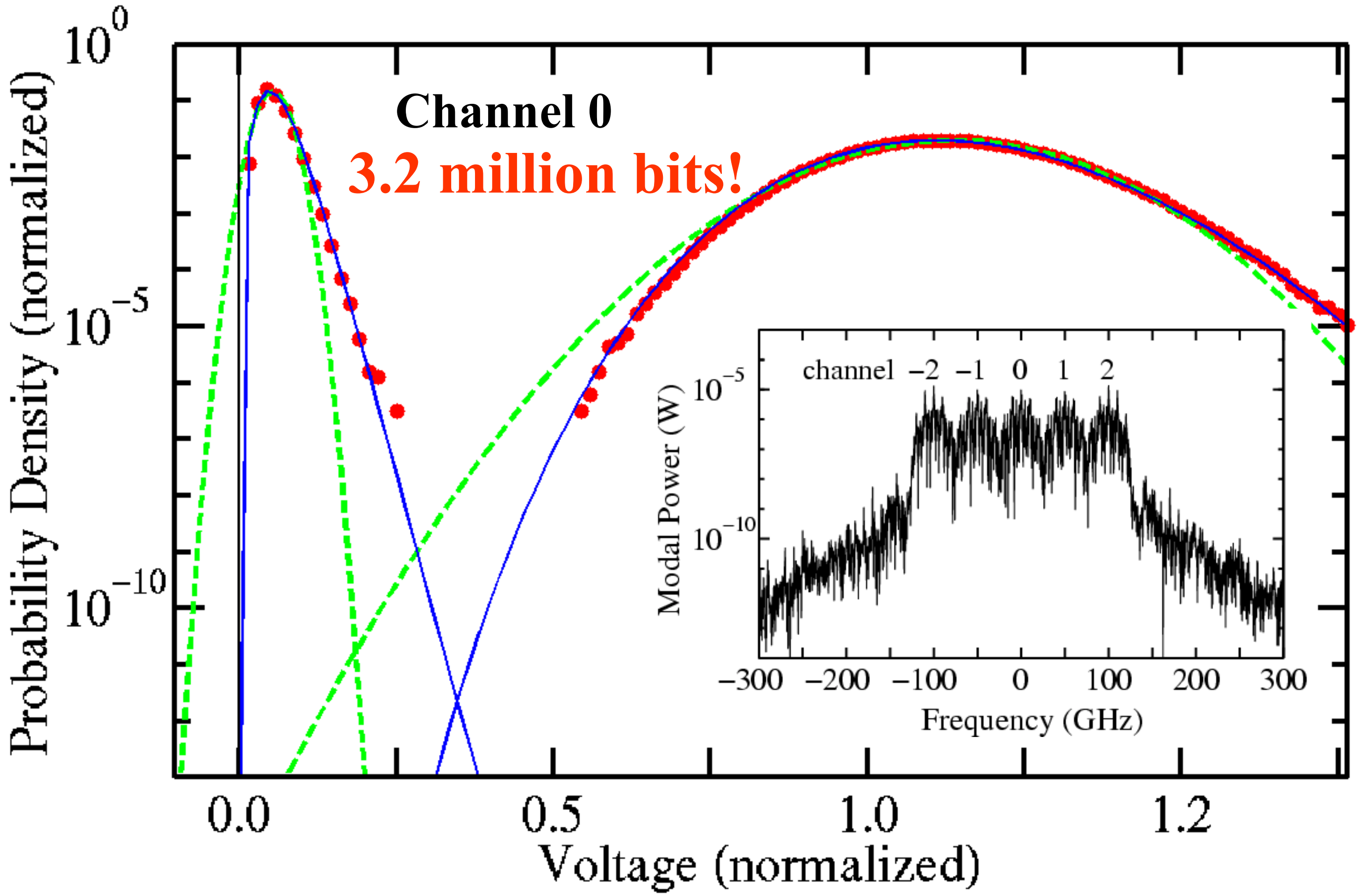
32 bits CRZ, 5 Channels, 50 GHz Spacing



Method: Include background field when computing Ψ, but omit in K

Conclusions

