Math 423/673, Spring 2012
Differential Geometry

Instructor: John Zweck
Office: MP 424
Email: zweck@umbc.edu
Webpage: I will maintain a web page for the course, linked from my web page www.math.umbc.edu/~zweck. I will also communicate with you using a class email list. (I do not use Blackboard.)
Phone: (410) 455 2424 (Do not leave messages, email instead!)
Fax: (410) 455 1066
Lectures: TuTh, 1:00-2:15 (MP 008)
Prerequisites: Math 221 (Introduction to Linear Algebra) and Math 251 (Multivariable Calculus). For general mathematical maturity, I strongly recommend that students have taken Math 301 (Introduction to Mathematical Analysis).
Office Hours: M 1:00-2:00, Tu 2:15-3:15 and by appointment. If you cannot come to my office hours please contact me in class or by email/phone to set up a time to meet. Also, you are encouraged to ask me questions by email/phone.

Course Summary and Learning Goals
The course focuses on the differential geometry of curves and surfaces in three-dimensional space. In Vector Calculus (Math 251), you studied the best linear approximations to curves and surfaces, which are tangent lines to curves and tangent planes to surfaces and you learned how to differentiate and integrate functions and vector fields on curves and surfaces. You also studied the Four Flavours of the Fundamental Theorem of Calculus (which include the theorems of Green, Stokes and Gauss).
In Math 423 we build on material from Vector Calculus and Linear Algebra (Math 221). The course will begin by relearning relevant material from Vector Calculus at a higher level. In addition to studying vector fields, we also study differential forms on curves and surfaces, which can be regarded as the most general mathematical objects that can be differentiated and integrated over curves, surfaces, and their higher-dimensional analogues. We will learn how the Four Flavours of the Fundamental Theorem of Calculus you learned in Math 251 are all special cases of The Fundamental Theorem of Calculus for Differential Forms. A second major focus is to study the curvature of curves and surfaces in space, which encodes information about the best quadratic approximations to curves and surfaces. The highlights of the course are two famous theorems about Gauss curvature. The first theorem is Gauss’s “Remarkable Theorem” that Gauss curvature can be computed simply by making measurements on the surface itself, rather than requiring information about how the surface is embedded in three-dimensional space. The second result is the Gauss-Bonnet Theorem which states that the integral of the Gauss curvature over the surface is a topological invariant: If you deform the surface, but don’t tear it, the total Gauss curvature doesn’t change.

Homework and exams will emphasize calculations for specific examples based on the theory discussed in class, as well as some more abstract proofs based on similar sorts of calculations to the ones performed in class. Exams will also test your understanding of definitions and theorems covered in class.

The course has the following specific **learning goals**.

1. Students will master the definitions, examples, calculations, theorems and proofs discussed in class and covered on homework.

2. Differential Geometry is field rich in examples which provide a wealth of intuition. Students will develop geometric intuition and learn how to transform this intuition into precise mathematical concepts using tools from vector calculus, linear algebra, and differential equations. They will learn how to formulate and perform calculations involving these concepts and to provide interpretations of the results of these calculations in terms of the motivating geometrical intuition.

3. An important course goal is to prepare students for higher level mathematics courses, for research and for professional work involving mathematics. This goal will be met by having students understand how mathematics involves gaining mathematical and scientific knowledge through the integration of concepts from a wide variety of subjects. In particular, students will learn how Differential Geometry integrates ideas from calculus, linear algebra, differential equations, analysis and physics to study the geometric structure of curves and surfaces in space. Students will learn to identify the abstract mathematical principles that unite apparently disparate examples and applications.

4. Finally, students will be encouraged to develop an appreciation for the many applications of Differential Geometry.
Academic Misconduct

I will not tolerate cheating in any form. All instances of cheating I discover will be reported to UMBC’s academic integrity committee. (See http://www.umbc.edu/integrity/) In particular, in this course, giving or receiving aid on exams will result in a grade of zero for that exam. Copying of homework solutions from other students in the class, from students who have previously taken this or an equivalent course, from a solutions manual, or from the web will treated as a serious offense and may result in a grade of zero for all homework for the semester. Plagarism or other cheating on projects (Math 673 only) will result in a grade of zero for the project. In addition, cheating on exams, homework or projects may be further penalized by a reduction in the final letter grade for the course, and could result in the student failing the course.

