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MATH 251 (Fall 2004) Exam 3, Nov 29th

No calculators, books or notes! Show all work and give **complete explanations** for all your answers. This 65 minute exam is worth 75 points.

(1) [15 pts]

(a) Suppose that $(0, 2)$ is a critical point of a function g with continuous second partial derivatives. What can you say about g if

$$g_{xx}(0, 2) = 1 \quad g_{xy}(0, 2) = 2 \quad g_{yy}(0, 2) = 7?$$

(b) Find the maximum rate of change of the function $f(x, y, z) = x^3y^2z^5$ at the point $(1, 1, 1)$, and the direction in which it occurs.

(2) [15 pts]

(a) Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y) = x^3y^2\mathbf{i} - y\sqrt{x}\mathbf{j}$, and where C is the curve parametrized by $\mathbf{r}(t) = t^2\mathbf{i} - t^3\mathbf{j}$ for $0 \leq t \leq 1$.

(b) Use Green's Theorem to evaluate $\int_C 2xy \, dx + \sqrt{1+y^3} \, dy$, where C is the curve that consists of straight lines joining $(0, 0)$ to $(2, 0)$, $(2, 0)$ to $(2, 4)$, and $(2, 4)$ to $(0, 0)$.

(3) [15 pts]

Find the absolute maximum and minimum of the function $f(x, y) = xy$ on the region $5x^2 + 5y^2 \leq 1$.

(4) [15 pts] Calculate the integral $\iint_D y \, dA$, where D is the region in the first quadrant that lies above the hyperbola $xy = 1$, above the line $y = x$, and below the line $y = 3$.

(5) [15 pts] Let \mathbf{F} be the vector field

$$\mathbf{F}(x, y) = (-y^2 \sin x - \sin y)\mathbf{i} + (2y \cos x - x \cos y)\mathbf{j}.$$

(a) Determine whether or not \mathbf{F} is conservative. If it is, find a function f so that $\mathbf{F} = \nabla f$.

(b) Let C be the curve that is the straight line from $(0, 0)$ to $(2, 2)$. What is $\int_C \mathbf{F} \cdot d\mathbf{r}$?

Pledge: *I have neither given nor received aid on this exam*

Signature: _____