New advances in modeling optical fiber communication systems

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1 Introduction

The design and optimization of modern optical fiber transmission systems relies on an increasingly close interaction between computer modeling and laboratory experiments. However, the full potential of modeling tools will not be realized as long as random processes, such as amplifier noise and fiber birefringence, are modeled using time-consuming Monte Carlo simulations. In practice Monte Carlo simulation is simply not capable of accurately computing error rates or outage probabilities, both of which are strongly influenced by extremely rare, worst-case events. By an outage probability we mean the probability that a system penalty exceeds an allowed margin. Even when Monte Carlo simulation is used to compute average measures of the system performance, such as the Q-factor, it can be too inefficient to be useful for optimizing the design of long-haul wavelength-division multiplexed (WDM) transmission systems. Moreover, a system that has been optimized to produce the best average behavior may not necessarily have the lowest possible error rate or outage probability [1].

A major focus of our research is to develop modeling tools that efficiently and accurately compute error rates and outage probabilities for realistic systems. In this paper we outline our recent progress on two important problems. First we have the computed accurate eye diagrams and bit error rates for a single-channel chirped return-to-zero (CRZ) system using linearization. Second, we have computed outage probabilities due to polarization effects in a WDM system using importance sampling.

2 Accurate eye diagrams and bit-error rates using linearization

Recently, we have developed a fully deterministic method to compute accurate probability density functions (pdfs) of the marks and spaces at the receiver taking into account the nonlinear and dispersive evolution of the signal and its interaction with the noise [2]. The method, which we call linearization, relies on the assumption that during transmission the nonlinear interaction of the noise with itself is negligible. We use a realistic receiver model that allows for arbitrary filter shapes and includes noise-noise beating. We have applied the method to a single-channel CRZ system propagating 32 bits over 6100 km, and validated it by comparison with results from Monte Carlo simulation. We focused on this system because it is well-characterized in simulations and it is similar to one of the channels in some commercial WDM submarine systems [3]. Very recently, we extended the method to a WDM CRZ system.

In the linearization method we first propagate a noise-free signal, u_0, through the transmission system. Under the linearization assumption, the noise that is added to the signal by the optical amplifiers evolves according to a linear equation whose coefficients depend on u_0. If the ASE noise that is input by the optical amplifiers is Gaussian white noise, the accumulated noise at the receiver is multivariate-Gaussian distributed, and is therefore determined by its covariance matrix. However, after a large enough number of nonlinear lengths, the accumulated noise is dominated by phase jitter. In other words, a large part of the noise just rotates the phase of u_0 and the linearization assumption breaks down. Nevertheless, for the CRZ system we studied, by removing the phase jitter component from the accumulated noise at each amplifier the distance over which the linearization is valid can be extended to over 6000 km. Because we use a square law receiver, the phase jitter does not affect the eye diagram or the bit-error rate (BER), and so we can ignore the phase jitter at the receiver.

The evolution of the noise covariance matrix $\mathcal{K}$ from one amplifier to the next is given by $\mathcal{K}_{n+1} = \Psi\mathcal{K}_n\Psi^T + \eta\mathcal{I}$, where Ψ is a propagator matrix and the term $\eta\mathcal{I}$ represents the lumped ASE noise input by the $(n+1)$-st amplifier. The k-th column of Ψ is the Fourier coefficient vector of the derivative at u_0, in the direction of the k-th Fourier mode, of the operator that is given by solving the nonlinear Schrödinger equation between the two amplifiers.

To ensure that the optical noise at the receiver is multivariate Gaussian distributed, the phase jitter must be correctly removed from the noise. The phase jitter is removed from the propagator matrix Ψ at each amplifier. Since each pulse in the signal can have a different phase, the phase jitter in the noise must be separately removed for each pulse. This task is easily accomplished for systems in which the pulses do not overlap. However, in the CRZ system there is a significant amount of pulse overlap. Nevertheless, the phase jitter can still be separately removed for each pulse in the signal, provided that we first separate the pulses from each other by linearly compensating for the total amount of accumulated dispersion. After the phase jitter is removed, the opposite dispersion is used to restore the signal shape.

