

# SUPERPIXEL-GUIDED SPARSE UNMIXING FOR REMOTELY SENSED HYPERSPECTRAL IMAGERY

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## ABSTRACT

Sparse representation-based approaches have been successfully applied to remotely sensed hyperspectral image unmixing. In recent years, sparse unmixing techniques have incorporated spatial information into the sparse unmixing model, achieving improved fractional abundance results. Most spatial-based sparse unmixing methods utilize regular-shaped neighborhoods (e.g., a cross or a square window) to characterize the spatial-contextual information around each pixel. However, the spatial characteristics of natural scenes are not always uniform, but vary according to the observed objects. Therefore, assuming uniform spatial neighborhoods may not be consistent with real spatial structures in the scene. Superpixels offer a good solution to this problem since they can better characterize such spatial structures. Based on this observation, in this paper we develop a new superpixel-guided sparse unmixing (SPGSU) method for hyperspectral scenes. The proposed SPGSU includes the spatial correlation through a superpixel-based technique rather than assuming predefined pixel grids. Each superpixel can be regarded as a small spatial region, whose shape and size can be adaptively changed to accommodate different spatial structures. Our experimental results, conducted using simulated data sets, quantitatively indicate that our newly proposed method produces better results than other advanced spectral unmixing methods.

**Index Terms**— Hyperspectral imaging, spatial-based sparse unmixing, superpixels, spatially-weighted unmixing.

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## 1. INTRODUCTION

Due to the (often limited) spatial resolution of imaging spectrometers, mixed pixels often dominate remotely sensed hyperspectral images [1]. To deal with this problem, spectral unmixing has been widely used to identify the spectrally pure components in a scene (endmembers) and to estimate the fractional abundances of such pure spectral signatures in each mixed pixel [2]. Conventional (linear) spectral unmixing algorithms may face difficulties when extracting the endmembers directly from the scene, since both the estimation of the number of endmembers and the actual presence of spectrally pure components in the scene may not be feasible in practice [1]. Sparse unmixing [3], as a semi-supervised approach in which mixed pixels are expressed in the form of combinations of a number of pure spectral signatures available *a priori* in a large spectral library, can alleviate the aforementioned drawbacks. For instance, the sparse unmixing algorithm via variable splitting and augmented Lagrangian (SUnSAL) [3], the collaborative SUnSAL (CLSUnSAL) [4], and the double reweighted sparse unmixing (DRSU) [5] algorithms have been successfully applied for spectral unmixing purposes. Although these methods obtained promising results, they consider pixels in the hyperspectral scene as independent entities, disregarding the spatial-contextual information in the hyperspectral image [6].

To address the aforementioned limitation, some spatial-based sparse unmixing algorithms have been proposed that include the spatial information on the final solution. Examples include the sparse unmixing via variable splitting augmented Lagrangian and total variation (SUnSAL-TV) [6], the nonlocal sparse unmixing (NLSU) [7] or the centralized collaborative sparse unmixing (CCSU) [8]. They generally adopt regular-shaped neighborhoods (e.g., a cross or a square window) to characterize the spatial-contextual information around each pixel. However, the spatial characteristics of natural scenes are not always uniform, but vary according to the

observed objects. Therefore, assuming uniform spatial neighborhoods may not be consistent with real spatial structures in the scene.

Superpixels offer a good solution to this problem since they can better characterize such spatial structures. For instance, the simple linear iterative clustering (SLIC) algorithm [9] has been widely used to construct superpixels in remotely sensed hyperspectral images. In this paper, we develop a new superpixel-guided sparse unmixing (SPGSU) method for hyperspectral scenes. The proposed SPGSU includes the spatial correlation using SLIC to construct superpixels that are later used to guide the sparse unmixing process. Each superpixel can be regarded as a small spatial region, whose shape and size can be adaptively changed to accommodate different spatial structures. The proposed SPGSU aims at obtaining a more accurate and reliable characterization of spatial-contextual information that is constrained simultaneously from the spectral and spatial domains under the  $\ell_1$  framework. The spectral weight, following previous developments [5, 10], enforces the sparsity of non-zero rows corresponding to the true endmembers in the estimated abundances. The superpixel-related spatial weight aims at promoting spatially homogenous regions in the image.

The remainder of the paper is organized as follows. Section 2 describes some related works. Section 3 describes the proposed SPGSU method in detail. Section 4 presents our experimental results with simulated hyperspectral scenes. Finally, section 5 concludes the paper with some remarks and hints at plausible future research lines.

