ACCURATE EYE DIAGRAMS AND ERROR RATES USING LINEARIZATION

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Abstract: We find that the signal evolution of a chirped return-to-zero (CRZ) pulse over 6,100 km is fully linearizable, and the noise Fourier components are multivariate Gaussian-distributed. We introduce a deterministic method to accurately calculate eye diagrams and error rates, avoiding Monte Carlo simulations altogether.

Introduction

In current optical fiber communication systems, amplifier spontaneous emission (ASE) noise sets the lower limit on the allowed system power [1], [2]. At the receiver, ASE noise leads to amplitude and timing jitter and deteriorates the bit error rate (BER). The noise spectrum is white over the spectral width of one channel. However, because of the optical fiber nonlinearity leading to cross-phase modulation and four-wave mixing, the noise interacts with the signal in a complex way that usually increases the signal degradation. The traditional computation of the probability distribution function (pdf) of the electrical signal in the receiver, based on Monte-Carlo simulations, only works for a limited range of BERs beyond which the BER has to be extrapolated [3]. Extrapolation methods to date assume that the noise power is Gaussian-distributed. This assumption sometimes yields good results in comparison of theory and experiment [4], but it is not always reliable.

In this contribution, we consider the transmission of a chirped return-to-zero (CRZ) pulse [5] over a distance of 6,100 km. Due to the low peak power, relative to the soliton format, we find that the Fourier components are all Gaussian-distributed. Both phase and timing jitter are very small. Hence, we infer that the noise-noise interaction is negligible, and the system is linearizable. In formats with higher optical peak powers such as in soliton systems, phase and timing jitter grow much faster and must be removed in order to maintain the linearity [6]. In a linearizable system, the distribution of the accumulated noise is multivariate Gaussian with zero mean. This pdf is completely determined by the covariance matrix $K_{kl} = \langle b_k b_l^* \rangle$ of the noise Fourier modes b_k. We employ the standard split-step algorithm to numerically compute the evolution of the covariance matrix. We may then calculate the pdf of the narrow-band filtered receiver voltage in the marks and the spaces. This last step is a generalization of [7], in that the diagonal elements of K are not equal and we take into account realistic electrical filtering. Note that our approach is fully deterministic and does not rely at all on Monte Carlo simulations. To verify our results, we compare them to a standard Monte Carlo simulation.

We are extending previous work that showed that the linearization is valid when calculating the amplitude and timing jitter for several different formats [8].

Theory and Results

Our simulated transmission line consists of 34 dispersion maps of length 180 km each, or 6,120 km in total [5]. Each map contains a normal span of 160 km and dispersion $D_n = -2.44$ ps/nm-km and an anomalous span of 20 km and $D_a = 16.55$ ps/nm-km. The average dispersion is $D_{\text{av}} = -0.33$ ps/nm-km. The fiber loss is compensated by four EDFAs per map. We use a pre- and postcompensating fiber of length 9.8 km each with $D_c = 93.5$ ps/nm-km. The spontaneous emission factor is $n_{\text{sp}} = 1.0$. A signal pulse fills the entire bit window of duration $T_{\text{bit}} = 100$ ps and has a chirped raised-cosine shape of the form $u(t) = [U_0(1 + \cos \Omega t)/2]^{1/2} \exp(iA\pi \cos \Omega t)$, where $\Omega = 2\pi/T_{\text{bit}}$ and $A = -0.6$. The signal has an initial peak power of 1 mW before the precompensation. We transmit a single pulse surrounded by seven zero bits, where the zeros are required to keep the pulse tails from interacting with each other due to the periodicity of of the Fourier transform.

We consider the normalized nonlinear Schrödinger equation

$$i\frac{\partial u}{\partial z} + \frac{D(z)}{2}\frac{\partial^2 u}{\partial t^2} + |u|^2 u = ig(z)u + \hat{F}(z), \qquad (1)$$

where u is the optical field envelope, z and t represent normalized distance and time, respectively, $D(z)$ is the local dispersion and $g(z)$ represents attenuation in the fiber and amplification in the EDFAs. The quantity $\hat{F}$ is amplified spontaneous emission (ASE) noise input and is only nonzero inside the amplifiers. We will now express the optical field u as $u = u_0 + \delta u$, where $u_0 = \langle u \rangle$ is the noise free field and δu represents accumulated noise. The Fourier decomposition of δu can be written $\delta u(t) = \sum_{k=1}^{N} [\alpha_k + i\beta_k] \exp(i\omega_k t)$, where $\omega_k = 2\pi k/T$ with the period T, and α_k and β_k represent the real and imaginary noise Fourier coefficients. We define a partitioned, real vector of length $2N$ to be $\mathbf{a} = (\alpha_1, \alpha_2, \ldots, \alpha_N, \beta_1, \beta_2, \ldots, \beta_N)^T$. We verified that the α_k and β_k are individually Gaussian-distributed using a statistical chi-square test, which is a necessary prerequisite for $\mathbf{a}$ to be multivariate Gaussian-distributed. We now linearize (1) around u_0 by only keeping terms up to first order

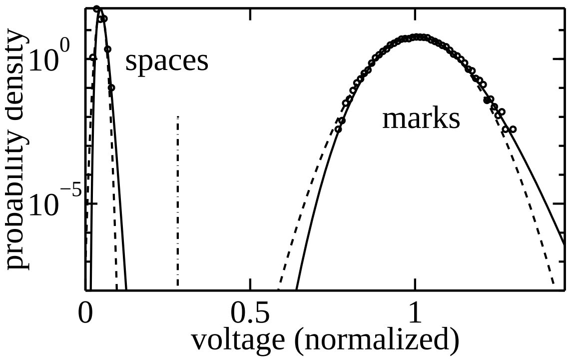

Figure 1: Pdf of the filtered receiver voltage. The solid lines show the deterministic pdf resulting from our approach and the dashed lines are Gaussian fits. The circles are results from a Monte Carlo simulation. The vertical dash-dotted line shows the optimal decision threshold.

