Parallel Hyperspectral Coded Aperture for Compressive Sensing on GPUs

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Abstract—The application of compressive sensing (CS) to hyperspectral images is an active area of research over the past few years, both in terms of the hardware and the signal processing algorithms. However, CS algorithms can be computationally very expensive due to the extremely large volumes of data collected by imaging spectrometers, a fact that compromises their use in applications under real-time constraints. This paper proposes four efficient implementations of hyperspectral coded aperture (HYCA) for CS, two of them termed P-HYCA and P-HYCA-FAST and two additional implementations for its constrained version (CHYCA), termed P-CHYCA and P-CHYCA-FAST on commodity graphics processing units (GPUs). HYCA algorithm exploits the high correlation existing among the spectral bands of the hyperspectral data sets and the generally low number of endmembers needed to explain the data, which largely reduces the number of measurements necessary to correctly reconstruct the original data. The proposed P-HYCA and P-CHYCA implementations have been developed using the compute unified device architecture (CUDA) and the cuFFT library. Moreover, this library has been replaced by a fast iterative method in the P-HYCA-FAST and P-CHYCA-FAST implementations that leads to very significant speedup factors in order to achieve real-time requirements. The proposed algorithms are evaluated not only in terms of reconstruction error for different compressions ratios but also in terms of computational performance using two different GPU architectures by NVIDIA: 1) GeForce GTX 590; and 2) GeForce GTX TITAN. Experiments are conducted using both simulated and real data revealing considerable acceleration factors and obtaining good results in the task of compressing remotely sensed hyperspectral data sets.

Index Terms—Coded aperture, compressive sensing (CS), graphics processing units (GPUs), high-performance computing, hyperspectral imaging.
an airborne instrument and sent to a ground station on Earth for subsequent processing. Usually, the bandwidth connection between the satellite/airborne platform and the ground station is reduced, which limits the amount of data that can be transmitted. As a result, there is a clear need for (either lossless or lossy) hyperspectral data compression techniques that can be applied onboard the imaging instrument [22]–[24]. In contrast, compressive sensing (CS) [25], [26] involves acquisition of the data in an already compressed form by computing inner products, also termed measurements, between known spectral vectors and the original data. This process is sometimes called “coded aperture” because inner products can be conceived as the total amount of light that is transmitted trough masks acting on the aperture of the instrument. In CS, the original data is inferred from the measurements by solving a convex optimization problem. A necessary condition to obtain good inferences is that the original data admits a sparse or compressible representation in a given basis or frame. This means that most of the coefficients of the representation in that basis or frame are zero or small and, thus, the data can be well approximated with just a small number of large coefficients. It happens that hyperspectral images are often highly compressible owing to a very high spatial and spectral correlation. Therefore, this imaging modality is a perfect candidate to apply the CS technology. Until now, there has been no effort to accelerate coded aperture algorithms for hyperspectral images using parallel techniques in the open literature.

In this paper, four computationally efficient implementations of an HYCA algorithm for CS on GPU platforms are proposed. HYCA [27] algorithm and its constrained version (CHYCA) are two algorithms that have been shown to be very successful from the viewpoint of using CS for improving the acquisition process of hyperspectral scenes. In this work, these algorithms are performed with several optimizations for accelerating their computational performance while maintaining their accuracy. The first one exploits the GPU architecture at low level, using shared memory and coalesced accesses to memory. The second one is an optimization focused on the use of a fast iterative method to solve a quadratic problem, avoiding the use of cuFFT library. The obtained new fast versions are called P-HYCA-FAST and P-CHYCA-FAST. The third one is focused on the configuration of a larger level one (L1) cache size and a smaller shared memory in the kernel developed on the P-HYCA-FAST and P-CHYCA-FAST implementations that lead to very significant speedup factors, thus taking full advantage of the computational power of GPUs. The considered implementations are intercompared in the context of real hyperspectral imaging applications. NVIDIA GeForce GTX 590 and GTX TITAN platforms have been used to test the proposed implementations with both synthetic and real hyperspectral scenes. Our study reveals that the NVIDIA GeForce GTX TITAN GPU can provide real-time CS performance. The implementations on GPUs have been carried out using NVIDIA CUDA and the cuBLAS library. The cuFFT library is only used on P-HYCA and P-CHYCA versions.

This paper is organized as follows. Section II describes the original HYCA, CHYCA, and the proposed fast optimization for both methods. Section III describes the proposed GPU implementations. Section IV presents an experimental evaluation of the proposed implementations in terms of both accuracy and parallel performance using synthetic and real hyperspectral data sets on two GPU platforms. Finally, Section V concludes with some remarks and hints at plausible future research lines.

II. DESCRIPTION OF THE METHODS

In this section, HYCA and CHYCA methods are introduced and described in the following sections. Moreover, a fast optimization for both cases is proposed to accelerate the CS process.

