MATH 251 (Fall 2004) Exam 1, Oct 1st

No calculators, books or notes!
Show all work and give **complete explanations** for all your answers.
This is a 65 minute exam. It is worth a total of 75 points.

(1) [20 pts]
(a) Find the dot product of two vectors if their lengths are 6 and $\frac{1}{3}$
and the angle between them is $\frac{\pi}{4}$.

(b) Find the area of the parallelogram with vertices $(1, 2, 3)$, $(0, 2, 5)$, $(4, 6, 8)$, and $(3, 6, 10)$.

(c) Find the scalar projection of the vector $\mathbf{v} = \mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$ onto the vector $\mathbf{w} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$.

(d) Draw a picture and write a sentence or two that clearly explain the geometrical meaning of the scalar projection of a vector $\mathbf{v}$ onto another vector $\mathbf{w}$.

(2) [15 pts] Sketch the following surfaces
(a) $y^2 - x^2 + 4z^2 = 1$. Also sketch some appropriately chosen traces (*i.e.*, slices) of this surface.

(b) $\phi = \frac{2\pi}{3}$

(3) [15 pts]
Consider the plane through $(0, 0, 0)$ with normal vector $(4, 2, 3)$.
(a) Find an equation of the form $ax + by + cz = d$ for this plane.

(b) Find a parametrization of this plane.

(c) Suppose that $\mathbf{r}(t)$ is a curve for which $\mathbf{r}(2) = (0, 1, -3)$, $\mathbf{r}'(2) = (-1, 2, 5)$, and $\mathbf{r}''(2) = (2, 4, -6)$. Find a parametrization of the tangent line to this curve at $t = 2$.

(4) [15 pts] Match the parametric equations (a)-(b) on the next page with the graphs labeled (I)-(VI). [Note that there are more graphs than equations!] *Carefully explain the reasons for your choices.*

(a) $x = t$, $y = t^2$, $z = e^{-t}$.

(b) $r = 1$, $\theta = t$, $z = \sin 5t$.

(5) [10 pts] Suppose that $\mathbf{r}$ is a curve that lies on the sphere of radius 1 centered at the origin. Prove that at each point on the curve, the velocity vector $\mathbf{r}'(t)$ to the curve is perpendicular to the position vector $\mathbf{r}(t)$ of the point.

Pledge: *I have neither given nor received aid on this exam*

Signature: _____