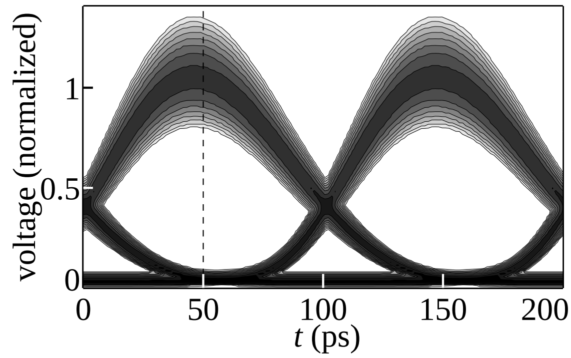

Figure 2: A contour plot of the voltage probability density displayed as an eye diagram. The dashed line at $\mathbf{t = 50}$ ps shows the location of the pdf in Figure 1. The logarithm of the pdf is displayed as different shades of gray. To obtain a more readable diagram, we only plot probability densities in the range $[\mathbf{10^{-4}, 10^{1}}]$. However our approach allows us to find the probability density for any voltage, thereby enabling us to calculate exact BERs.

in δu. Inside the fiber the linearized equation is homogeneous since $\hat{F} = 0$. Consequently, its Fourier transform must be linear and homogeneous as well, and can be written in terms of $\mathbf{a}$ as

$$\frac{\partial \mathbf{a}}{\partial z} = \mathcal{R}\mathbf{a}, \tag{2}$$

where $\mathcal{R}$ is a distance dependent real matrix of dimension $2N \times 2N$. We write the solution of (2) as $\mathbf{a}(z) = \Psi(z)\mathbf{a}(0)$ where Ψ is a propagator matrix. Defining the $2N \times 2N$ covariance matrix $\mathcal{K} = \langle \mathbf{a}\mathbf{a}^T \rangle$, we infer that the evolution of $\mathcal{K} = \mathcal{K}_{\text{in}}$ over one fiber leg followed by an EDFA is given by

$$\mathcal{K}_{\text{out}} = \Psi\, \mathcal{K}_{\text{in}}\, \Psi^T + \eta \mathcal{I}, \tag{3}$$

where $\mathcal{I}$ is the identity matrix and $\eta\mathcal{I}$ represents the lumped ASE noise input. We choose a perturbative method to compute Ψ rather than solving (2) directly. Let $u_0(0)$ and $u_0(L)$ be the noise free optical field at the beginning and end of a given fiber leg of length L, respectively. We then perturb the initial field in a single frequency mode $u^{(k)}(0) = u_0(0) + \Delta \exp(i\omega_k t)$, solve the NLS again for the perturbed field and obtain $u^{(k)}(L)$. The resulting noise vector $\mathbf{a}^{(k)}$ corresponding to $\delta u^{(k)} = u^{(k)}(L) - u_0^{(k)}(L)$. We then divide by Δ, which yields the k-th column of the matrix Ψ. We find this method to be very stable and accurate in comparison with the Monte Carlo simulation. The approach corresponds to the Lyapunov method described by Bennetin et al. [9]. We find the value of Ψ to be independent of Δ over a wide range.

Figure 1 shows the pdf of the voltage out of the square-law detector after passing through a 4.3 GHz Bessel filter, calculated from the covariance matrix $\mathcal{K}$. The final calculation of the pdf keeps the quadratic nonlinearity in the receiver and is a generalization of [7]; it yields a generalized chi-square distribution [10]. The spread of the pdf in the marks is much larger than in the spaces, reflecting the impact of signal-noise beating. The optimal decision level in Fig. 1 lies at 0.28 (see the dash-dotted line) and yields a BER of 1.2×10^{-35}. The low BER is due to the low value of $n_{\text{sp}} = 1.0$ and the lack of inter-symbol interference. Figure 2 shows the corresponding eye diagram.

Conclusion

We investigate the noise propagation in a chirped return-to-zero (CRZ) pulse over 6,100 km similar to [5]. We find that the real and imaginary Fourier coefficients are multivariate Gaussian-distributed and the signal-noise beating can be neglected, implying that the noise propagation is linearizable. Phase and timing jitter are both small compared to formats with higher peak powers such as soliton systems [6]. To determine the multivariate Gaussian noise distribution, we compute the covariance matrix of the noise Fourier modes by numerically solving the linearized noise propagation equation. Using these results, we can accurately calculate eye diagrams and the bit error rates, keeping the quadratic nonlinearity in the receiver. Our approach is fully deterministic and does not rely at any point on Monte Carlo simulations. Thus, it has great potential to speed up the design of optical transmission systems (the numerical computation of the eye diagram using our approach took about 15 minutes, while the Monte Carlo simulation of 23,000 realizations ran for several days). In the future, we will extend our approach to realistic pulse sequences and values of n_{sp}.

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