A. HYCA Algorithm

The original HYCA method for CS was developed in [27]. This approach compresses the data on the acquisition process, then the compressed signal is sent to Earth and stored in compressed form. Later, the original signal can be recovered by taking advantage of two key properties of hyperspectral images: 1) the spectral vectors live systematically in low-dimensional subspaces [28]; and 2) the spectral bands present a high correlation in the spatial domain. The former property allows to represent the data vectors using a reduced set of spectral end-members due to the mixing phenomenon and also exploits the high spatial correlation of the fractional abundances associated to the spectral endmembers. Let $x_i \in \mathbb{R}^{n_b}$, for $i = 1, \ldots, n_p$, denote the $n_p := n_r \times n_c$ spectral vectors of a hyperspectral image, where $n_r$, $n_c$, and $n_b$ denote, respectively, the number of rows, columns, and bands of the hyperspectral image, and $x := [x_1, \ldots, x_{n_p}]^T$ [the operator $(\cdot)^T$ stands for transpose] denote, in a vector format, the hyperspectral image. In order to perform the compression of the original signal $x$, and as in [27], for each pixel $i \in \{1, \ldots, n_p\}$, a set of $q$ inner products between $x_i$ and samples of i.i.d. Gaussian random vectors is performed. The total number of measurements is therefore $q \times n_b$ yielding an undersampling factor of $q/n_b$. This measurement operation can be represented as a matrix multiplication

$$y = Ax$$

where $A$ is a block-diagonal matrix containing the matrices $A_j \in \mathbb{R}^{q \times n_b}$ acting on the pixel $x_i$, for $i \in \{1, \ldots, n_p\}$. For reasons linked with 1) the computational management of the sampling process and 2) the spatial correlation length of hyperspectral images (see [27] for more details), matrices $A_j$ are organized into spatial windows of size $ws \times ws = m$. Each window contains the same set of matrices. All windows have the same spatial configuration of $H_j$, for $j = 1, \ldots, m$.

The HYCA method also takes advantage of the fact that the hyperspectral vectors $x_i$ generally live in a low-dimensional subspace. This fact can be modeled by $x_i = E z_i$, where $E \in \mathbb{R}^{n_b \times p}$ is a matrix whose columns spans the signal subspace and $z_i \in \mathbb{R}^p$ denotes the vector of coordinates with respect to the
columns of E. Defining $z := [z_1^T, \ldots, z_{n_p}^T]^T$, for $i = 1, \ldots, n_p$, we have

$$x = (\mathbf{I} \otimes \mathbf{E})z \quad (2)$$

where $E \in \mathbb{R}^{1 \times n_p}$, with $p \ll n_b$, $\otimes$ stands for Kronecker product, and $I$ is the identity matrix of suitable size. In this work, we assume that the linear mixing model is a good approximation to the spectral vectors $x_i$ [4] and, therefore, matrix $E$ contains in its columns the spectral signatures of the $p$ endmembers. We use the VCA algorithm [29] to infer $E$. Since $E$ is the mixing matrix, hence $z$ contains the fractional abundances associated to each pixel.

Let $K = A(\mathbf{I} \otimes E)$. If matrices $E$ and $A$ are available, one can formulate the estimation of $z$ from $(q \times n_x)$-dimensional vector of measurements. Since the fractional abundances in hyperspectral images exhibit a high spatial correlation, we exploit this feature for estimating $z$ using the following optimization problem:

$$\min_{z} \frac{1}{2}||y - Kz||^2 + \lambda_{TV} TV(z) \quad (3)$$

subject to: $z \geq 0$

where $TV(z)$ stands for the sum of nonisotropic total variations (TVs) [30], [31] associated to $z$, one per image of abundance. Defined as

$$TV(z) := \phi(Dz)$$

where $D := [D^T_h D^T_v]^T$, $D_h, D_v$ compute the horizontal and vertical backward differences, assuming a cyclic boundary, and

$$\phi(\vartheta) := \sum_{i=1}^{p} \sum_{j=1}^{n_p} ||\vartheta[i, j]||$$

with $\vartheta := [\vartheta^T_h, \vartheta^T_v]$, $\vartheta_h$ standing for horizontal differences and $\vartheta_v$ standing for vertical differences. The TV regularizer promotes piecewise abundance images $z$. Therefore, the minimization (2) aims at finding a solution which is a compromise between the fidelity to the measured data, enforced by the quadratic term $(1/2)||y - Kz||^2$, and the properties enforced by the TV regularizer, that is piecewise smooth image of abundances. The relative weight between the two characteristics of the solution is set the regularization parameter $\lambda_{TV} > 0$.

To solve the convex optimization problem in (3), a methodology closely related with the one presented in [32] is adopted. The solution of this problem is obtained by an instance of the alternating direction method of multipliers (ADMM) [33], which decomposes very hard problems into a cyclic sequence of simpler problems. With this in mind, an equivalent way of writing the optimization problem in (3) is

$$\min_{z \geq 0} \frac{1}{2}||y - Kz||^2 + \lambda_{TV} \phi(Dz) + \epsilon_{R+}(z) \quad (4)$$

where $\epsilon_{R+}(z) = \sum_{i=1}^{n_p} \epsilon_{R+}(z_i)$ is the indicator function ($z_i$ represents the $i$th element of $z$ and $\epsilon_{R+}(z_i)$ is zero if $z_i$ belongs to the nonnegative orthant and $+\infty$ otherwise). Given the objective function in (4), we can write the following equivalent formulation:

$$\min_{x,v_1,v_2,v_3,v_4} \frac{1}{2}||y - K(d_1 + v_2 + d_2) + d_3 + d_4||^2 + \epsilon_{R+}(v_2) + \lambda_{TV} \phi(Dz)$$

subject to $v_1 = z$

$$v_2 = z$$

$$(v_3, v_4) = Dz.$$

Algorithm 1 shows the pseudocode of the HYCA algorithm to solve the problem in (5) and how to reconstruct the data using (2).