❶ Covariance matrix method was validated for DMS and CRZ

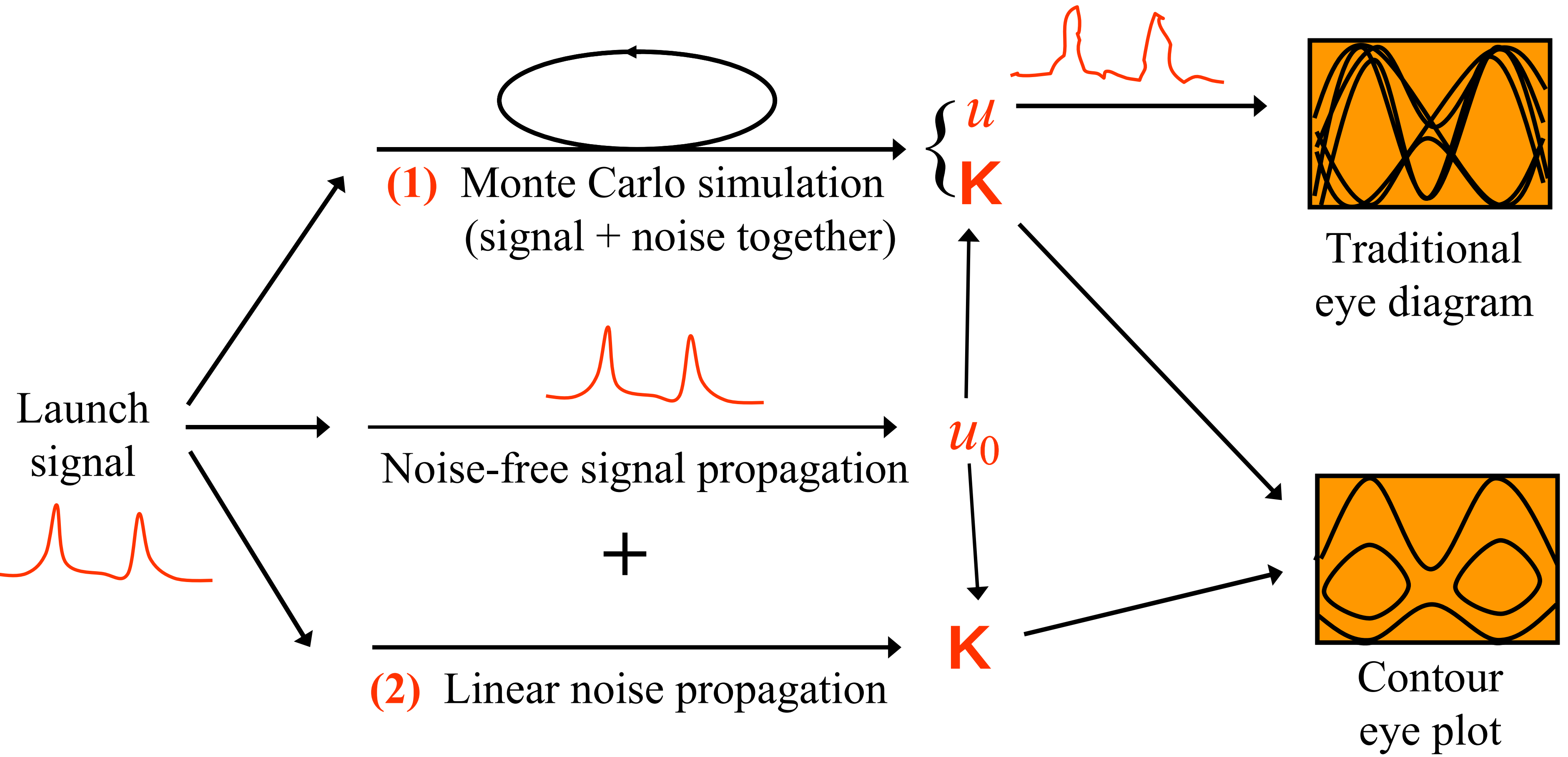
❷ Critical step: Phase and timing jitter separation

