

# A MULTI-MEASUREMENT VECTOR APPROACH FOR ENDMEMBER EXTRACTION IN URBAN ENVIRONMENTS

Marian-Daniel Iordache<sup>1</sup>, Akpona Okujeni<sup>2</sup>, Sebastian van der Linden<sup>2</sup>, José M. Bioucas-Dias<sup>3</sup>, Antonio Plaza<sup>4</sup> and Ben Somers<sup>5,1</sup>

<sup>1</sup>Flemish Institute for Technological Research, Centre for Remote Sensing and Earth Observation Processes (TAP), Boeretang 200, 2400 Mol, Belgium

<sup>2</sup>Geography Department, Humboldt-Universität zu Berlin, Unter den Linden 6, 10099 Berlin, Germany

<sup>3</sup>Instituto de Telecomunicações and Instituto Superior Técnico, TULisbon, 1049-001, Lisbon, Portugal

<sup>4</sup>Hyperspectral Computing Laboratory, Department of Technology of Computers and Communications, University of Extremadura, E-10071 Caceres, Spain

<sup>5</sup>Department Earth and Environmental Sciences, Division Forest, Nature and Landscape, Katholieke Universiteit Leuven, Celestijnenlaan 200E - bus 2411, B-3001 Leuven, Belgium

## ABSTRACT

Hyperspectral images pose many challenges in endmember extraction applied to urban environments. The heterogeneity of the scenes and the relative high spatial resolution of the acquisition sensors lead to difficulties in the inference of the correct number of endmembers in the scene and possibly to the existence of a large number of pure pixels for one class of materials (*e.g.*, grass, tree, roof). Classical approaches, based on iterative schemes, aim mainly at selecting a set of distinct signatures from the image, which introduces a relative high instability of the solutions in the subsequent abundance estimation (unmixing) process. This paper introduces a method to build a large mixing matrix accounting for the variability of the endmembers in urban environments. The method, inspired from the multi-measurement vector problem, does not impose a fixed number of endmembers in the mixing matrix. Our experiments with a hyperspectral urban image acquired over Berlin, Germany, indicate that the method is able to select with high accuracy validated endmember spectra on the ground and ensures a high variability of the extracted signatures.

**Index Terms**— Hyperspectral, endmember extraction, urban, convex optimization, sparse regression

## 1. INTRODUCTION

The potential of hyperspectral remote sensing in support of urban management has been amply demonstrated [1]. Yet, the performance of image classification algorithms within heterogeneous environments is often impaired by mixed pixels [2, 3]. Urban areas are generally very diverse in terms of the number and extent of man-made and natural materials and the different materials are intimately mixed. Spectral unmixing then becomes an alternative processing step (and necessary when mixing complexity in the image is amplified) [3]. It amounts to estimating the abundance of pure spectral

This work has been supported by the European Community's Marie Curie Research Training Networks Programme under contract MRTN-CT-2006-035927, Hyperspectral Imaging Network (HYPER-I-NET). Funding from the Spanish Ministry of Science and Innovation (CEOS-SPAIN project, reference AYA2011-29334-C02-02) is also gratefully acknowledged.

constituents (endmembers) in each (possibly mixed) pixel (a comprehensive review of available unmixing algorithms can be found in [4]). In recent years, several endmember extraction algorithms have been proposed for automated endmember extraction from hyperspectral data sets. Traditionally, endmember extraction algorithms extract/select only one single standard endmember spectrum for each of the presented endmember classes or scene components. The use of fixed endmember spectra, however, is a simplification since in many cases the conditions of the scene components are spatially and temporally variable (*e.g.*, due to ageing, physical/chemical composition, surface anisotropy effects). As a result, variation in endmember spectral signatures is not always accounted for and, hence, spectral unmixing can lead to poor accuracy of the estimated endmember fractions [5]. Here, we address this issue by developing a simple strategy to select multiple endmembers (or bundles) per scene component. The methodology can be applied to update existing libraries with new spectra, such that variability is taken into account, or to build entirely new libraries by *a posteriori* refining the extracted signatures, which are then used as input to existing unmixing algorithms.