### Algorithm 1. Pseudocode of HYCA algorithm

1. **Initialization**: set $k = 0$, choose $\mu > 0$, $E, z^0$,

$v_1^0, v_2^0, v_3^0, v_4^0, d_1^0, d_2^0, d_3^0, d_4^0$

2. **repeat**:

3. $aux = v_1^k + d_1^k + v_2^k + d_2^k$

$z^{k+1} \leftarrow (D^TD + 2I)^{-1} \times$

$$\left( aux + D_h(v_3^k + d_3^k) + D_v(v_4^k + d_4^k) \right)$$

4. $v_1^{k+1} \leftarrow (K^TK + \mu I)^{-1} \times$

$$\left( K^Ty + \mu(z^{k+1} - d_1^{k}) \right)$$

5. $v_2^{k+1} \leftarrow \max(0, z^{k+1} - d_2^{k})$

6. $v_3^{k+1} \leftarrow \text{soft}(D_h(z^{k+1}) - d_3^{k}, \lambda_{TV}/\mu)$

7. $v_4^{k+1} \leftarrow \text{soft}(D_v(z^{k+1}) - d_4^{k}, \lambda_{TV}/\mu)$

8. **Update Lagrange multipliers**:

$d_1^{k+1} \leftarrow d_1^{k} - z^{k+1} + v_1^{k+1}$

$d_2^{k+1} \leftarrow d_2^{k} - z^{k+1} + v_2^{k+1}$

$d_3^{k+1} \leftarrow d_3^{k} - D_h(z^{k+1}) + v_3^{k+1}$

$d_4^{k+1} \leftarrow d_4^{k} - D_v(z^{k+1}) + v_4^{k+1}$

9. **Update iteration**: $k \leftarrow k + 1$

10. **until** $k = \text{MAX ITERATIONS}$

11. **Reconstruction** $\hat{x} = (\mathbf{I} \otimes E)z^k$

B. CHYCA Algorithm

In order to avoid tuning the $\lambda_{TV}$ parameter, another algorithm called CHYCA was proposed in [27]. In this algorithm, the reconstruction error term is constrained to $||y - Kz||^2 \leq \delta$ instead of being part of the objective function. The advantage of this formulation is that $\delta$ may be set with basis on the noise characteristics of the data set which are likely to be known beforehand.

As in HYCA, a set of new variables per term of the objective function are introduced and the ADMM methodology [33] is used to solve the CHYCA minimization problem

$$\min_{z \geq 0} TV(z) \quad \text{subject to: } ||y - Kz||^2 \leq \delta$$

where $\delta$ is a scalar value linked to the noise statistics.
Algorithm 2. Pseudocode of CHYCA algorithm

1. Initialization: set $k = 0$, choose $\mu > 0$, $\mathbf{z}^{(0)}$, $\mathbf{v}_1^{(0)}$, $\mathbf{v}_2^{(0)}$, $\mathbf{v}_3^{(0)}$, $\mathbf{v}_4^{(0)}$, $\mathbf{v}_5^{(0)}$, $\mathbf{d}_1^{(0)}$, $\mathbf{d}_2^{(0)}$, $\mathbf{d}_3^{(0)}$, $\mathbf{d}_4^{(0)}$, $\mathbf{d}_5^{(0)}$

2. repeat:
   3. aux = $v_1^{(k)} + d_1^{(k)} + v_2^{(k)} + d_2^{(k)}$
   4. $z^{(k+1)} = (D^T D + 2I)^{-1} \times 
   \left( \text{aux} + D^T_h (v_3^{(k)} + d_3^{(k)}) + D^T_v (v_4^{(k)} + d_4^{(k)}) \right)$
   5. $v_1^{(k+1)} = \langle (K^T K + I)^{-1} \times 
   \left( z^{(k+1)} - d_1^{(k)} + K^T (-v_5^{(k)} + y - d_5^{(k)}) \right) \rangle$
   6. $v_2^{(k+1)} = \max \left( 0, z^{(k+1)} - d_2^{(k)} \right)$
   7. $v_3^{(k+1)} = \text{soft} \left( D_h (z^{(k+1)}) - d_3^{(k)} + 1/\mu \right)$
   8. $v_4^{(k+1)} = \text{soft} \left( D_v (z^{(k+1)}) - d_4^{(k)} + 1/\mu \right)$
   9. $y_{aux} = y - K_v v_1^{(k+1)} - d_5^{(k)}$
   10. $v_5^{(k+1)} = \left\{ \begin{array}{ll} y_{aux} & \text{if } \|y_{aux}\| \leq \delta \\ \frac{y_{aux}}{\|y_{aux}\|} & \text{otherwise,} \end{array} \right.$
   11. Update Lagrange multipliers:
       $d_1^{(k+1)} = d_1^{(k)} - z^{(k+1)} + v_1^{(k+1)}$
       $d_2^{(k+1)} = d_2^{(k)} - z^{(k+1)} + v_2^{(k+1)}$
       $d_3^{(k+1)} = d_3^{(k)} - D_h z^{(k+1)} + v_3^{(k+1)}$
       $d_4^{(k+1)} = d_4^{(k)} - D_v z^{(k+1)} + v_4^{(k+1)}$
       $d_5^{(k+1)} = d_5^{(k)} + v_5^{(k+1)} - (y - K_v v_1^{(k+1)})$
   12. Update iteration: $k = k + 1$

III. GPU IMPLEMENTATIONS

By a careful choice of the new variables, the problem is converted into a sequence of much simpler problems. With this in mind, $\ell_B(\varepsilon)(z) = 0$ if $\|z\| \leq \varepsilon$ and $+\infty$ otherwise. With these definition in place, an equivalent way of writing the optimization problem in (6) is

$$
\min_{v_1, v_2, v_3, v_4, v_5} \phi(Dz) + \ell_B(\varepsilon)(v_5) + \ell_{R^+}(v_2)
$$

subject to:

\begin{align}
&v_1 = z \\
v_2 = z \\
(v_3, v_4) &= Dz \\
v_5 &= y - K_v
\end{align}

which we solve via ADMM in a way similar to Algorithm 1. Algorithm 2 shows the pseudocode of the CHYCA algorithm to solve the problem in (7) and how to reconstruct the data using (2).