Here is a summary of UMBC’s official policy on academic misconduct, which I fully endorse:

By enrolling in this course, each student assumes the responsibilities of an active participant in UMBC’s scholarly community in which everyone’s academic work and behavior are held to the highest standards of honesty. Cheating, fabrication, plagiarism, and helping others to commit these acts are all forms of academic dishonesty, and they are wrong. Academic misconduct could result in disciplinary action that may include, but is not limited to, suspension or dismissal. To read the full Student Academic Conduct Policy, consult the UMBC Student Handbook, the Faculty Handbook, or the UMBC Policies section of the UMBC Directory.

Grading

Grades (423): Study Plan Essay 0%, Homework 25%, Midterm One 25%, Midterm Two 25%, Final 25%
Grades (673): Study Plan Essay 0%, Homework 15%, Project 10%, Midterm One 25%, Midterm Two 25%, Final 25%

Study Plan Essay: Due at start of class on Tuesday Jan 31st. Students who do not complete this assignment will receive a grade of ZERO for all homework for the entire course. See http://www.umbc.edu/zweck/M423S12/StudySkillsHwk.html.

Homework: The homework problems will be posted on the course web page for each day of class. Problems assigned on Tuesday and Thursday will be due at the start of class the following Thursday. At least some of them will be graded. Make sure your homework paper is stapled. No late homework will be accepted! Your lowest two homework grades will be dropped. You may
ask me questions about the homework and you may collaborate with another student in the class. In fact you are encouraged to do so! However the final write up is your own – two (almost) identical solutions may both be given zero. I do not encourage large groups of people to work together on homework. Do not miss class to complete a homework. I will not accept homework that is handed in after the first few minutes of class.

Midterm Exams: There will be two midterm exams.

- Midterm 1: Tuesday March 6th
- Midterm 2: Thursday April 12th

Final Exam: Tuesday May 15th from 1-3pm in MP 008. The final will be based on the whole course and may be harder than midterms.

Math 673: The graduate level version of the course (Math 673) will differ from the undergraduate version (Math 423) in that students will write a report on an application of Differential Geometry based on a journal or review article chosen by the student in close consultation with the instructor. The topic and guidelines for the report must be agreed upon by Tuesday March 13th and the final report is due on Thursday May 3rd.

Making up an exam you missed

If you miss one of the midterms you may be given the chance to take a make up exam. To request a make up you should speak with me no later than 48 hours after the exam time. Generally speaking, you will be offered a make up if you are sick or if a close relative or friend is gravely injured/sick or dies. However I will listen to all reasonable requests. Be prepared to bring appropriate evidence in support of your request. There will be no make ups for the final exam.

How I assign final grades

For each exam I work out how many points I expect a student who has a solid understanding of the material to get. I tend to put the bottom B near this score. Then I work out where to place the bottom A,C,D using the grade distribution and by looking at individual exams. I also work out the bottom A,B,C,D for the homework, and presentations. Then I take an imaginary student who got the bottom B (say) for each component of the course and calculate their score. If your score is higher than the imaginary student’s you get a B. If it is a little less than the imaginary student’s score I look carefully at your work to decide whether you deserve a B or a C. Most importantly I look at your final exam and your homework. In particular, students on the borderline between two grades and who show mastery of the
material on the final are more likely to receive the higher grade. *However, students who do very poorly on the final might find that their course grade is lower than they had expected!*

As you can see I place quite a bit of emphasis on the final exam. In short I reward “strong finishers” who can show me they have a solid understanding of the entire course.

### Study Tips

1. Read the results of a survey I did on “Study Habits and the Transition from High School to UMBC” which can be found on my web page\(^1\) and find a strategy that works for you.