## 2. RELATED WORKS

Let  $\mathbf{Y} = [y_1, \dots, y_n] \in \mathbb{R}^{d \times n}$  denote a hyperspectral image, where  $n$  is the number of pixel vectors and  $d$  is the number of bands. Let  $\mathbf{A} \in \mathbb{R}^{d \times m}$  be a large spectral library, where  $m$  is the number of spectral signatures in  $\mathbf{A}$ , and  $\mathbf{X} = [x_1, \dots, x_n]$  denotes the abundance maps corresponding to library  $\mathbf{A}$  for the observed data  $\mathbf{Y}$ . With the aforementioned definitions in mind, sparse unmixing finds a linear combination of endmembers for  $\mathbf{Y}$  from the spectral library  $\mathbf{A}$ :

$$\mathbf{Y} = \mathbf{A}\mathbf{X} + \mathbf{N} \quad \text{s.t.:} \quad \mathbf{X} \geq 0, \quad (1)$$

where  $\mathbf{N} \in \mathbb{R}^{d \times n}$  is the error, and  $\mathbf{X} \geq 0$  is the so-called non-negativity constraint (ANC). It should be noted that we explicitly enforce the ANC constraint without the sum-to-one constraint (ASC), due to some criticisms about the ASC in the literature [3].

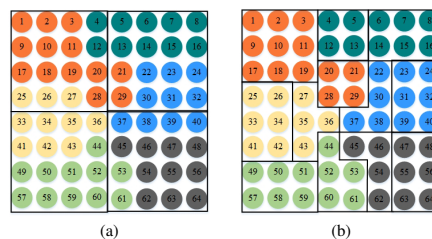
As the number of endmembers involved in a mixed pixel is usually very small when compared with the size of the spectral library, the abundance matrix  $\mathbf{X}$  is sparse. Then the unmixing problem can be formulated as an  $\ell_2 - \ell_0$  or  $\ell_2 - \ell_1$  optimization (as in the SUnSAL algorithm). However, SUnSAL focuses on analyzing the spectral information without considering the spatial-contextual information. Currently, several

spatial-based sparse unmixing algorithms have been shown to obtain better results by including spatial-contextual information. However, most of these methods rely on regular-shaped neighborhoods (e.g., a cross or a square window) to model the spatial-contextual information around each pixel, which limits the characterization of real spatial structures in the scene.

## 3. PROPOSED METHOD

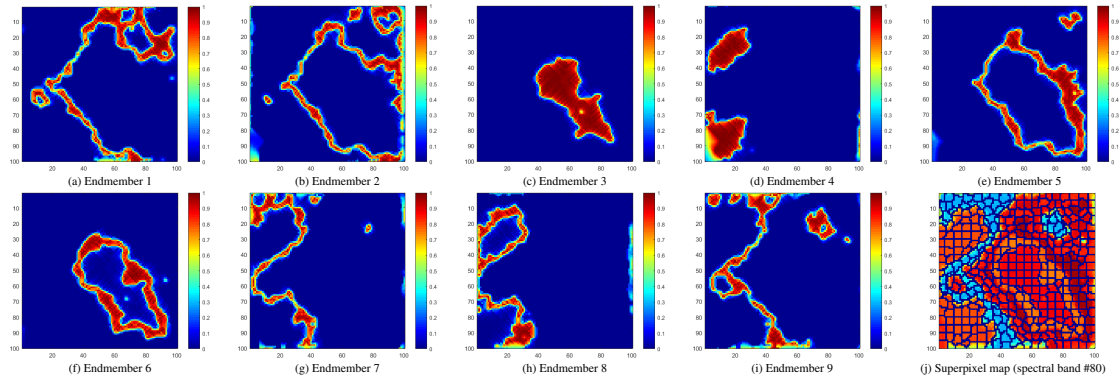
With the goal of exploiting the spatial-contextual information more efficiently in the sparse unmixing process, we develop a new SPGSU method that exploits the spatial correlation present in remotely sensed hyperspectral images by using superpixels. Among several available superpixel algorithms, we have selected the SLIC due to its capacity to generate high quality, compact, and nearly uniform superpixels. It has been successfully applied to hyperspectral images in recent works [11]. As a result, the SLIC method was used to generate superpixels in this work.

