Optimization of the split-step Fourier method in modeling optical fiber communications systems

Oleg V. Sinkin, Ronald Holzlöhner, John Zweck, and Curtis R. Menyuk

Dept. of Computer Science & Electrical Engineering
University of Maryland Baltimore County
1000 Hilltop Circle, Baltimore, MD 21250

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Introduction

- Nonlinear Schrödinger equation – optical fiber transmission

$$i\frac{\partial u}{\partial z}-\frac{\beta''}{2}\frac{\partial^2 u}{\partial t^2}+\gamma|u|^2 u=0$$

- Split-step Fourier method
  - Derivatives in the frequency domain
  - Multiplication in the time domain
  - Easily applied to modifications of the NLS

Efficiency of the split-step method

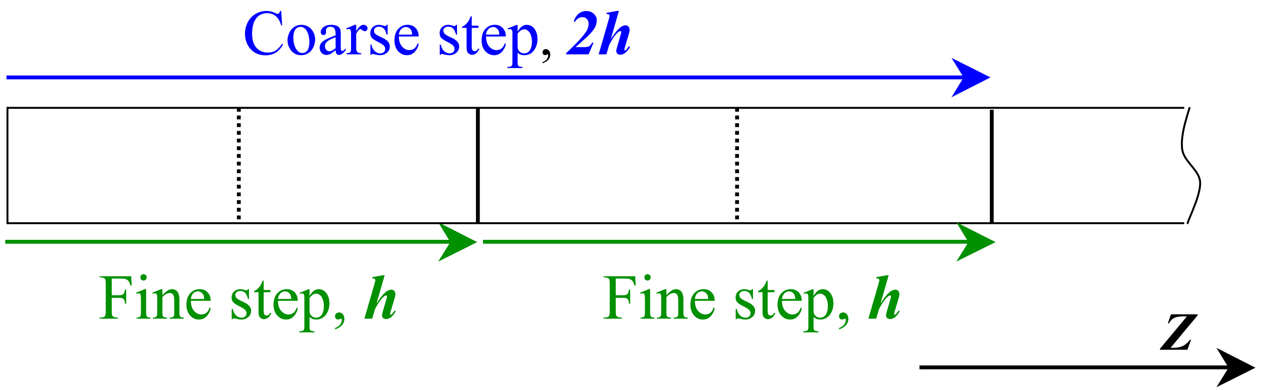
- Number of Fourier modes
  - Number of bits
  - Frequency bandwidth of the signal
- Step size distribution along the fiber
  - Step size selection based on physical intuition
  - System-independent adaptive scheme

We focus on the step size distribution along the fiber

Physical step size selection methods

- Constant step size method
- Nonlinear phase rotation method
  - Step size is inversely proportional to maximum power
  - Designed for solitons
- Logarithmic step size distribution [Bosco *et al*., PTL **12**, (2000)]
  - Reduces the spurious four-wave mixing
- Walk-off method
  - Step size is inversely proportional to local dispersion
  - Designed for WDM systems