## 2. PROPOSED METHODOLOGY

The method we propose here is an adaptation of a recent sparse regression approach to hyperspectral unmixing, which exploits the fact that the observed data is generated by a relative small number of endmembers. This means that all the observed pixels can be expressed as linear combinations of only a few endmembers, possibly available in a spectral library (a large collection of pure spectra).

Let us assume that a spectral library  $\mathbf{A} = [\mathbf{a}_1, \dots, \mathbf{a}_m]$ , with  $m$  spectral signatures and  $L$  spectral bands, is available, and the data set contains  $n$  pixels organized in the matrix  $\mathbf{Y} = [\mathbf{y}_1, \dots, \mathbf{y}_n]$  (where each column  $\mathbf{y}_i$  stores the observed spectrum of one pixel). We may write then

$$\mathbf{Y} = \mathbf{A}\mathbf{X} + \mathbf{N}, \quad (1)$$

where  $\mathbf{X} = [\mathbf{x}_1, \dots, \mathbf{x}_n]$  is the abundance fraction matrix and  $\mathbf{N} = [\mathbf{n}_1, \dots, \mathbf{n}_n]$  is the noise matrix. The constraints  $\mathbf{x} \geq \mathbf{0}$  and  $\mathbf{1}^T \mathbf{x} = 1$  termed, in hyperspectral jargon, *abundance non-negativity con-*

straint (ANC) and *abundance sum-to-one constraint* (ASC), respectively, are often imposed into the model described in Eq. (1) [6]. When a spectral library is available, the unmixing problem can be tackled *via* a Multiple-Measurement Vector approach [7, 8, 9, 10], by solving the following optimization problem:

$$\begin{aligned} \min_{\mathbf{X}} \quad & \|\mathbf{X}\|_{\text{row}-0} \\ \text{subject to:} \quad & \mathbf{A}\mathbf{X} = \mathbf{Y} \\ & \mathbf{X} \geq 0. \end{aligned} \quad (2)$$

where  $\|\mathbf{X}\|_{\text{row}-0}$  represents the so-called row- $\ell_0$  norm of  $\mathbf{X}$ , which simply counts the non-zero rows of the matrix. In [11], the objective function (2) was adapted for endmember extraction. The spectral library  $\mathbf{A}$  can be replaced by the image itself [12],  $\mathbf{Y}$ , it means that every pixel in the image is treated as a potential endmember and the solution of the optimization problem (2) should provide a reduced set of pixels which explain with high accuracy the entire dataset, *i.e.* the image endmembers. The work [11] proposes two greedy algorithms (modifications of the Simultaneous Orthogonal Matching Pursuit (SOMP) [7, 8] and the Reduce MMV and Boost (ReMBo) [13], called SD-SOMP and SD-ReOMP, respectively, where "SD" stands for "Self Dictionary") to solve the optimization problem (2), and provides theoretical guarantees for the correct recovery of the endmember set in the noiseless case, for any number of distinct endmembers.

However, the greedy algorithms like the aforementioned ones update iteratively the set of endmembers and select, at each iteration, an endmember which is orthogonal to the previously selected ones. This means that the extracted endmembers are ideally orthogonal, so no variability is ensured. This might be a drawback in the consequent unmixing process, in which the extracted spectra can only approximate the ones on the ground. In this work, we propose a method which allows variability of endmembers as shown further.

By employing the image  $\mathbf{Y}$  as a self-dictionary, the optimization problem (2) can be rewritten in an equivalent form as follows:

$$\begin{aligned} \min_{\mathbf{C}} \quad & \|\mathbf{Y}\mathbf{C} - \mathbf{Y}\|_F^2 + \lambda \sum_{k=1}^n \|\mathbf{c}^k\|_2 \\ \text{subject to:} \quad & \mathbf{C} \geq 0, \end{aligned} \quad (3)$$