❸ Noise cross-correlations are significant in some systems

❹ Computational cost equals 200–300 noise-free simulation runs

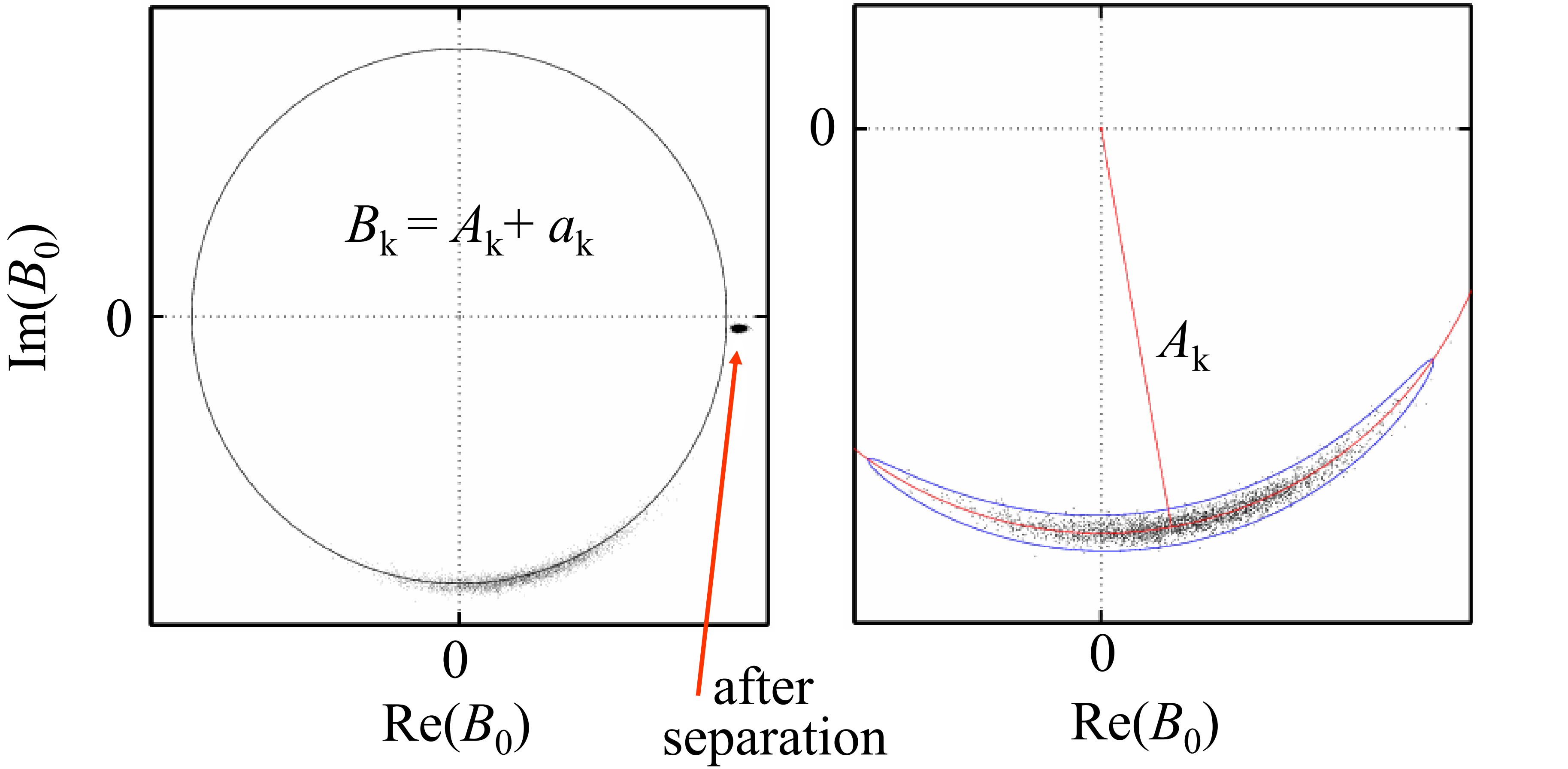
Covariance matrix method is a validation tool for other methods