In Fig. 1(a) we show the pdfs of the marks and spaces at the receiver for the single-channel CRZ system. The results for the linearization method, shown as solid lines, are based on a covariance matrix of dimension 280×280. These results are in excellent agreement with a histogram, shown using dots, obtained from a traditional Monte Carlo simulation with $65,000$ noise realizations. The dashed lines show a Gaussian fit to the Monte Carlo data. The large deviation between the solid and dashed curves is obvious. In summary, the method can compute accurate pdfs and the BER in a fraction of the time required to compute the Q-factor using a Monte Carlo simulation.

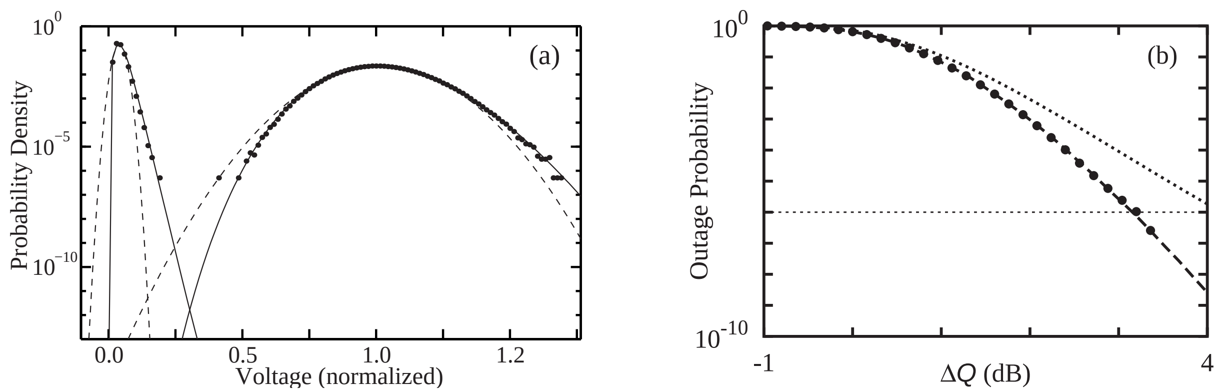

Fig. 1. (a) Probability distribution of the receiver voltage (b) Outage probability due to polarization effects

3 Computation of outage probabilities due to polarization effects using importance sampling

The polarization effects of polarization-mode dispersion (PMD) and polarization-dependent loss (PDL) play a significant role in long-haul WDM transmission systems. As the propagation distance increases, the random nature of the PMD causes the Stokes vectors of the different channels to gradually drift apart from each other on the Poincaré sphere. If the random realization of the fiber birefringence causes the Stokes vector of a given channel to be repeatedly aligned with the high-loss axes of the PDL elements, then the gain saturation of the optical amplifiers causes the channel to lose power relative to the other channels. The polarization penalty, ΔQ, is the amount by which the Q-factor of a channel is decreased due to polarization effects. Since ΔQ depends on the particular random realization of the fiber birefringence we need to compute the distribution of ΔQ values. We define the outage probability due to polarization effects to be the probability that ΔQ exceeds an allowed margin, such as 2 dB. Because outages are so rare, it has been difficult to obtain them from experiments or from standard Monte Carlo simulations.

We compute the ΔQ-distribution using the reduced Stokes model which follows the average power and Stokes parameters of the signal and noise in each WDM channel due to the polarization effects and the noise and gain saturation in optical amplifiers [4]. The model is valid as long as the PMD is not so large that it distorts the pulses within a single channel. The reduced model decreases the computational time of simulations of the polarization effects by several orders of magnitude compared to full time-domain simulations. Even so, until now efficient computation of outage probabilities as small as 10^{-6} has only been carried out by using Monte Carlo simulations to compute the mean and standard deviation of the ΔQ-distribution from which the tails of the distribution can be estimated using Gaussian extrapolation.