C. Algorithmic Improvements

The step 3 of Algorithms 1 and 2 corresponds to the solution of a system of equations $Mz^{(k+1)} = b$ where $M = (D^T D + 2I)$ and $b = \left( v_1^{(k)} + d_1^{(k)} + v_2^{(k)} + d_2^{(k)} + D^T_h (v_3^{(k)} + d_3^{(k)}) + D^T_v (v_4^{(k)} + d_4^{(k)}) \right)$. Given that the matrices $D_h$ and $D_v$ are block circulant, corresponding to two-dimensional (2-D) cyclic convolutions, then the computation of $b$ and the solution of the linear system of equations may be implemented efficiently in the frequency domain with a complexity of $O(pn_p \log(n_p))$. However, because the complexity involved in the matrix–vector multiplications of the form $D_h x$ and $D_v x$ is of $O(pn_p)$, it may be advantageous to solve the system $Mz = b$ with a first-order stationary iterative procedure [34], which has the form

$$
\text{for } t = 0, 1, \ldots
$$

$$
r_t = Mz_t - b \tag{8}
$$

$$
z_{t+1} = z_t - \beta r_t.
$$

Let $0 < \lambda_{min} < \lambda_{max}$ denotes, respectively, the smallest and the largest eigenvalues of $M$. Therefore, the sequence $z_{t+1}$ converges to the solution of the system $Mz = b$ provided that $0 < \beta < 2/\lambda_{max}$ [34]. In addition, the optimal convergence factor is given by

$$
\rho_{opt} = \frac{1 - \lambda_{min}/\lambda_{max}}{1 + \lambda_{min}\lambda_{max}} \tag{9}
$$

and is obtained with $\beta_{opt} = 2/(\lambda_{min} + \lambda_{max})$. For the problem in hands, we have $\lambda_{min} = 2$ and $\lambda_{max} = 8$ and, therefore, $\beta_{opt} = 1/6$ and $\rho_{opt} = 2/3$. In these conditions, the convergence rate, i.e., the number of iterations to attenuate the error $\|z_t - z_\ast\|$, where $z_\ast = M^{-1}b$, by a factor of 10, is $-1/\log(2/3) = 5.57$. In practice, it is not necessary to solve exactly the linear system of equations in each ADMM iteration as far as the errors are summable [33]. In ADMM iteration, we initialize (8) with $z^{(k)}$ and run only a few iterations.

We now derive faster versions of HYCA and CHYCA. The modified versions of HYCA and CHYCA are termed, respectively, HYCA-FAST and CHYCA-FAST hereinafter. The pseudocode with the main modification in line 3 of Algorithm 1 and 2 is shown in Algorithm 3. In this way, the use of the fast Fourier transform to solve the $z$ optimization is avoided and a fastest way to solve the optimization is provided.

Algorithm 3. Pseudocode of FAST optimization

3. Fast optimization:
   3.1. set $\beta = 1/6$ (optimum value)
   3.2. aux = $v_1^{(k)} + d_1^{(k)} + v_2^{(k)} + d_2^{(k)}$
       $g^{(k)} = \left( \text{aux} + D^T_h (v_3^{(k)} + d_3^{(k)}) + D^T_v (v_4^{(k)} + d_4^{(k)}) \right)$
   3.3. repeat:
       3.3.1. $z^{(f)} = 2 z^{(f)} + D^T_h (D_h (z^{(f)})) + D^T_v (D_v (z^{(f)})) - g^{(k)}$
       3.3.2. $z^{(f+1)} = z^{(f)} - \beta z^{(f)}$
   3.4. until $f = \text{DESIRED\_ITERATIONS}$

GPUs can be abstracted as an array of highly threaded streaming multiprocessors (SMs), where each multiprocessor is characterized by a single instruction multiple data (SIMD)
architecture, i.e., in each clock cycle each processor executes the same instruction while operating on multiple data streams. Each SM has a number of streaming processors that share a control logic and instruction cache and have access to a local shared memory and to local cache memories in the multiprocessor, while the multiprocessors have access to the global GPU (device) memory. Fig. 1 presents a typical architecture and the data flow communication between CPU and GPU.

The algorithms are constructed by chaining the so-called kernels which operate on entire streams and which are executed by a multiprocessor, taking one or more streams as inputs and producing one or more streams as outputs. Thereby, data-level parallelism is exposed to hardware, and kernels can be concurrently applied without any sort of synchronization. The kernels can perform a kind of batch processing arranged in the form of a grid of blocks where each block is composed by a group of threads that share data efficiently through the shared local memory and synchronize their execution for coordinating accesses to memory. As a result, there are different levels of memory in the GPU for the thread, block, and grid concepts. There is also a maximum number of threads that a block can contain (depending on the GPU model), however, the number of threads that can be concurrently executed is much larger due to the fact that several blocks executed by the same kernel can be managed concurrently. With the above ideas in mind, the proposed implementations for HYCA, CHYCA, and their fast optimizations are detailed as follows.

### A. P-HYCA Implementation and Its Fast Optimization

The implementation of P-HYCA algorithm starts with an initialization step. The cuBLAS and cuFFT libraries are first initialized. After that the compressed hyperspectral image is loaded band by band from the hard disk to the main memory of the GPU. This arrangement intends to access consecutive pixels in the same wavelength in parallel by the processing kernels (coalesced accesses to memory). This means that the $i$th thread of a block will access the $r$th pixel for a given wavelength. This technique is used to maximize global memory bandwidth and minimize the number of bus transactions.

