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NAME.

MATH 251 (Fall 2004) Exam 2, Oct 27th

No calculators, books or notes!
Show all work and give **complete explanations** for all your answers.
This is a 65 minute exam. It is worth a total of 75 points.

(1) [12 pts] Let $z = f(x,y) = 3x^2 - y^2 - y\sin(\frac{\pi x^2}{2})$.

(a) Find the first partial derivatives of f at the point $(1,-2,1)$.

$$\frac{\partial f}{\partial x} = 6x - y\cos\left(\frac{\pi x^2}{2}\right) \cdot \frac{\pi \cdot 2x}{2} \quad 2$$

$$\frac{\partial f}{\partial x}(1,-2) = 6 + 2\cos\left(\frac{\pi}{2}\right) \cdot \pi = 6 \quad 1$$

$$\frac{\partial f}{\partial y} = -2y - \sin\left(\frac{\pi x^2}{2}\right) \quad 2 \qquad \frac{\partial f}{\partial y}(1,-2) = 4 - 1 = 3 \quad 1$$

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(b) Find an equation of the form $z = ax + by + c$ for the tangent plane to the graph of f at $(1,-2,1)$.

$$z = f(a,b) + \nabla f(a,b) \cdot (x-a, y-b) \quad 3$$

$$(a,b) = (1,-2) \qquad f(1,-2) = 3 - 4 + 2\sin(\pi/2) = 1 \checkmark$$

$$\nabla f(1,-2) = (6,3) \text{ from (a)}$$

So

$$z = 1 + (6,3) \cdot (x-1, y+2)$$

$$= 1 + 6x - 6 + 3y + 6$$

$$\boxed{z = 6x + 3y + 1} \quad 3$$

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(2) [15 pts] In each case, evaluate the limit or show that it does not exist.

(a) $\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^2+y^2} = \lim_{r\to 0} \frac{2r^2\cos^2\theta \, r\sin\theta}{r^2}$

$= \lim_{r\to 0} 2r\cos^2\theta\sin\theta = 0$

So limit exists and is 0

NOTE SHOWING THAT limit is same ~~do~~ if you approach origin along several different curves ~~is NOT enough to~~ ? and always get same limit is NOT enough to show limit exists. How do you know that you couldn't find another curve to come in along which approaches a different limit ?!

(b) $\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^4+y^2}$

① $y = x^2$.

$\lim_{x\to 0} \frac{2x^2 \cdot x^2}{x^4+x^4} = \lim_{x\to 0} \frac{2x^4}{2x^4} = 1$

② $y=0$

$\lim_{x\to 0} \frac{0}{x^4} = \lim_{x\to 0} 0 = 0.$

So limit DNE

8

(4) [24 pts] Consider the parametrized surface

$$\begin{aligned} x &= u\cos v \\ y &= 2u\sin v \qquad (\star) \\ z &= u^2 \end{aligned}$$

(a) Find a parametrization for the tangent plane to this surface at $(u, v) = (2, \frac{\pi}{4})$.

$$\vec{r}\,(2, \pi/4) = \left(2\cos \pi/4,\ 2.2 \sin \pi/4,\ 4\right)$$

$$= \left(\sqrt{2},\ 2\sqrt{2}, 4\right)$$

$$\frac{\partial \vec{r}}{\partial u} = (\cos v,\ 2\sin v,\ 2u)$$

$$\frac{\partial \vec{r}}{\partial u}(2, \pi/4) = \left(\frac{1}{\sqrt{2}},\ \frac{2}{\sqrt{2}},\ 4\right)$$

$$\frac{\partial \vec{r}}{\partial v}(u, v) = (-u\sin v,\ 2u\cos v,\ 0)$$

$$\frac{\partial \vec{r}}{\partial v}(2, \pi/4) = (-\sqrt{2},\ 2\sqrt{2},\ 0)$$

6

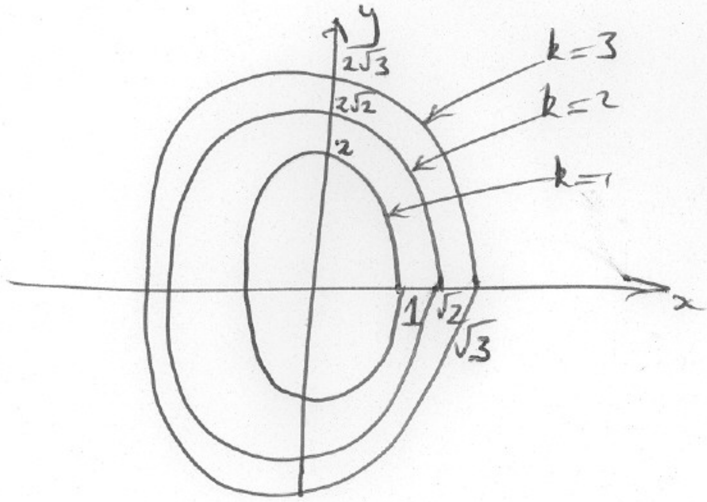
$$L(s,t) = (\sqrt{2},\ 2\sqrt{2},\ 4) + s\left(\frac{1}{\sqrt{2}},\ \frac{2}{\sqrt{2}},\ 4\right)$$