where  $\lambda$  is a regularization parameter weighting the two terms of the objective function,  $\mathbf{c}^k$  denotes the  $k$ -th line of the coefficients matrix  $\mathbf{C}$  and  $\mathbf{C} \geq 0$  is to be understood componentwise. The convex term  $\sum_{k=1}^n \|\mathbf{c}^k\|_2$  is the so-called  $\ell_{2,1}$  mixed norm which promotes sparsity among the lines of  $\mathbf{C}$ , *i.e.*, it promotes solutions of (3) with small number of nonzero lines of  $\mathbf{C}$ . We use the *Collaborative Sparse Unmixing via variable Splitting and Augmented Lagrangian* (CLSUnSAL) [14], based on the Alternating Direction Method of Multipliers (ADMM) [15], to solve the optimization problem (3). CLSUnSAL was originally designed to solve an unmixing problem when spectral libraries are available. Here, we use it in a similar fashion, the only difference being that the observed image is employed as a self-dictionary. By solving the optimization problem (3), CLSUnSAL provides a coefficients matrix  $\mathbf{C}$  with a set of non-zero rows corresponding to the endmembers in the image. Note that, while we do not enforce a fixed number of endmembers to be extracted, the value of  $\lambda$  can influence the number of extracted endmembers (higher lambda enforces more sparsity, thus less extracted endmembers).