Local error method



Coarse solution $$u_c = u_e + \kappa(2h)^3 + O(h^4)$$

Fine solution $$u_f = u_e + 2\kappa h^3 + O(h^4)$$

Higher-order solution $$u_4 = \frac{4}{3}u_f - \frac{1}{3}u_c = u_e + O(h^4)$$

Step size selection

Relative local error of the higher-order solution

$$\delta_4 = \frac{\left\| u_4 - u_e \right\|}{\left\| u_e \right\|}$$

$$\left\| u \right\| = \sqrt{\quad \left| u(t) \right|^2 dt}$$

Step size selection

Relative local error of the higher-order solution

$$\delta_4 = \frac{\left\| u_4 - u_e \right\|}{\left\| u_e \right\|}$$

Relative local error

$$\delta = \frac{\left\| u_f - u_c \right\|}{\left\| u_f \right\|}$$

estimate of the local error δ_4

$$\left\| u \right\| = \sqrt{\int \left| u(t) \right|^2 dt}$$

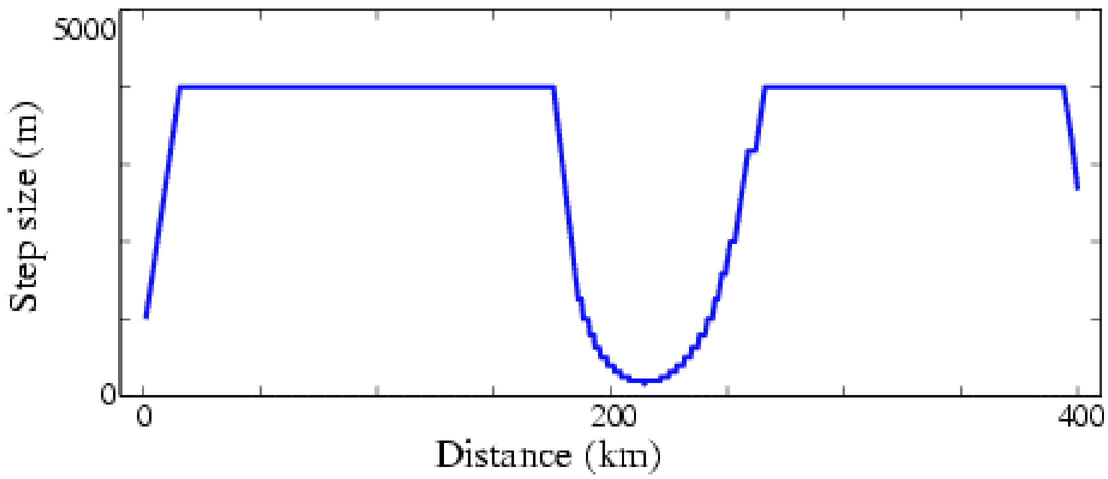
Step size chosen to keep δ within a specified range

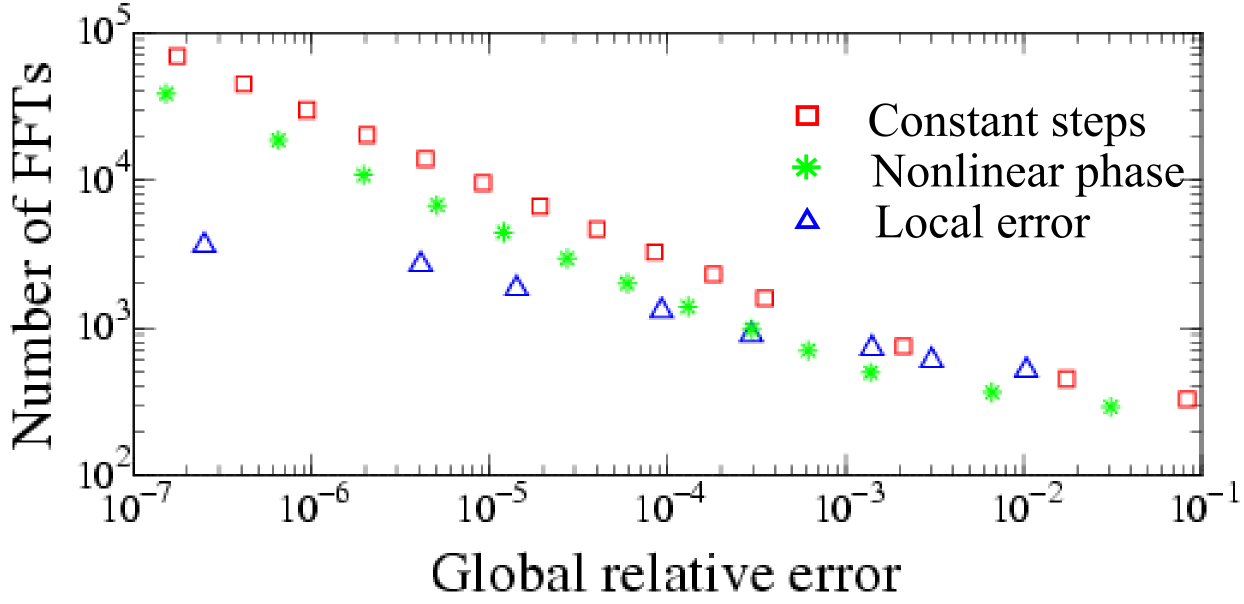
Comparison of different step size selection methods

