NAME:

1 /12	2 /15	3 /12	4 /24	5 /12	T /75

MATH 251 (Fall 2004) Exam 2, Oct 27th

No calculators, books or notes! Show all work and give **complete explanations** for all your answers. This 65 minute exam is worth 75 points.

(1) [12 pts] Let $z = f(x, y) = x^2 - 3y^2 - x\cos(\pi y^2)$.
(a) Find the first partial derivatives of f at the point $(1, -2, -12)$.

(b) Find an equation of the form $z = ax + by + c$ for the tangent plane to the graph of f at $(1, -2, -12)$.

(2) [15 pts] In each case, evaluate the limit or show that it does not exist.

(a) $\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^4+y^2}$

(b) $\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^2+y^2}$

(3) [12 pts]
(a) Suppose $z = f(x, y)$ is a function and $\mathbf{r}(t) = (x(t), y(t))$ is a parametrized curve. State the version of the Chain Rule that you would use to differentiate the composition $f \circ \mathbf{r}$.

(b) Suppose now that $\mathbf{r}(t) = (e^t + \cos t, e^t + \sin t)$. Calculate $(f \circ \mathbf{r})'(0)$ using the following table of values.

(a, b)	$f(a, b)$	$f_x(a, b)$	$f_y(a, b)$
$(1, 2)$	8	7	6
$(2, 1)$	6	-3	11

(4) [24 pts] Consider the parametrized surface

$$\begin{aligned} x &= 3u\cos v \\ y &= u\sin v \qquad (\star) \\ z &= u^2 \end{aligned}$$

(a) Find a parametrization for the tangent plane to this surface at $(u, v) = (1, \frac{\pi}{3})$.

(b) Find an equation of the form $z = f(x, y)$ for the parametrized surface given by $(\star)$ and carefully sketch the level curves of this function at levels $k = 1, 2, 3$.

(c) Use the equation $z = f(x, y)$ in (b) to sketch the graph of the surface. Also sketch the grid curves $u = 1$ and $v = \frac{\pi}{3}$ on the surface.

(d) Suppose that $w = f(x, y, z) = x^2 + y^2 + z$ is temperature at (x, y, z) and that x, y, z are the functions of (u, v) given by $(\star)$ above. Use the Chain Rule to compute $\frac{\partial w}{\partial v}$ at $(u, v) = (1, \frac{\pi}{3})$.

Also: Explain the geometrical meaning of $\frac{\partial w}{\partial v}$ at $(u, v) = (1, \frac{\pi}{3})$.

(5) [12 pts] Suppose that

$$z = f(x, y) = y \sin(xy^7 + \sqrt{x^3 y + \tan y}) + e^{\sin x + \cos y}.$$

Find $\frac{\partial f}{\partial x}(0, 0)$.

[Hint: There is an hard way and an easy way to do this calculation. *You will get zero points for doing the problem the hard way!!*]

Pledge: *I have neither given nor received aid on this exam*

Signature: _____