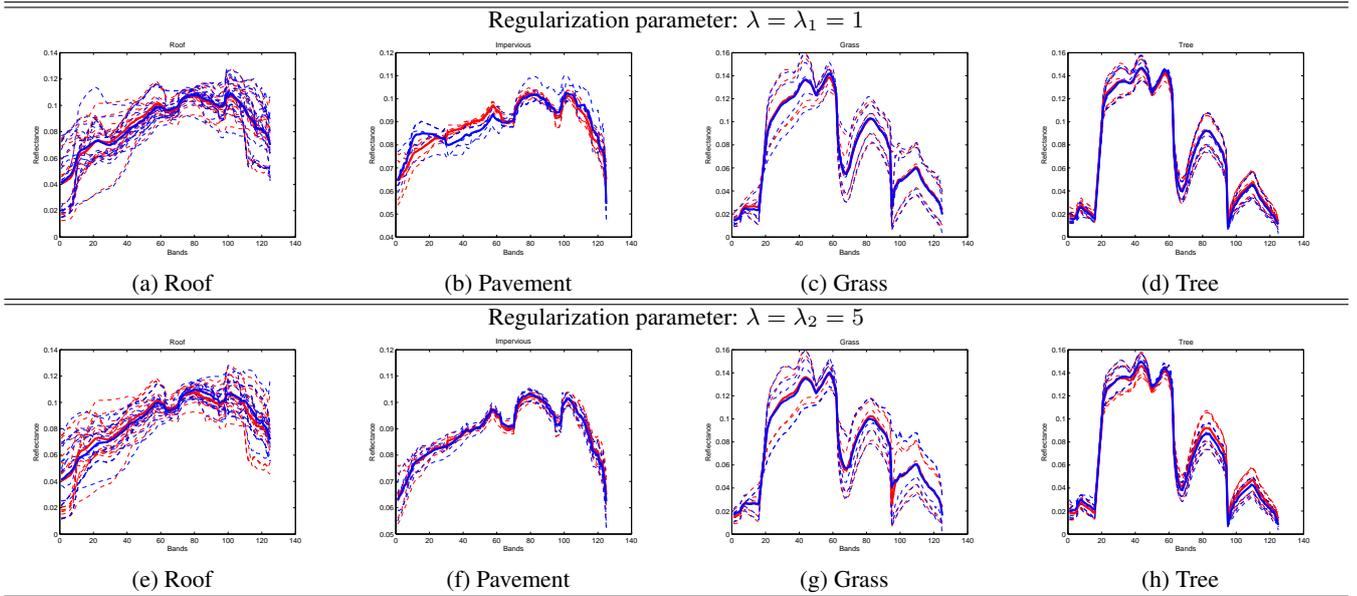
### 3. EXPERIMENTAL RESULTS

The dataset used in our experiments is a  $516 \times 514$  subset of a hyperspectral image covering the city of Berlin, Germany (see Fig. 2 – left). The scene was acquired by the HyMap sensor in 125 spectral bands, with a spatial resolution of 3.6m. A total number of 41 spectra from a comprehensive spectral library were selected as pure signatures of endmembers on the ground. The spectral library was developed from the image on the basis of expert knowledge and auxiliary information (field mapping data, Google Street View). The library spectra are categorized into four spectrally complex and similar urban land cover classes, *i.e.* roof and pavement as well as grass and tree. An extensive overview of the characteristics of the image, image-preprocessing and library development can be found in [16]. In the programming environment used for experimental tests (Matlab 7.10), the use of the entire subset as input to the endmember extraction algorithm was not possible due to memory usage issues. The overall computational complexity of the algorithm is  $\mathcal{O}(nL^2)$ . Thus, we split the image in 100 sub-images of similar size (approximately 2500 pixels each). The endmember extraction is performed in each of these sub-images and, at the end, the selected spectra from all sub-images are put together to form the matrix of extracted endmembers. We expect to build large libraries, partly because the endmembers are likely to repeat themselves in several parts of the image. This matrix can be further refined through different strategies (*e.g.*, pruning, clustering, sampling), which are out of the scope of this paper. Firstly, we are interested to see if the extracted set of endmembers contains spectra closely related to the reference ones, which were selected manually using *a priori* information.

We use, in turn, two distinct values of the regularization parameter  $\lambda$ :  $\lambda_1 = 1$  and  $\lambda_2 = 5$ , for all cases. After building the extracted spectral libraries, the total number of selected spectra was 2129 and 1472, respectively. For the two parameters and five example endmembers from the four classes, Fig. 1 shows the following features: dashed red lines - the manually selected reference spectra, dashed blue lines - the most similar extracted signatures to the reference spectra, thick red line - mean of the reference spectra, thick blue line - mean of the extracted signatures. The similarity is measured in terms of spectral angle distance (SAD). Note, in Fig. 1 that the extracted libraries contain indeed very similar spectra to the reference ones. The same behavior is observed when the spectra similarity is measured by the Euclidean Distance (ED). It is also interesting to see that the extracted endmembers for the pavement class seem to be less accurate for  $\lambda = 1$ . However, in the previous work [16], in which the same reference spectra were used for unmixing, it was shown that the pavement class is unmixed with less accuracy than other classes, due to spectral similarities to other classes, ambiguities and shadowing.

We have seen, so far, that the method is able to identify with good accuracy the reference spectra in the image. However, due to the sub-image based processing employed, the number of identified endmembers is high, as the same endmember is likely to appear in distinct blocks (*e.g.*, pavement spectra are extracted in most of the sub-images). Although we do not investigate here optimal methods for retaining the most useful extracted spectra, some possible options are: library pruning, clustering, averaging, direct identification using reference spectra, re-running CLSUnSAL using the extracted pixels as input. In this paper, we make use of the available reference spectra to retain, from the extracted set of pixels, the most similar five to each reference spectrum, in terms of spectral angle. This way, we build an extended library with 205 atoms.

The ground-truth abundance fractions corresponding to the ref-



**Fig. 1.** Quality of the extracted endmembers when  $\lambda = \lambda_1 = 1$  (top) and  $\lambda = \lambda_2 = 5$  (bottom).



**Fig. 2.** HyMap subset and reference polygons for validation.

reference spectra are known in the considered image at a block level, as shown in Fig. 2 (left – positioning of the polygons in the image; right – dominant classes inside the polygons; for more details, see [16]). Here, we are interested to evaluate the impact of using extracted spectra on the unmixing accuracy, as compared to the one obtained using reference spectra.

To evaluate the impact of using the extracted spectra on the mapping accuracy, we used the reference library, denoted as  $\mathbf{A}_r$ , and the extracted library, denoted as  $\mathbf{A}_s$ , as input to a per-pixel sparse unmixing algorithm, from which CLSUnSAL is derived, called *Sparse Unmixing via variable Splitting and Augmented Lagrangian (SUnSAL)* [17]. SUnSAL estimates the vector of fractional abundances  $\mathbf{x}$  by solving the following  $\ell_2 - \ell_1$  norm optimization problem:  $\min_{\mathbf{x}} \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2^2 + \lambda_S \sum_{k=1}^n \|\mathbf{x}\|_1$  subject to:  $\mathbf{x} \geq 0$ , where  $\mathbf{y}$  is the current observed pixel,  $\mathbf{A}$  is the spectral library and  $\lambda_S$  is a regularization parameter which weights the two terms of the objective function. In our experiments,  $\lambda_S$  was manually tuned for near-optimal performance. The accuracy of fraction maps was evaluated by comparing modeled versus reference fractions using polygon-wise averages (Fig. 3). Reference data is shown in Fig. 2. Reference fractions per polygon were available for 35 polygons, includ-

ing building blocks (black squares), streets (with triangles) and green spaces (black triangles).

