

MATH 251 (Spring 2004) Exam 1, Feb 25th

No calculators, books or notes!

Show all work and give **complete explanations** for all your answers.

This is a 65 minute exam. It is worth a total of 75 points.

(1) [10 pts] Let \mathbf{u} and \mathbf{v} be the vectors shown in the sketch and suppose that $|\mathbf{u}| = 1$.

Use the geometric definition of the dot and cross products to find

(a) $\mathbf{u} \bullet \mathbf{v} =$

(b) $\mathbf{u} \times \mathbf{v} =$

(2) [20 pts]

Consider the plane through the points

$P = (4, 5, 6)$, $Q = (1, -2, -5)$, and $R = (3, 0, 7)$.

(a) Find a vector that is perpendicular to this plane.

(b) Write down an equation of the form $ax + by + cz = d$ for this plane.

(c) Find a parametrization of this plane.

(d) Using a sketch and a couple of sentences, explain why $\mathbf{r}(s, t) = \mathbf{r}_0 + s\mathbf{u} + t\mathbf{v}$ is a parametrization of the plane through the endpoint of the vector \mathbf{r}_0 containing the vectors \mathbf{u} and \mathbf{v} .

(3) [10 pts] Let P be the point with spherical coordinates $\rho = 8$, $\theta = \pi/2$, $\phi = \pi/6$. Find the rectangular and cylindrical coordinates of P .

(4) [15 pts] Let $\mathbf{r}(t)$ be the parametrized curve $\mathbf{r}(t) = (2 \cos t, t, \sin t)$.

(a) Sketch the curve. [Hint: First find and sketch a surface on which the curves lies.]

(b) Compute the velocity vector of the curve \mathbf{r} at $t = 0$.

(c) Compute the parametric equation of the tangent line to the curve at $t = 0$.

(5) [20 pts] Match the equations (a)-(e) with the graphs labeled (I)-(VI). [Note that there are more graphs than equations!] *Give reasons for your choices.*

(a) $x^2 + y^2 + z^2 = 2y$

(b) $\rho = 1$

(c) $r = 2$

(d) $y^2 = x^2 + z^2$

(e) $y^2 = x^2 + z^2 + 1$

Pledge: *I have neither given nor received aid on this exam*

Signature: _____