MATH 251 (Fall 2009) Hwk on Dot Product (10.3)

(1) Let $\mathbf{v} = (1, -2, 4)$ and $\mathbf{w} = (-2, 3, 1)$. Compute

(a) $3\mathbf{v} - 5\mathbf{w}$
(b) $\mathbf{v} \cdot \mathbf{w}$
(c) $|\mathbf{v} - \mathbf{w}|$ and provide a geometric intrepretation for the answer.
(d) $\frac{\mathbf{v}}{|\mathbf{v}|}$ and provide a geometric intrepretation for the answer
(e) The angle between $\mathbf{v}$ and $\mathbf{w}$ in degrees
(f) The vector projection, $\text{Proj}_{\mathbf{v}}(\mathbf{w})$, of $\mathbf{w}$ onto $\mathbf{v}$
(g) The component, $\text{Comp}_{\mathbf{v}}(\mathbf{w})$, of $\mathbf{w}$ along $\mathbf{v}$.

(2) Find the equation of the sphere one of whose diameters is the line segement joining $(1, -2, 4)$ to $(-2, 3, 1)$.

(3) Consider the sphere $x^2 + y^2 + z^2 + 4x - 6y = 0$. Let $\mathbf{r} = (x, y, z)$ be an arbitrary point on this sphere. Find a vector $\mathbf{a}$ and a scalar c so that the vector equation of the sphere is $|\mathbf{r} - \mathbf{a}| = c$. What do $\mathbf{a}$ and c represent geometrically?

(4) Determine whether the vectors $(4, -2, 6)$ and $(4, 2, 2)$ are perpendicular.

(5) Use the dot product to construct a nonzero vector perpendicular to both $(1, 2, -3)$ and $(2, 0, 1)$.

(6) Given vectors $\mathbf{a}$ and $\mathbf{b}$, let $a = |\mathbf{a}|$ and $b = |\mathbf{b}|$. Use dot products to show that the vector

$$\mathbf{c} = \frac{b\mathbf{a} + a\mathbf{b}}{a + b}$$

bisects the angle between $\mathbf{a}$ and $\mathbf{b}$.