- Total number of FFTs as a measure of computational cost
- Compute accurate solution u_a
- Compute solution u_n for each method
- Global relative error:

$$\varepsilon = \frac{\left\| u_a - u_n \right\|}{\left\| u_a \right\|}$$

Soliton collision





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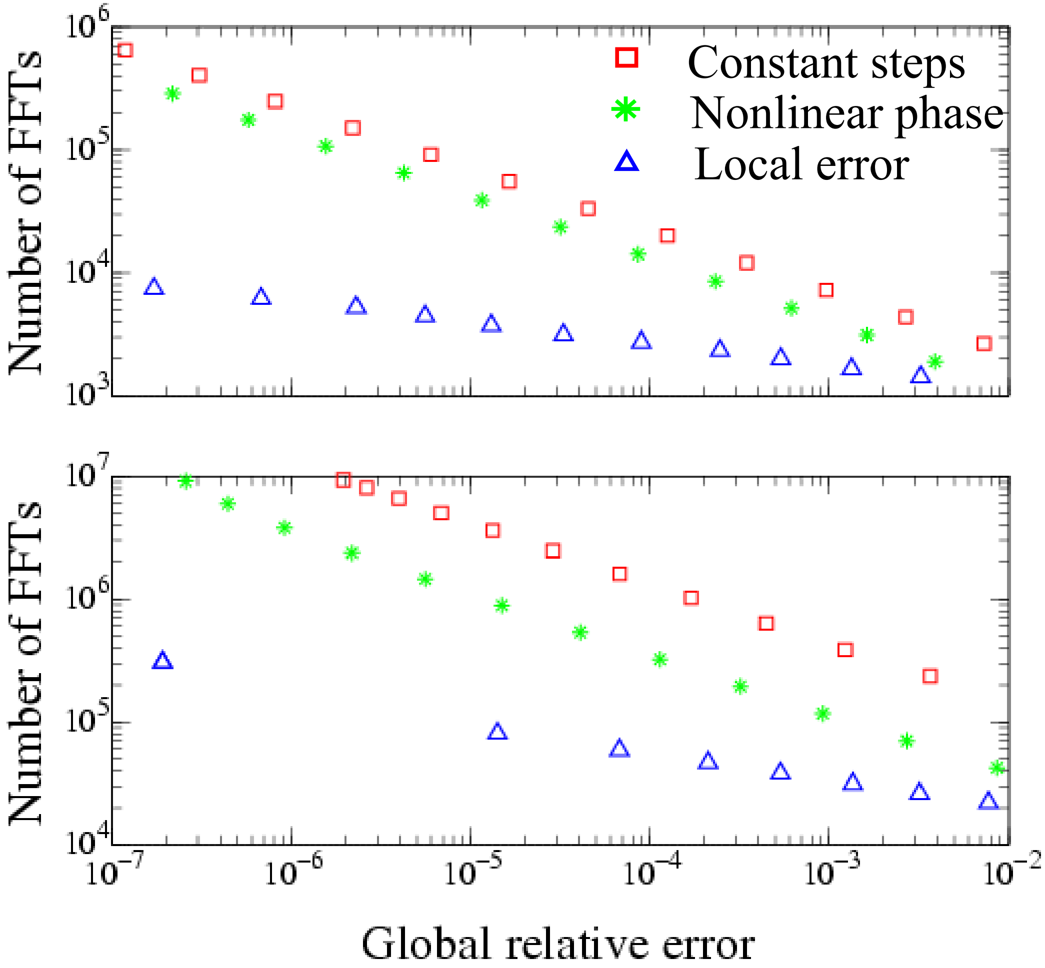
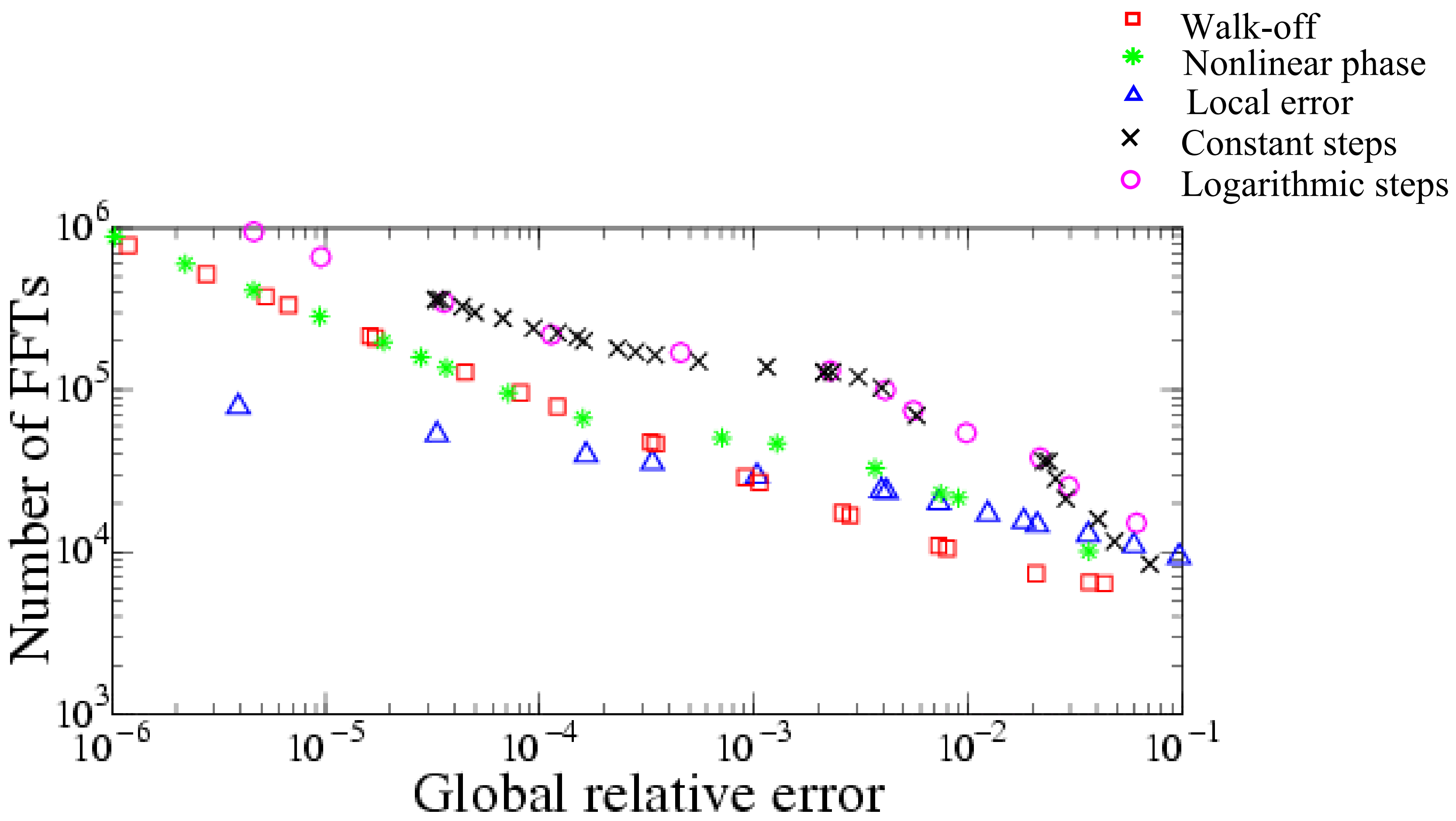
Numerical results: higher-order solitons

Second-order soliton

Fifth-order soliton



Five-channel CRZ system



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Conclusions

- Local error method
  - Step size adaptation does not rely on system knowledge
  - Robust for arbitrary systems
  - Most efficient for many systems
- Nonlinear phase rotation and logarithmic step size methods are inefficient for typical WDM systems
- Walk-off method performs well for WDM systems

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