Fig. 1 illustrates the difference between fixed spatial neighborhoods and superpixels, where the pixels that belong to the same set of endmembers are depicted in the same color. In the figure, we consider a toy example made up of a scene with  $8 \times 8$  pixels, and each pixel is represented by a circle. Fig. 1(a) shows the spatial regions that are defined by a fixed-size window, while Fig. 1(b) shows the ideal superpixels. As shown in Fig. 1(a), regular grids cannot perfectly characterize the spatial-contextual information. In contrast, superpixels can explicitly and naturally represent spatial neighborhoods, where the shape and size of the resulting superpixels are fully adaptive. Furthermore, each superpixel is considered to be a spatially homogeneous region, which preserves the consistency and uniformity of abundance maps.



**Fig. 1.** Toy example illustrating the difference between fixed spatial neighborhoods (a) and superpixels (b).

Inspired by the success of weighted  $\ell_1$  minimization in sparse signal recovery [10, 12], and also by the success of the double reweighted sparse unmixing with total variation (DRSU-TV) [13] algorithm, which uses weighting factors to improve the unmixing performance, our SPGSU simultaneously exploits the spectral dual sparsity as well as the spatial



**Fig. 2.** True fractional abundances of the endmembers in our simulated dataset, and obtained superpixel map (the number of superpixels is set to 400).

smoothness of fractional abundance maps as follows:

$$\min_{\mathbf{X}} \frac{1}{2} \|\mathbf{A}\mathbf{X} - \mathbf{Y}\|_F^2 + \lambda \|\mathbf{W}_1 \mathbf{W}_{spg} \odot \mathbf{X}\|_{1,1}, \quad \text{s.t.: } \mathbf{X} \geq 0, \quad (2)$$

where  $\|\cdot\|_F$  is the Frobenius norm,  $\|\mathbf{X}\|_{1,1} = \sum_{j=1}^n \|\mathbf{x}_j\|_1$ , with  $\mathbf{x}_j$  being the  $j$ th column of  $\mathbf{X}$ . The operator  $\odot$  denotes the element-wise multiplication of two variables, and  $\lambda$  is a regularization parameter. Last but not least,  $\mathbf{W}_1 = \text{diag}(w_{1,11}, \dots, w_{1,ii}, \dots, w_{1,mm}) \in \mathbb{R}^{m \times m}$ , ( $i = 1, \dots, m$ ) is the spectral weight, aimed at promoting nonzero row vectors and defined as follows:

$$w_{1,ii}^{t+1} = \frac{1}{\|\mathbf{X}^t(i, :)\|_2 + \varepsilon}, \quad (3)$$

where  $\mathbf{X}^t(i, :)$  is the  $i$ -th row in the estimated abundance map at the  $t$ -th iteration, and  $\varepsilon > 0$  is a small positive value. Notice that, as shown in Eq. (3), large weights can be used to discourage non-zero entries in the recovered signal, while small weights encourage non-zero entries [13].

As mentioned earlier, our method characterizes the spatial-contextual information in hyperspectral images by using superpixels. For this purpose, we introduce a superpixel-based spatial weighting factor  $\mathbf{W}_{spg}$ . Let  $w_{spg,ij}^{t+1}$  be the element of the  $i$ -th line and  $j$ -th row in  $\mathbf{W}_{spg}$  at iteration  $t+1$ , expressed as follows:

$$w_{spg,ij}^{t+1} = \frac{1}{f(x_{ij}^t) + \varepsilon}, \quad (4)$$

where  $f(\cdot)$  is a function that explicitly exploits the spatial correlation through the superpixel system as follows:

$$f(x_{ij}) = \frac{\sum_{i'j' \in S(h)} x_{S(h)}}{k}, \quad (5)$$

where  $S(h)$  denotes the superpixel set,  $h \in \{1, 2, \dots, c\}$  denotes the number of superpixel sets obtained by the SLIC algorithm, and  $k$  denotes the total number of elements in the superpixel set  $S(h)$ .  $x_{S(h)}$  denotes the abundance of all the

elements in the superpixel set  $S(h)$ . For each superpixel, the average value of element vectors in the superpixel serves as the new value for each element. In other words, all elements in the same superpixel set  $S(h)$  are assigned the same value. The optimization problem associated to our method can be easily solved by using the alternating direction method of multipliers (ADMM) [14].

#### 4. EXPERIMENTAL RESULTS

The spectral library that we use in our synthetic image experiments is a dictionary of minerals extracted from the United States Geological Survey (USGS) library<sup>1</sup>. Such library, denoted by  $\mathbf{A}$ , contains  $m = 222$  materials (different mineral types), with spectral signatures with reflectance values consisting of  $L = 221$  spectral bands and distributed uniformly in the interval  $0.4\text{-}2.5\mu\text{m}$ . Following the work in [15], a database of  $100 \times 100$ -pixel synthetic hyperspectral scenes has been created using fractals to generate distinct spatial patterns often found in nature. Nine spectral signatures are chosen from  $\mathbf{A}$  to generate the synthetic hyperspectral images. For illustrative purposes, Fig. 2(a)-(i) shows the true abundance maps of the considered endmembers.

