

MATH 251 (Fall 2009) Hwk on Cross Product (10.4)

(1) Find $\mathbf{a} \times \mathbf{b}$ when

(a) $\mathbf{a} = \mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = -2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$,

(b) $\mathbf{a} = (2, -3)$ and $\mathbf{b} = (4, 5)$.

(2) Find two different unit vectors both of which are perpendicular to both $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$.

(3) Show that the cross product is not associative by comparing $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$ to $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$ in the case that $\mathbf{a} = \mathbf{i}$, $\mathbf{b} = \mathbf{i} + \mathbf{j}$, and $\mathbf{c} = \mathbf{i} + \mathbf{j} + \mathbf{k}$.

(4) Find non-zero vectors \mathbf{a} , \mathbf{b} , and \mathbf{c} so that $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$ but $\mathbf{b} \neq \mathbf{c}$.

(5) Suppose that three vectors \mathbf{a} , \mathbf{b} , and \mathbf{c} are mutually perpendicular. Explain why $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = \mathbf{0}$.

(6) Find the area of the triangle with vertices $P = (1, 3, -2)$, $Q = (2, 4, 5)$, and $R = (-3, -2, 2)$.

(7) Find the volume of the parallelepiped determined by the vectors $\mathbf{a} = \mathbf{i} - \mathbf{j} + 3\mathbf{k}$, $\mathbf{b} = -2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$, and $\mathbf{c} = 3\mathbf{i} - 4\mathbf{k}$.