Once the original image is loaded in the global memory, the kernel Compute_compression performs the projections between the random vectors and the image pixels in order to compress the data. For this purpose each thread will compute the multiplication of the matrix $H_i$ with its corresponding pixel, so that the total number of threads will be equal to the number of pixels in the data set. The grid configuration of this kernel in the GPU will contain $Num_b$ blocks with the maximum number of threads supported by the architecture (1024 for the GPU considered). Thus, the number of blocks $Num_b$ will be given by the expression

$$Num_b = \left\lceil \frac{n_T}{1024} \right\rceil. \tag{10}$$

Due to the fact that we have a small set of $H_i$ matrices which are systematically multiplied by the image pixels located inside spatial windows, the compression matrices $H_i$ are stored in shared memory in order to optimize the memory access to the $H_i$ matrices. At the end of this process, the threads will store the compressed measurements $y$ in the global memory.

The next step precomputes the fixed terms $(K^T K + \mu I)^{-1}$ and $(K^T y)$ of Algorithm 1 in order to avoid repeated computations inside the main loop from lines 2 to 10. Herein, the term $(K^T K + \mu I)^{-1}$ in line 4 of Algorithm 1 is computed in the CPU using the LAPACK$^3$ package due to the fact that the size of these matrices is small and it is not worth to perform this computation in the GPU. However, $(K^T y)$ is computed using a kernel called Compute_KTY, which will perform the multiplication of the matrix $K^T$ by its corresponding pixel. The grid configuration for this kernel is given by expression (10) as explained before. In this kernel, the matrix $K$ is stored in shared memory to optimize the memory access to the elements of this matrix. It is important to emphasize that we have declared shared memory dynamically in all kernel launch configurations. In this case, the shared memory allocation (size per thread block) must be specified (in bytes) using an optional third execution configuration parameter. Inside the kernel, the shared memory array is declared by means of an unsized extern array syntax. _extern__ double $s[]$. The size is simply determined from the third execution configuration parameter when the kernel is launched.

The optimization of $z$ in line 3 of Algorithm 1 is carried out in two steps. First, a kernel computes the right side of the equation: $v_1^{(k)} + d_1^{(k)} + v_2^{(k)} + d_2^{(k)} + D_p(v_3^{(k)} + d_3^{(k)}) + D_d(v_4^{(k)} + d_4^{(k)})$; here each thread computes one element and the grid configuration is the same than the previous kernels. Later, the optimization with respect to $z$ is performed using the cuFFT. Herein, two Fourier transform types were used: real—to—complex (R2C) and complex—to—real (C2R). Finally, the result is stored in global memory.

The optimization of $v_1$ in line 4 of Algorithm 1 is carried out with a kernel called Optimize_v1. This kernel uses the same grid configuration as the previous ones. This kernel also makes

use of the shared memory to optimize the memory access to the matrix \((K^T K + \mu I)^{-1}\), which was precomputed before. The resulting \(v_1\) is stored in the global memory.

Line 5 in Algorithm 1 is carried out with a kernel called \text{Optimize}_{v2}, which computes the maximum between 0 and \(z^{(k+1)} - d_5^{(k)}\). This kernel uses as many threads as the number of elements of the vector \((n_p)\), with the same grid configuration as the previous kernels.

The optimization of \(v_3\) is carried out jointly by a single kernel called \text{Optimize}_{v3 \_v4}, which computes the lines 6 and 7 in Algorithm 1. This kernel uses the same configuration as the previous kernels with as many threads as the image pixels. In this kernel each thread computes the horizontal and vertical differences of one element and performs the soft function for the corresponding element, where \(\text{soft}(\cdot, \tau)\) denotes the application of the soft-threshold function \(b \mapsto b \max[[|b| - \tau, 0]] + \tau\).

Finally, Lagrange multipliers update is computed with two kernels called \text{Compute} \_d12 and \text{Compute} \_d34 which, respectively, compute the update of the variables \(d_1, d_2\) and \(d_3, d_4\).

The algorithm repeats this process until a number of iterations \(k\) is reached. Once the estimated fractional abundances \(z\) are computed, the algorithm reconstructs the original hyperspectral data set multiplying by the endmember matrix. This process is performed using the cuBLAS library. Specifically, the cublasSgemm function of cuBLAS was used to reconstruct the image \(x\).

Finally, a fast optimized version termed P-HYCA-FAST has been implemented following the previous strategies except the implementation of the line 3 from Algorithm 1. In this version, a new kernel called \text{Compute} \_z\_Fast is used to compute lines 3.3.1 and 3.3.2 of Algorithm 3. The kernel uses the same grid configuration as the previous ones (one thread per pixel). This kernel is called \(f\) times and performs the update of both \(r\) and \(z\) variables. This kernel optimizes the memory access by using the L1 cache memory. On CUDA devices with a compute capability 2.0 or later (our case), both the L1 cache and shared memory use the same hardware resources, but CUDA allows setting a preferred size for the L1 cache memory. The device sets a preference of larger L1 cache and smaller shared memory using the CUDA directive cudaFuncSetCacheConfig("name_kernel," cudaFuncCachePreferL1). Due to the fact that this kernel does not use shared memory, the memory access can be optimized with a larger L1 cache memory in order to increase the kernel performance.

\[N_{numb} = \left\lceil \frac{n_p \times q}{512} \right\rceil.\]

Once the optimization \(v_5\) is completed and stored in global memory, the Lagrange multipliers are updated following the P-HYCA scheme and the reconstruction of the original hyperspectral data set are performed. Note that the \(y - K v_1^{(k+1)}\) operation has been calculated previously.