From Fig. 3, it can be seen that SUnSAL performance is slightly degraded when the extracted library is used. This was expected, as the performance of sparse regression algorithms is greatly influenced by the library size. However, it is obvious that the results follow the same trend and the abundances estimated using  $\mathbf{A}_s$  are closely related to the ones obtained using  $\mathbf{A}_r$ . We should also mention that the accuracy assessment is performed in regions where the reference spectra are known to exist, thus the superior performance of SUnSAL when using the reference library might not be so obvious in other parts of the image, where the extracted spectra can be more representative than the reference ones.

#### 4. CONCLUSIONS

In this paper, we introduced a Multiple-Measurement Vector approach to endmember extraction for urban hyperspectral images. The endmember extraction is formulated as a convex optimization problem and a sparse regression algorithm is employed to retrieve the endmembers. Our experimental results show that the method is able to extract endmembers highly correlated to the ground-truth ones and it is able to ensure a high variability of the extracted endmembers, which is useful in the subsequent unmixing procedure.

#### 5. REFERENCES

- [1] U. Heiden, W. Heldens, S. Roessner, K. Segl, T. Esch, and A. Mueller, “Urban structure type characterization using hyperspectral remote sensing and height information,” *Land-scape and Urban Planning*, vol. 105, pp. 361–375, 2012.
- [2] S. Roessner, K. Segl, U. Heiden, and H. Kaufmann, “Automated differentiation of urban surfaces based on airborne hyperspectral imagery,” *IEEE Transactions on Geoscience and Remote Sensing*, vol. 39, no. 7, pp. 1525–1532, 2001.

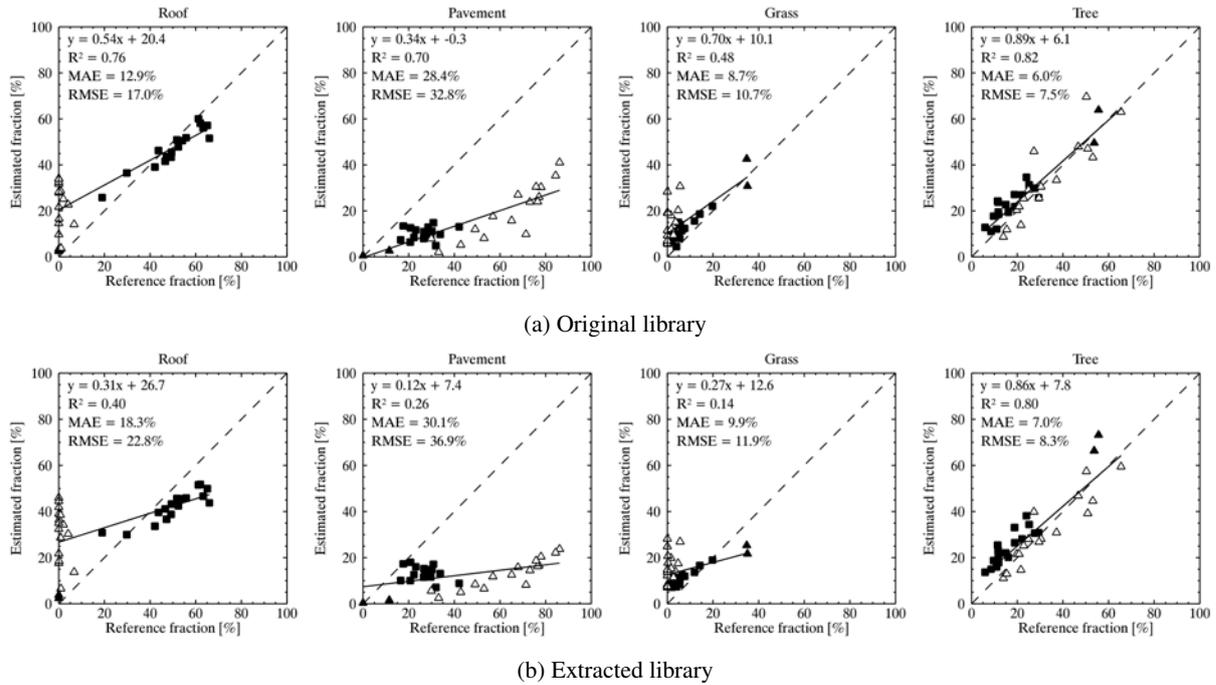


Fig. 3. SUNSAL performance when using as input: a)  $A_R$  and b)  $A_S$ .