Recently, we applied importance sampling to resolve the tails of the ΔQ-distribution, which enabled us to obtain a more accurate computation of the outage probability due to PMD and PDL. Importance sampling is a statistical technique that makes efficient use of Monte Carlo simulations to accurately calculate the probabilities of rare events [5]. The idea is to bias the random process so that the rare events of interest occur more frequently than they would in a standard Monte Carlo simulation. It is important to understand that standard Monte Carlo is the same as unbiased importance sampling. For our application, the random process is the random fiber birefringence and the rare events of interest are large ΔQ values. For a given channel we bias the fiber birefringence so that at each amplifier the random angle θ between the polarization state of the channel and the polarization state with the greatest loss due to PDL tends to be much smaller than in the unbiased case. Specifically, we select $\cos\theta$ using the pdf $f_\alpha(\cos\theta) = \frac{\alpha}{2}[(\cos\theta + 1)/2]^{\alpha-1}$. By varying α we can statistically resolve the ΔQ-distribution in any desired range.

We tested the method by comparing it to standard Monte Carlo simulations using a trans-oceanic WDM system with eight 10 Gb/s return-to-zero channels spaced 124 GHz apart. The propagation distance was 8,910 km with an amplifier spacing of 33 km. The PDL was 0.2 dB per amplifier and the PMD was 0.1 ps/km$^{1/2}$. The reduced model has been validated for this system by comparison with a full time-domain model [4]. In Fig. 1(b) we show the outage probability versus the allowed ΔQ margin. The agreement between the importance sampling method shown as a dashed line and the standard Monte Carlo method shown as filled circles is excellent. On the other hand, when Gaussian extrapolation is used to estimate the outage probability, as shown with the dotted curve, the ΔQ-margin at an outage probability of 10^{-6} is overestimated by about 1 dB. The standard Monte Carlo simulation used 1.5×10^7 samples. By contrast, the Monte Carlo simulation using importance sampling used only 3×10^4 samples, which is a tiny fraction of the number of samples necessary to obtain an equivalent statistical resolution using a standard Monte Carlo simulation.

4 Conclusion

Modeling tools play a crucial role in the design of fiber-optic transmission systems. Accurate and efficient methods that supercede standard Monte Carlo simulations will soon be commonplace. In the past year we have accurately calculated the pdfs of the marks and spaces for a single-channel quasi-linear system with multiple bits, and used importance sampling to calculate outage probabilities due to polarization effects.

References

1. I. T. Lima Jr., G. Biondini, B. Marks, W. L. Kath, and C. R. Menyuk, "Analysis of polarization mode dispersion compensators using importance sampling," in *Proc. OFC'01*, Anaheim, CA, 2001, paper MO4.
2. R. Holzlöhner, C. R. Menyuk, V. S. Grigoryan, and W. L. Kath, "Accurate calculation of eye diagrams and error rates in long-haul transmission systems using linearization," *J. Lightwave Technol.*, (accepted, to appear in March 2002).
3. R.-M. Mu, T. Yu, V. S. Grigoryan, and C. R. Menyuk, "Convergence of the CRZ and DMS formats in WDM systems using dispersion management," in *Proc. OFC'00*, Baltimore, MD, 2000, paper FC1.
4. D. Wang and C. R. Menyuk, "Calculation of penalties due to polarization effects in a long-haul WDM system using a Stokes parameter model," *J. Lightwave Technol.*, vol. 19, pp. 487–494, 2001.
5. G. Biondini and W. L. Kath, "Non-Maxwellian DGD distributions of PMD emulators," in *Proc. OFC'01*, Anaheim, CA, 2001, paper ThA5.