Finally, a fast optimized version termed P-CHYCA-FAST has been implemented as shown in Algorithm 3 using the same strategy as in P-HYCA-FAST implementation.

**IV. Experimental Results**

**A. Hyperspectral Image Data**

The experiments are carried out using three hyperspectral images. The first two synthetic data sets used in our experiments, denoted hereinafter as synthetic\_1 and synthetic\_2 were generated from spectral signatures randomly selected from the U.S. Geological Survey (USGS).\(^4\) Synthetic\_1 consists of a set of 5 × 5 squares of 10 × 10 pixels each one, yielding a total size of 110 × 110 pixels (10.8 MB). The first row of squares contains the endmembers, the second row contains mixtures of two endmembers, the third row contains mixtures of three endmembers, and so on. Fig. 2 displays the ground-truth abundance maps used for generating the simulated imagery. Synthetic\_2 consists of a set of 30 signatures from the USGS library and it is generated using the procedure described in [35] to simulate natural spatial patterns, composed by a total size of

Fig. 2. Ground-truth abundance maps of endmembers in the synthetic_1 hyperspectral data represented in grayscale. (a) Gray scale bar (white and black represent 1 and 0, respectively), endmembers (b) #1, (c) #2, (d) #3, (e) #4, and (f) #5.

Fig. 3. Gray-scale composition and three examples of ground-truth abundance maps of endmembers in the synthetic_2 hyperspectral data. (a) Gray scale bar (white and black represent 1 and 0, respectively), (b) gray-scale composition, endmembers (c) #5, (d) #23, (e) #29.

600 × 512 pixels (275 MB). Fig. 3 displays a gray-scale composition and three examples of ground-truth abundance maps from the simulated image.

The third hyperspectral image considered in experiments is the well-known AVIRIS Cuprite scene, available online in reflectance units after atmospheric correction. This scene has been widely used to validate the performance of endmember extraction algorithms. The portion used in experiments corresponds to a 250 × 190 pixels subset of the sector labeled as f970619t01p02_r02_sc03.a.rfl in the online data, which comprises 188 spectral bands in the range from 400 to 2500 nm and a total size of around 36 MB. Water absorption bands as well as bands with low signal-to-noise ratio (SNR) were removed prior to the analysis. The site is well understood mineralogically, and has several exposed minerals of interest, including Alunite, Buddingtonite, Calcite, Kaolinite, and Muscovite. For this subimage, the number of endmembers were estimated by Hysime method [28] and their signatures were extracted in a very fast way through VCA algorithm [29].

B. Analysis of Accuracy

In order to evaluate the accuracy of the proposed implementations in terms of reconstruction error, the normalized mean squared error (NMSE) between the original image and the reconstructed image (after data compression and decompression) and the peak signal-to-noise ratio (PSNR) have been adopted as performance indicators. The first one is given by

\[ \text{NMSE} = \frac{\| \hat{x} - x \|^2_F}{\| x \|^2_F} \] (12)

where \( x \) and \( \hat{x} \) denote the original and reconstructed hyperspectral images, respectively, and \( \| \cdot \|^2_F \) denotes the Frobenius norm. The second performance indicator is given by

\[ \text{PSNR} = 10 \log_{10} \max(\|x\|^2_F / \|\hat{x} - x\|^2_F) \] (13)

With the aim of evaluating the proposed implementations at different levels of compression we defined the number of measurements \( q = p = 5 \) in the case of the synthetic_1 data set, \( q = p/2 = 15 \) in the case of synthetic_2, and finally \( q = 5 \ll p \) for the real Cuprite data set. Thus, the compression ratios \( (n_b/q) \) are 44.8, 14.93, and 37.60. The window sizes considered were \( w_s = 2 \times 2, 4 \times 4 \), and \( 2 \times 2 \), respectively. These parameter values were defined empirically.

Fig. 4 shows the PSNR achieved for the proposed implementations as a function of the number of iterations. In this experiment one can see that P-HYCA or P-CHYCA and their fast versions provide very similar values when the number of iterations is high. However, in the first few iterations we can appreciate the higher performance of the P-HYCA and P-CHYCA versions. On the other hand, P-CHYCA presents highest SNR in the AVIRIS Cuprite scene demonstrating their good performance in real datasets.

Fig. 5 presents the NMSE maps for each data set for the proposed implementations with \( k = 175 \) iterations. Note that the proposed methods provide very good results with very low NMSE values. Note also that the scale of these figures is the same in order to compare the P-HYCA or P-CHYCA and their fast optimizations for each considered data set.

C. Analysis of Parallel Performance

The proposed parallel versions have been tested on two different GPUs:

1) The first GPU (denoted hereinafter as GPU1) is the NVidia GeForce GTX 590, which features 1024 processor cores operating at 1.215 GHz, total dedicated memory of 3072 MB (1536 MB by GPU), 1707 MHz memory clock (with 384-bits GDDR5 interface per GPU) and memory bandwidth of 327.7 GB/s. This GPU is


connected to a multicore Intel i7-2600 CPU at 3.40 GHz with four physical cores and 16 GB of DDR3 RAM memory.

2) The second GPU (denoted hereinafter as GPU2) used is the NVidia GeForce GTX TITAN, which features 2688 processor cores operating at 876 MHz, total dedicated memory of 6144 MB, 6.0 Gbps memory clock (with 384-bit GDDR5 interface) and memory bandwidth of 288.4 GB/s. This GPU is connected to a multicore Intel i7-4770K CPU at 3.50 GHz with four physical cores and 32 GB of DDR3 RAM memory.

