MATH 251 (Spring 2004) Exam 2, March 31st

No calculators, books or notes!
Show all work and give **complete explanations** for all your answers.
This is a 65 minute exam. It is worth a total of 75 points.

(1) [12 pts] Let $\mathbf{r}(t) = (\cos(\frac{3}{5}t), \sin(\frac{3}{5}t), \frac{4}{5}t)$ be a parametrization of a helix.
(a) Show that $\mathbf{r}$ is a unit speed curve

(b) Calculate the curvature of $\mathbf{r}$.

(2) [12 pts] Let $z = f(x, y) = 3x^2 + 4xy + 5y^2$ and let $\mathbf{r}(t) = (x(t), y(t))$ be a parametrization of a curve in the plane such that

$$\begin{array}{rclrcl} \mathbf{r}(0) & = & (1,2), & \mathbf{r}(-2) & = & (-6,8), \\ \mathbf{r}(7) & = & (-1,3), & \mathbf{r}(4) & = & (9,1), \\ \mathbf{r}'(0) & = & (-2,7), & \mathbf{r}'(7) & = & (-1,3), \\ \mathbf{r}'(1) & = & (4,5), & \mathbf{r}'(2) & = & (-3,6). \end{array}$$

Let $g = f \circ \mathbf{r}$. Find $g'(0)$.

(3) [8 pts] Does the limit exist? Explain why, and if it does exist evaluate it.

$$\lim_{(x,y)\to(0,0)} \frac{3xy}{2x^2+y^2}$$

(4) [15 pts] Suppose a function $z = f(x, y)$ has continuous second partial derivatives and that

(a, b)	$f(a, b)$	$\nabla f(a, b)$	$f_{xx}(a, b)$	$f_{xy}(a, b)$	$f_{yy}(a, b)$
$(1, 2)$	3	$(0, 0)$	5	3	2
$(3, 4)$	0	$(1, 4)$	6	4	5
$(5, 6)$	0	$(0, 0)$	8	4	2
$(7, 8)$	-5	$(2, 3)$	1	5	2
$(9, 10)$	1	$(0, 0)$	-2	4	-3
$(11, 12)$	2	$(0, 0)$	-2	1	-3

Which of the points (a, b) are local maxima, minima or saddle points of f? Why?

(5) [18 pts] Let S the the surface which is the graph of the function $z = f(x, y) = 4x^2 + y^2$.

(a) Use the fact that $\mathbf{r}(u, v) = (\frac{1}{2}u\cos v, u\sin v, u^2)$ is a parametrization of S to find a parametrization of the tangent plane to S at the point where $(u, v) = (1, \frac{\pi}{3})$.

(b) Sketch the level curves of f at levels $z = 0, 1, 4$.

(c) In what direction in the xy-plane is the rate of change of f minimized at $(x, y) = (1, 1)$. What is the value of this minimum rate of change?

(6) [10 pts] Let $z = f(x, y)$. Prove that the gradient vector $\nabla f(a, b)$ is perpendicular to the level curve of f through the point (a, b).

Hint: Let $\mathbf{r}(t)$ be a parametrization of the level curve to f through (a, b). What do you know about $f(\mathbf{r}(t))$?

Pledge: *I have neither given nor received aid on this exam*

Signature: _____