Monte Carlo and Linearization



Jitter Separation at Work

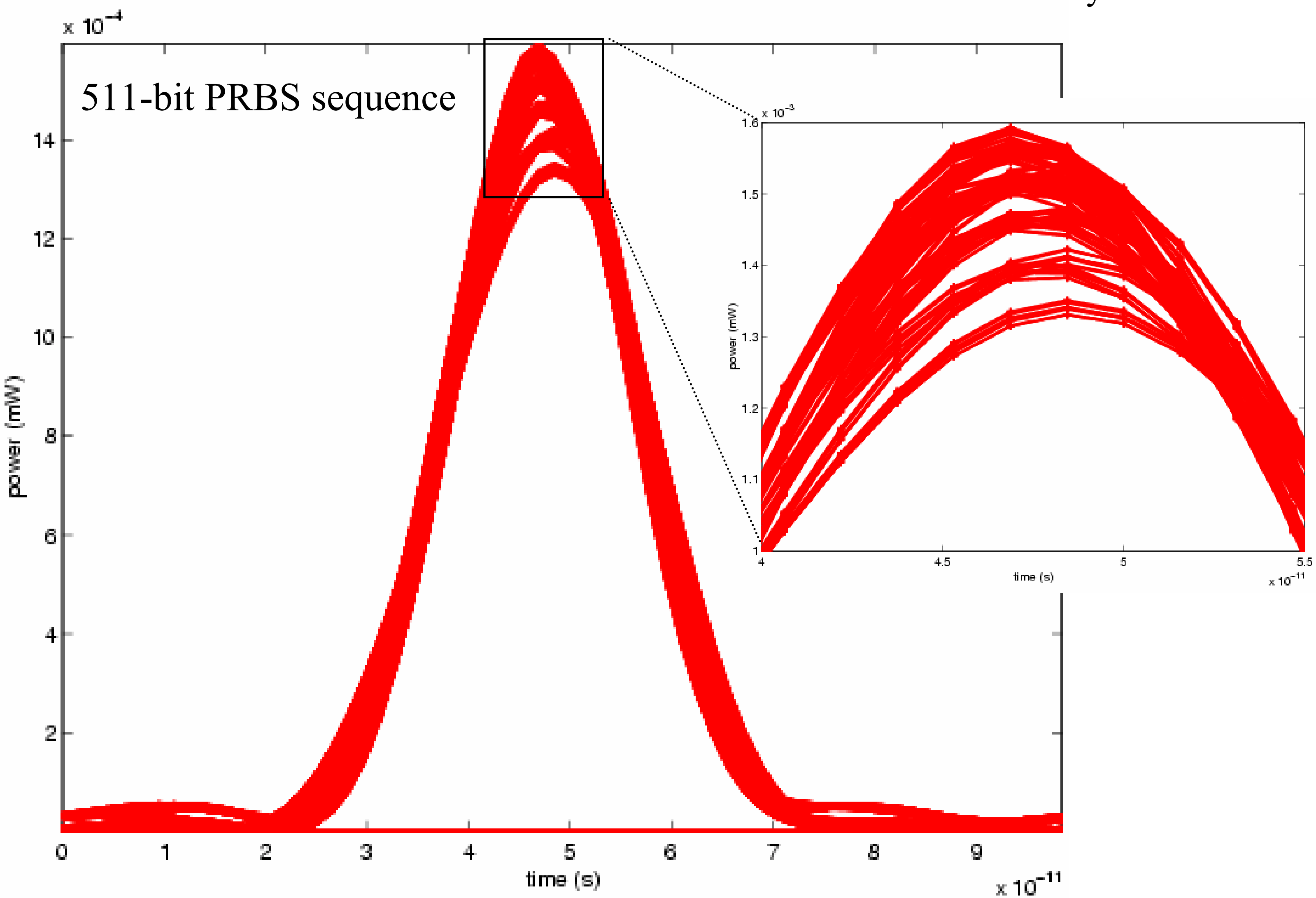
DMS system, 2550 Monte Carlo noise realizations, $n_{sp} = 1.2 \times 10^{-3}$