Before describing the parallel algorithm performance results, it is important to emphasize that the GPU versions provide exactly the same results as the serial versions of the implemented algorithms, using the gcc-4.8.2 (gnu compiler default) with optimization flags -O3 (for the single-core version) to exploit data locality and avoid redundant computations. Note that for the serial HYCA implementation, the 3.3.4 FFTW library version has been used. Hence, the only difference between the serial and parallel versions is the time they need to complete their calculations. The serial algorithms were executed in one of the available CPU cores, whereas the parallel version developed for the GPU GTX 590 has been executed using one of the two GPUs available in the system. For each experiment, 10 runs were performed and the mean values were reported (these times were always very similar, with differences on the order of a few milliseconds).

Table I shows the execution time of the compression method [see expression (1)] for the synthetic and real data sets with the compression ratios shown in Section IV-B. These results

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report speedups higher than $56\times$, achieved on the GPU2 device. As expected the speedup is higher for the largest image (Synthetic_2), this is mainly because the parallelization is performed in a pixel-based scheme, thus the speedup grows with the number of pixels of the image. Notice that the speedup results for GPU2 are better than GPU1 since GPU2 has more processing cores and more dedicated memory on the device.

Tables II–V summarize the time results and speedups measured after processing synthetic and real hyperspectral images on the considered GPU platforms during 175 iterations. The processing times for the P-HYCA method and P-HYCA-FAST methods are presented in Tables II and III, respectively. It is worth noting that P-HYCA-FAST achieves higher speedups factors when compared with P-HYCA method. After comparing P-CHYCA and P-CHYCA-FAST processing times, which are presented in Tables IV and V, one can conclude that P-CHYCA-FAST achieves better performance.

It should be also noted that the cross-track line scan time in AVIRIS, a push-broom instrument [2], is quite fast (8.3 ms to collect 512 full pixel vectors). This introduces the need to process the considered scenes in less than 0.196, 0.770, and 4.980 s, for Synthetic_1, AVIRIS Cuprite, and Synthetic_2 datasets, respectively, in order to achieve real-time performance. As shown in Tables II–V, all the scenes could be processed in real-time using method P-HYCA-FAST on GPU2 device, which results in speedups higher than $100\times$, while all other methods only could be processed in near real time. This is due to the optimizations that include the iterative method in order to calculate the $z$ coefficients, being one of the most time consuming part for both parallel implementations.
TABLE I
PROCESSING TIMES (IN SECONDS) AND SPEEDUPS ACHIEVED FOR THE PROPOSED COMPRESSION METHODOLOGY IN TWO DIFFERENT PLATFORMS AND TESTED WITH SYNTHETIC AND REAL DATA SETS

<table>
<thead>
<tr>
<th></th>
<th>Synthetic_1</th>
<th>AVIRIS Cuprite</th>
<th>Synthetic_2</th>
</tr>
</thead>
<tbody>
<tr>
<td>RAM → GlobalMem.</td>
<td>– 0.0024</td>
<td>– 0.0109</td>
<td>– 0.0112</td>
</tr>
<tr>
<td>Compute compression</td>
<td>0.0127</td>
<td>0.0018</td>
<td>0.0097</td>
</tr>
<tr>
<td>RAM → GlobalMem.</td>
<td>– 0.0003</td>
<td>– 0.0101</td>
<td>– 0.0112</td>
</tr>
<tr>
<td>Total time</td>
<td>0.0127</td>
<td>0.0045</td>
<td>0.0134</td>
</tr>
<tr>
<td>Speedup (CPU time / GPU time)</td>
<td>2.82×</td>
<td>3.76×</td>
<td>4.89×</td>
</tr>
</tbody>
</table>

V. CONCLUSION AND FUTURE RESEARCH LINES

In this work, we have developed computationally efficient implementations of HYCA and CHYCA methods for hyperspectral CS on GPU platforms. The significant speedups reported in the experiments is expected to bridge the gap toward real-time CS of hyperspectral data sets, which is a highly desirable requirement for many remote sensing applications. The performance of the proposed implementations has been evaluated (in terms of the quality of the solutions provided and their parallel performance) using a real data set collected by...
TABLE IV
PROCESSING TIMES (IN SECONDS) AND SPEEDUPS ACHIEVED FOR THE P-CHYCA IN TWO DIFFERENT PLATFORMS AND TESTED WITH SYNTHETIC AND REAL DATA SETS