$$+\, t\,(-\sqrt{2},\ 2\sqrt{2},\ 0)$$

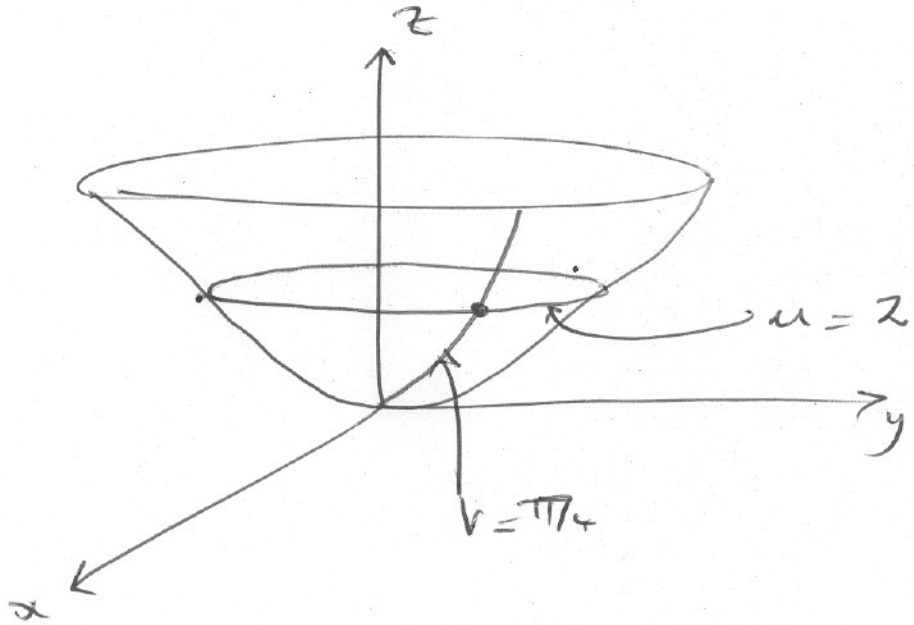
(b) Find an equation of the form $z = f(x, y)$ for the parametrized surface given by $(\star)$ and carefully sketch the level curves of this function at levels $k = 1, 2, 3$.

$$x^2 + \left(\frac{y}{2}\right)^2 = u^2 \cos^2 v + u^2 \sin^2 v = u^2 = z$$

So $z = x^2 + \left(\frac{y}{2}\right)^2$



(c) Use the equation $z = f(x, y)$ in (b) to sketch the graph of the surface. Also sketch the grid curves $u = 2$ and $v = \frac{\pi}{4}$ on the surface.



(d)

$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\frac{\partial w}{\partial u} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial u}$$

$$= 2x \cos v + 2y\, 2\sin v + 1\, 2u$$

$$\frac{\partial w}{\partial u} = 2u \cos^2 v + 8u \sin^2 v + 2u$$

$$\frac{\partial w}{\partial u}(2, \pi/4) = 4.\left(\frac{1}{\sqrt{2}}\right)^2 + 8.2.\left(\frac{1}{\sqrt{2}}\right)^2 + 4$$

$$= 2 + 8 + 4 = 14$$

/4

R of C of Temp at $\vec{r}(2, \pi/4)$

as go along grid curve $v = \pi/4$

in dirn of $\uparrow u$

/2

(5) [12 pts] Suppose that

$$z = f(x, y) = x\cos(xy^2 + \sqrt{x^2y^4 + \tan y}) + e^{x^2+\sin y}.$$

Find $\frac{\partial f}{\partial y}$ at $(x, y) = (0, 0)$.

[Hint: There is an hard way and an easy way to do this calculation. *You will get zero points for doing the problem the hard way!!*]

$\frac{\partial f}{\partial y}(0,0) = ?$

Let $g(y) = f(0, y) = e^{\sin y}$ 4

Then $\frac{\partial f}{\partial y}(0,0) = g'(0)$ 4

And $g'(y) = e^{\sin y} \cos y$ 2

So $\frac{\partial f}{\partial y}(0,0) = e^{\sin 0} \cos 0 = \underline{\underline{1}}$ 2

Pledge: *I have neither given nor received aid on this exam*

Signature: ______________________________