After generating the simulated data cubes, they were contaminated with i.i.d. Gaussian noise, using three levels of signal-to-noise ratio (SNR): 30, 40 and 50 dB. We compare our SPGSU algorithm with other advanced algorithms for sparse unmixing, i.e. SUnSAL [3], SUnSAL-TV [6], DR-SU [5] and DRSU-TV [13].

For quantitative assessment, the signal-to-reconstruction error (SRE), measured in dB, and the probability of success ( $p_s$ ) are used to evaluate the unmixing accuracy. Let  $\hat{\mathbf{x}}$  be the estimated abundance, and  $\mathbf{x}$  be the true abundance. The SRE(dB) can be computed as  $\text{SRE(dB)} = 10 \cdot \log_{10}(E(\|\mathbf{x}\|_2^2)/E(\|\mathbf{x} - \hat{\mathbf{x}}\|_2^2))$ , and  $p_s$  is given by

<sup>1</sup>Available online: <https://speclab.cr.usgs.gov/spectral-lib.html>

**Table 1.** SRE(dB) and  $p_s$  scores achieved after applying different unmixing methods to our simulated data (the optimal parameters for which the reported values were achieved are indicated in the parentheses).

Algorithm	SNR=30dB		SNR=40dB		SNR=50dB	
	SRE(dB)	$p_s$	SRE(dB)	$p_s$	SRE(dB)	$p_s$
SUnSAL	10.0847	0.9038	17.7578	0.9961	26.3684	1
	$(\lambda = 1e-2)$		$(\lambda = 3e-3)$		$(\lambda = 1e-3)$	
SUnSAL-TV	14.7077	0.9945	23.1831	0.9999	31.4355	1
	$(\lambda = 3e-3; \lambda_{TV} = 4e-3)$		$(\lambda = 1e-3; \lambda_{TV} = 1e-3)$		$(\lambda = 9e-4; \lambda_{TV} = 3e-4)$	
DRSU	20.4370	0.9983	33.6210	1	43.4449	1
	$(\lambda = 3e-3)$		$(\lambda = 3e-3)$		$(\lambda = 5e-4)$	
DRSU-TV	24.6846	0.9998	35.2502	1	43.4449	1
	$(\lambda = 2e-3; \lambda_{TV} = 4e-3)$		$(\lambda = 3e-3; \lambda_{TV} = 6e-4)$		$(\lambda = 5e-4; \lambda_{TV} = 0)$	
SPGSU	24.9950	1	35.9939	1	44.1235	1
	$(\lambda = 5e-3)$		$(\lambda = 3e-3)$		$(\lambda = 6e-4)$	

$p_s \equiv P(\|\hat{\mathbf{x}} - \mathbf{x}\|^2 / \|\mathbf{x}\|^2 \leq 3.16)$  [3], where  $E(\cdot)$  denotes the expectation function. The larger the SRE(dB) or the  $p_s$ , the more accurate the unmixing results.

Table 1 shows the SRE(dB) and  $p_s$  results achieved by the different tested algorithms under different SNR levels. For all the tested algorithms, the input parameters have been carefully tuned for optimal performance. As shown in Fig. 2(j), the number of superpixels is set to 400 for oversegmenting the hyperspectral scene. From Table 1, we can observe that the proposed SPGSU algorithm obtains better SRE(dB) results than all other tested algorithms, in all cases. The  $p_s$  obtained by the proposed approach are also better than those obtained by other algorithms in the case of low SNR, which reveals that the inclusion of spatial information leads to high robustness. Based on the aforementioned results, we can conclude that our superpixel-based spatial weighted strategy offers the potential to improve sparse unmixing performance.

## 5. CONCLUSIONS AND FUTURE WORK

In this work, we have introduced a new superpixel-guided sparse unmixing algorithm (SPGSU) for remotely sensed hyperspectral images. Our SPGSU exploits the spatial-contextual information contained in hyperspectral images by using superpixel-guided spatial weights. Our experiments with simulated hyperspectral data reveal that the SPGSU algorithm consistently achieves robust spectral unmixing performance in comparison with other advanced algorithms. Future work will be focused on conducting additional experiments with real hyperspectral images.

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