<table>
<thead>
<tr>
<th></th>
<th>Synthetic _1</th>
<th>AVIRIS Cuprite</th>
<th>Synthetic _2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CPU</td>
<td>GPU1</td>
<td>GPU2</td>
</tr>
<tr>
<td>RAM → GlobalMem.</td>
<td>–</td>
<td>0.0002</td>
<td>0.0001</td>
</tr>
<tr>
<td>((\mathbf{K}^T \mathbf{K} + \mu I)^{-1})</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
</tr>
<tr>
<td>RAM → GlobalMem.</td>
<td>–</td>
<td>(\approx 0.0000)</td>
<td>(\approx 0.0000)</td>
</tr>
<tr>
<td>Compute (x) (line 3)</td>
<td>0.3097</td>
<td>0.0845</td>
<td>0.0877</td>
</tr>
<tr>
<td>Compute (v_1) (line 4)</td>
<td>0.1841</td>
<td>0.0173</td>
<td>0.0121</td>
</tr>
<tr>
<td>Compute (v_2) (line 5)</td>
<td>0.0093</td>
<td>0.0033</td>
<td>0.0030</td>
</tr>
<tr>
<td>Compute (v_3), (v_4) (lines 6–7)</td>
<td>0.0554</td>
<td>0.0076</td>
<td>0.0061</td>
</tr>
<tr>
<td>Compute (v_5) (line 8)</td>
<td>0.0928</td>
<td>0.0120</td>
<td>0.0106</td>
</tr>
<tr>
<td>Compute (d) (line 9)</td>
<td>0.0755</td>
<td>0.0145</td>
<td>0.0115</td>
</tr>
<tr>
<td>Reconstruction (line 12)</td>
<td>0.0099</td>
<td>0.0008</td>
<td>0.0004</td>
</tr>
<tr>
<td>RAM → GlobalMem.</td>
<td>–</td>
<td>0.0036</td>
<td>0.0031</td>
</tr>
<tr>
<td>Total time</td>
<td>0.7389</td>
<td>0.1439</td>
<td>0.1346</td>
</tr>
<tr>
<td>Speedup (CPU time / GPU time)</td>
<td>5.13(\times)</td>
<td>5.49(\times)</td>
<td>14.15(\times)</td>
</tr>
</tbody>
</table>

TABLE V
PROCESSING TIMES (IN SECONDS) AND SPEEDUPS ACHIEVED FOR THE P-CHYCA-FAST IN TWO DIFFERENT PLATFORMS AND TESTED WITH SYNTHETIC AND REAL DATA SETS

<table>
<thead>
<tr>
<th></th>
<th>Synthetic _1</th>
<th>AVIRIS Cuprite</th>
<th>Synthetic _2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CPU</td>
<td>GPU1</td>
<td>GPU2</td>
</tr>
<tr>
<td>RAM → GlobalMem.</td>
<td>–</td>
<td>0.0002</td>
<td>0.0001</td>
</tr>
<tr>
<td>((\mathbf{K}^T \mathbf{K} + \mu I)^{-1})</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
</tr>
<tr>
<td>RAM → GlobalMem.</td>
<td>–</td>
<td>(\approx 0.0000)</td>
<td>(\approx 0.0000)</td>
</tr>
<tr>
<td>Compute (x) (line 3)</td>
<td>0.2098</td>
<td>0.0196</td>
<td>0.0159</td>
</tr>
<tr>
<td>Compute (v_1) (line 4)</td>
<td>0.1663</td>
<td>0.0153</td>
<td>0.0116</td>
</tr>
<tr>
<td>Compute (v_2) (line 5)</td>
<td>0.0091</td>
<td>0.0032</td>
<td>0.0029</td>
</tr>
<tr>
<td>Compute (v_3), (v_4) (lines 6–7)</td>
<td>0.0537</td>
<td>0.0076</td>
<td>0.0060</td>
</tr>
<tr>
<td>Compute (v_5) (line 8)</td>
<td>0.0928</td>
<td>0.0120</td>
<td>0.0103</td>
</tr>
<tr>
<td>Compute (d) (line 9)</td>
<td>0.0788</td>
<td>0.0145</td>
<td>0.0114</td>
</tr>
<tr>
<td>Reconstruction (line 12)</td>
<td>0.0098</td>
<td>0.0008</td>
<td>0.0005</td>
</tr>
<tr>
<td>RAM → GlobalMem.</td>
<td>–</td>
<td>0.0036</td>
<td>0.0030</td>
</tr>
<tr>
<td>Total time</td>
<td>0.6194</td>
<td>0.0770</td>
<td>0.0617</td>
</tr>
<tr>
<td>Speedup (CPU time / GPU time)</td>
<td>8.05(\times)</td>
<td>10.03(\times)</td>
<td>22.01(\times)</td>
</tr>
</tbody>
</table>

the AVIRIS instrument and synthetic scenarios. The experimental results reported in this paper indicate that remotely sensed hyperspectral imaging can greatly benefit from the development of efficient implementations of CS algorithms in specialized hardware devices for better exploitation of high-dimensional data sets. In this case, real-time performance could be obtained using the P-HYCA-FAST version and the NVIDIA GeForce GTX TITAN device, one of the latest GPU models characterized by the integration of a significant number of processing cores.

Although the results reported in this paper are very encouraging, in future work we will continue exploring additional strategies for optimization, such as splitting the original hyperspectral image into subimages and applying a multiGPU implementation to each of them. We are also investigating the use of OpenCL as a computing standard for multicore architectures. Moreover, other high-performance computing architectures such as digital signal processors (DSPs) or FPGAs will be also explored due to their capacity to be used as onboard processing modules in airborne and, particularly, spaceborne Earth observation missions.

REFERENCES


José M. P. Nascimento (S’03–M’06) received the B.S. degree in electronics and telecommunications engineering and E.E. degree in electrical engineering from the Instituto Superior de Engenharia de Lisboa, Instituto Politecnico of Lisbon, Lisbon, Portugal, in 1993 and 1995, respectively, and the M.Sc. and Ph.D. degrees in electrical and computer engineering from the Instituto Superior Técnico, Technical University of Lisbon, Lisbon, Portugal, in 2000 and 2006, respectively. Currently, he is a Professor with the Department of Electronics, Telecommunications, and Computer Engineering, Instituto Superior de Engenharia de Lisboa, Lisbon, Portugal. He is also a Researcher with the Instituto de Telecomunicações, Lisbon, Portugal. Recently, he has been a Principal Researcher of two projects in the field of high-performance computing for hyperspectral imagery. His research interests include remote sensing, image processing, and high-performance computing.

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