- [3] J. Franke, D.A. Roberts, K. Halligan, and G. Menz, "Hierarchical multiple endmember spectral mixture analysis (mesma) of hyperspectral imagery for urban environments," *Remote Sensing of Environment*, vol. 113, no. 8, pp. 1712–1723, 2009.
- [4] J. M. Bioucas-Dias, A. Plaza, N. Dobigeon, M. Parente, Q. Du, P. Gader, and J. Chanussot, "Hyperspectral unmixing overview: Geometrical, statistical and sparse regression-based approaches," *IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing*, vol. 5, no. 2, pp. 354–379, 2011.
- [5] B. Somers, G.P. Asner, L. Tits, and P. Coppin, "Endmember variability in spectral mixture analysis: a review," *Remote Sensing of Environment*, vol. 115, no. 7, pp. 1603–1616, July 2011.
- [6] D. Heinz and C.-I. Chang, "Fully constrained least squares linear mixture analysis for material quantification in hyperspectral imagery," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 39, pp. 529–545, 2001.
- [7] J. Chen and X. Huo, "Theoretical results on sparse representations of multiple measurement vectors," *Mathematical Programming*, vol. 55, no. 12, pp. 4634–4643, 2006.
- [8] A. Gilbert J. Tropp and M. Strauss, "Algorithms for simultaneous sparse approximation. Part I: Greedy pursuit," *Signal Processing*, vol. 86, no. 3, pp. 572–588, 2006.
- [9] J. Tropp, "Algorithms for simultaneous sparse approximation. part ii: Convex relaxation," *Signal Processing*, vol. 86, no. 3, pp. 589–602, 2006.
- [10] M.-D. Iordache, J. Bioucas-Dias, A. Plaza, and B. Somers, "Music-csr: Hyperspectral unmixing via multiple signal classification and collaborative sparse regression," *IEEE Transactions on Geoscience and Remote Sensing (accepted)*, 2014.
- [11] X. Fu, W.-K. Ma, T.-H. Chan, J.M. Bioucas-Dias, and M.-D. Iordache, "Greedy algorithms for pure pixels identification in hyperspectral unmixing: a multiple-measurement vector viewpoint," *21st European Signal Processing Conference*, Marrakech, Morocco, 2013.
- [12] Ernie Esser, Michael Moller, Stanley Osher, Guillermo Sapiro, and Jack Xin, "A convex model for nonnegative matrix factorization and dimensionality reduction on physical space," *Image Processing, IEEE Transactions on*, vol. 21, no. 7, pp. 3239–3252, 2012.
- [13] M. Mishali and Y. Eldar, "Reduce and boost: Recovering arbitrary sets of jointly sparse vectors," *IEEE Transactions on Signal Processing*, vol. 56, no. 10, pp. 4692–4702, October 2008.
- [14] M.-D. Iordache, J.M. Bioucas-Dias, and A. Plaza, "Collaborative sparse regression for hyperspectral unmixing," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 52, pp. 341–354, 2014.
- [15] J. Eckstein and D. Bertsekas, "On the Douglas–Rachford splitting method and the proximal point algorithm for maximal monotone operators," *Mathematical Programming*, vol. 5, pp. 293–318, 1992.
- [16] A. Okujeni, S. van der Linden, L. Tits, B. Somers, and P. Hostert, "Support vector regression and synthetically mixed training data for quantifying urban land cover," *Remote Sensing of Environment*, vol. 137, pp. 184–197, 2013.
- [17] J. Bioucas-Dias and M. Figueiredo, "Alternating direction algorithms for constrained sparse regression: application to hyperspectral unmixing," *2nd Workshop on Hyperspectral Image and Signal Processing: Evolution in Remote Sensing*, vol. 1, pp. 1–4, Reykjavik, Iceland, 2010.