Phase jitter is Gaussian distributed in polar coordinates

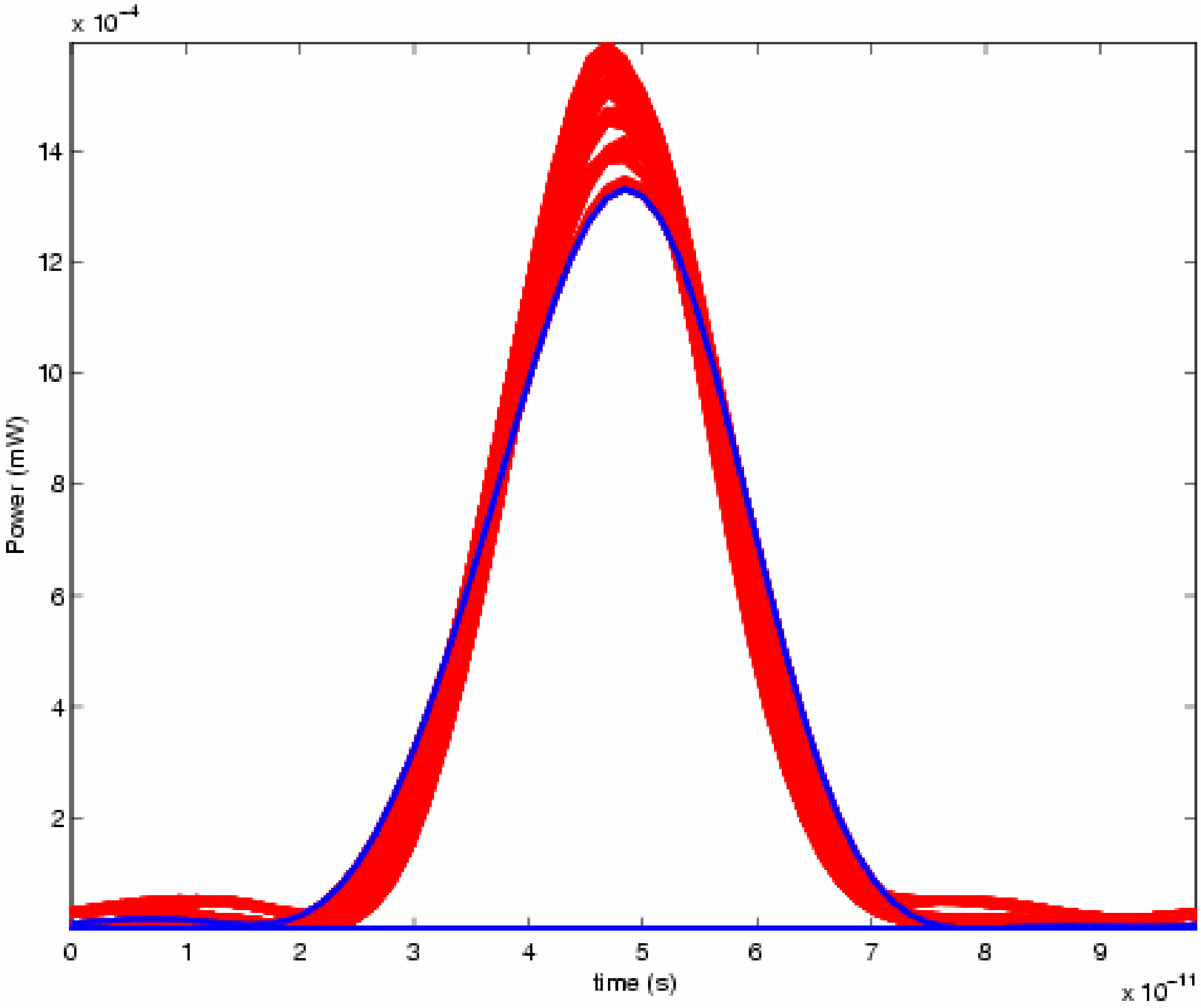
Multiple Bits in CRZ: Patterns

by Brian Marks

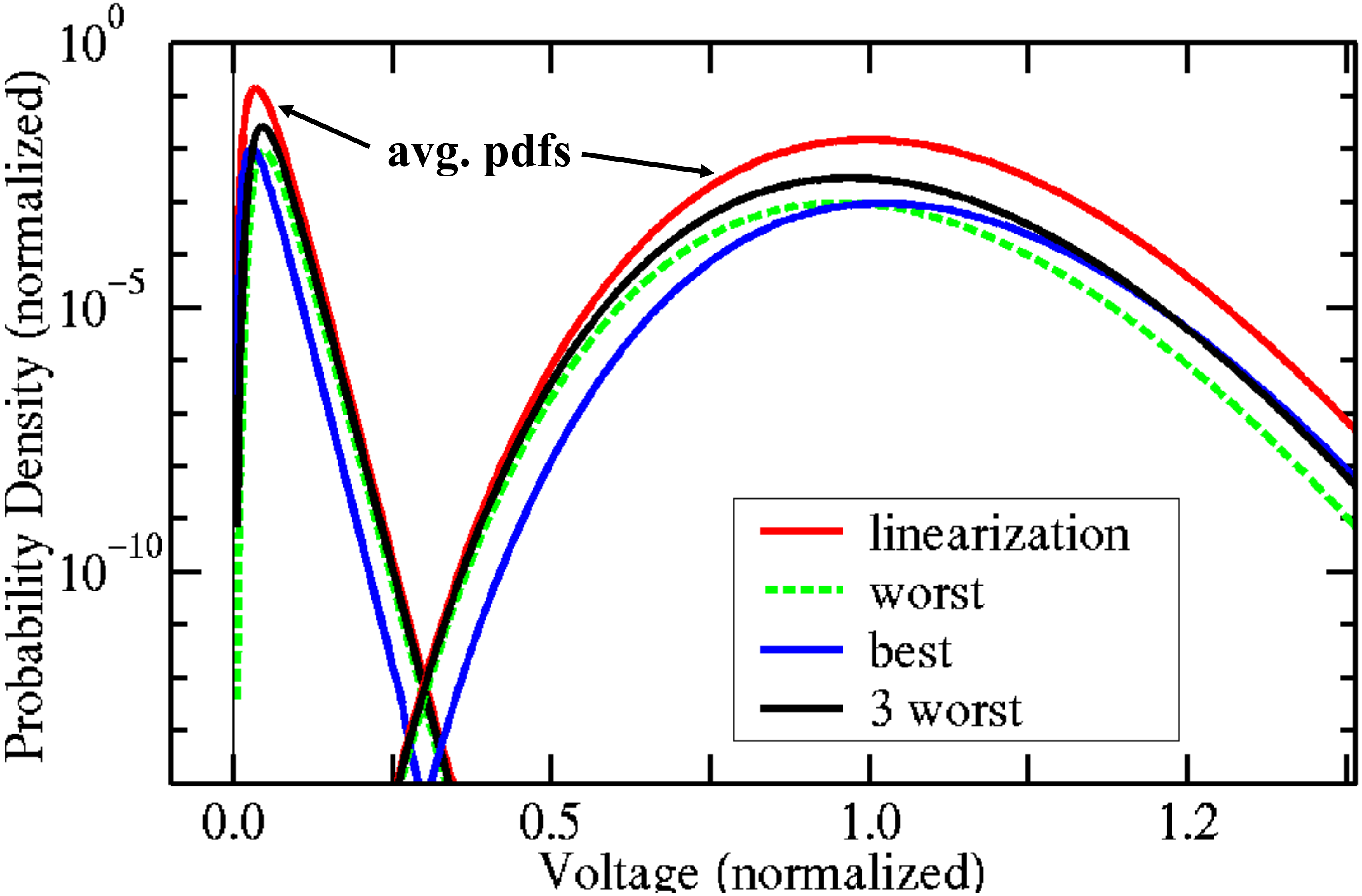


Isolation of Worst Pattern

by Brian Marks