2. **Warning:** This course gets harder as the semester progresses. My experience is that student who receive a C on the midterms are in grave danger of getting D/F on the final and in the course. **To do as well as you can, I strongly encourage you to come to see me with specific questions on a regular basis.**

3. On the course web page I will post the sections that we will cover each day. **You are expected to read the section ahead of time.**

4. It is very important to keep the main definitions, statements of theorems, and simpler examples on the forefront of your minds throughout the course, since we will refer back to them many times. You will need to digest the material several times to master it — before class, in class, reading through material after class, rederiving for yourself without any aid results discussed in class, and doing the assigned problems.

5. This is a fast paced course. Do not get behind. Do not miss class. If you miss a class or start to get lost, it will only be a week before you are totally lost. So ask for help from me and from your fellow students immediately!

6. I encourage you to **ask questions** both in and out of class. If you are dazed and confused most likely most of your class mates are too! So you’ll be doing everyone a favor by asking your question.

7. In class I call on people by name to answer questions. This is to keep you involved and on your toes. It also helps me find out whether you are understanding what’s going on. **If you do not feel comfortable being called on in class, please come and talk with me, and we will find another way to actively involve you.**

8. Come and talk with me in my office. Talk math with your fellow students, don’t work in isolation.

9. Learn the art of taking good notes. My lectures often present a complementary perspective to that in the textbook. **I test what I teach!** So you need a detailed record of what was discussed in class.

### Advice for Homework

1. Never start your homework the day before it is due!!

2. **Do all** the hwk problems. Learn from your mistakes on the graded hwk.

3. My Dad used to say “You can’t do maths on a postage stamp”, so use lots of paper. Write your solutions up neatly after working out the problem on scrap paper. Apart from anything else, this helps you organize your thoughts and therefore learn the material better.

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\(^1\)See [http://www.math.umbc.edu/~zweck/TATrain/StudyHabits.html](http://www.math.umbc.edu/~zweck/TATrain/StudyHabits.html)
4. I’ll teach you by example how to write up your solutions in a connected step-by-step fashion with explanatory sentences. You should aim to write up solutions so that you’ll easily understand them in a month’s time when you’re studying for the exam!

5. If your homework grades are not as high as you’d like you should arrange to meet with me for 15 minutes at a fixed time each week. We will use this time to discuss what you did wrong on past homeworks and also check how you are doing on the current homework. Don’t wait until exam time!

6. The only way to learn math is to do it: Struggle to solve problems for yourself.

7. However, if you get stuck on a problem for too long get help and get it before you waste too much time!! Here are some places you can go for help.

- Carefully read your notes from lectures and the book (again!).
- Draw a schematic picture to help you think about the problem.
- Ask me for help by email or in person.
- Ask a fellow class member – often two heads are better than one! I encourage you to find a study partner for this class. First attempt the hwk yourself, then discuss them with your study partner, and finally carefully write the solutions up in your own words.
- Sleep on it. Some of my best ideas come when I wake up in the morning.

Advice for Exams

A large collection of past exams are on the course web page together with some solutions. Do as many as you can!

Study in small groups for exams and learn from each other. Presenting material to someone else is often the best way to work out whether you really know it yourself. Exams will cover theory covered in class (definitions, theorems, examples) and problems similar to those in the homework and in the textbook. Old exams (some with solutions) are available on the course web page.

I encourage you to first master the theory and memorize calculation methods and formulae you need to know and then use this knowledge to work a range of problems without looking at your notes. To learn theory, calculation methods, and formulae go through your notes and the book and write down a detailed list of topics you need to know. Then with your lecture notes and book closed write down what you know about each topic, as precisely and succinctly as you can. Only when you get stuck should you look at your lecture notes. If you do this about 4 times in the 10 days prior to the exam you should be in good shape. Don’t forget to work lots of problems as well!

You should also spend some but not all of your preparation time studying in small groups to learn from each other. Presenting material to someone else is often the best way to work out whether you really know it yourself.