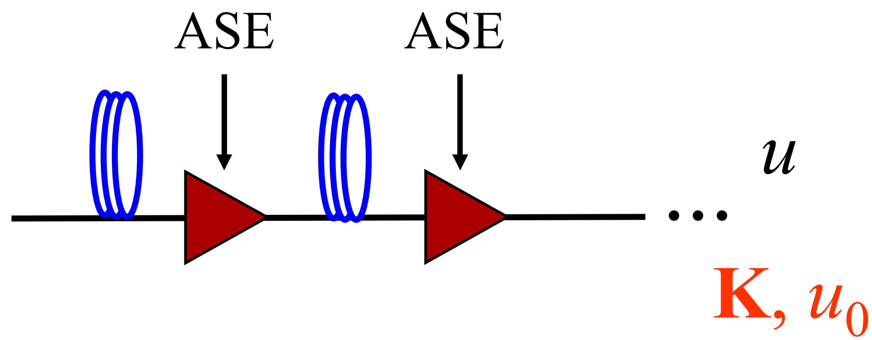


32 Bits CRZ: Focus on Worst Patterns

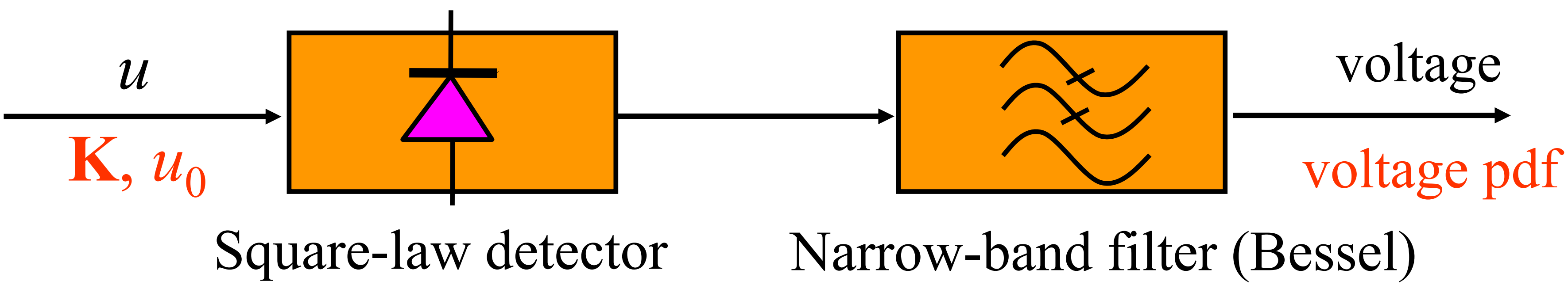


Simulation Setup

Propagation:



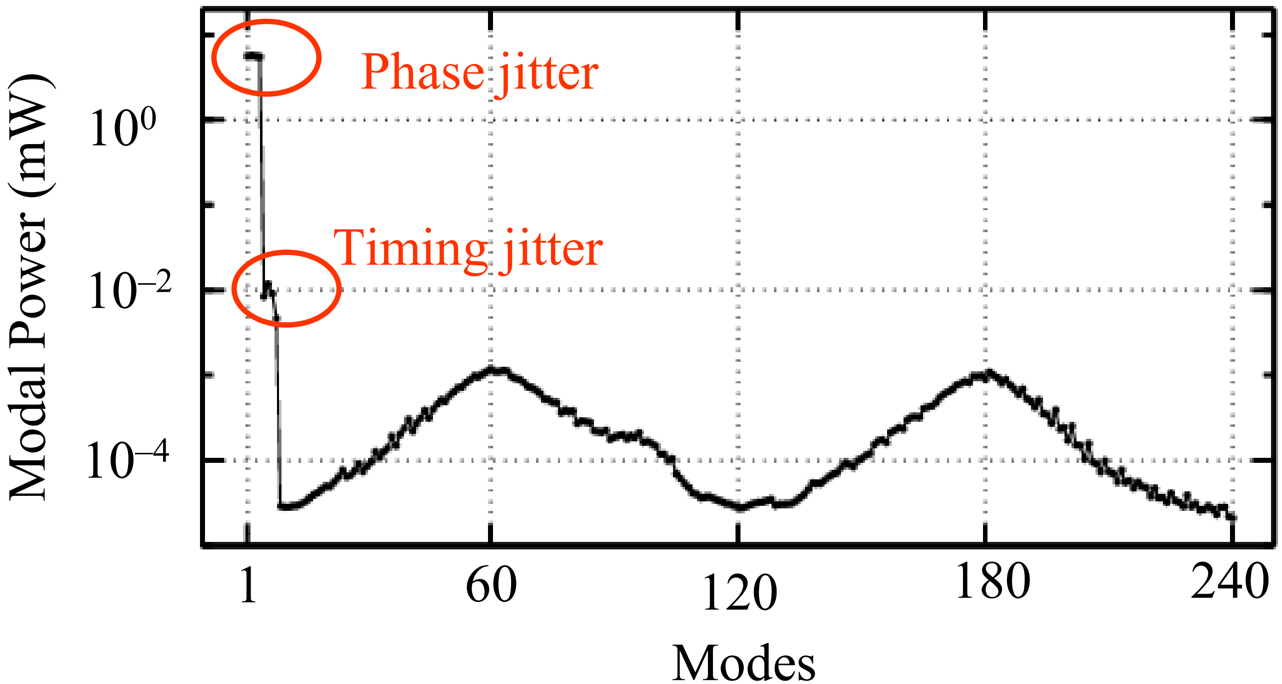
Receiver model:



Quadratic noise-noise terms in the receiver cannot be neglected !

Noise Modes

Diagonal elements in covariance matrix without jitter separation, jitter basis



Phase and timing jitter are significantly stronger than other modes