MATH 251 (Fall 2009) Hwk on Derivatives of Curves (11.2)

(1) Suppose that the position of a particle in space at time t is $\mathbf{r}(t) = (t+1)\mathbf{i} + (t^2-1)\mathbf{j} + 2t\mathbf{k}$. Find the velocity and speed of the particle at time $t = 1$.

(2) Consider the parametrized curve $\mathbf{r}(t) = \sin(t)\mathbf{i} + (t^2 - \cos t)\mathbf{j} + e^t\mathbf{k}$. Find a parametric equation for the tangent line to this curve at $t = 0$.

(3) Each of the equations parts (a)-(d) describes the motion of a particle around the circle $x^2 + y^2 = 1$. Although the path of each particle is the same, the behavior of the four particles is different. For each particle answer the following questions.

(i) Does the particle have constant speed? If so what is its speed?

(ii) Is the particle's acceleration vector always perpendicular to its velocity vector?

(iii) Does the particle move clockwise or counterclockwise around the circle?

(iv) Does the particle start at the point $(1, 0)$.

(a) $\mathbf{r}(t) = \cos(t)\mathbf{i} + \sin(t)\mathbf{j}$, $t \geq 0$

(b) $\mathbf{r}(t) = \sin(2t)\mathbf{i} + \cos(2t)\mathbf{j}$, $t \geq 0$

(c) $\mathbf{r}(t) = \cos(t - \pi/2)\mathbf{i} + \sin(t - \pi/2)\mathbf{j}$, $t \geq 0$

(d) $\mathbf{r}(t) = \cos(t^2)\mathbf{i} + \sin(t^2)\mathbf{j}$, $t \geq 0$

(4) Show that

$$\mathbf{r}(t) \;=\; 2\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \cos(t)(\frac{1}{\sqrt{2}}\mathbf{i} - \frac{1}{\sqrt{2}}\mathbf{j}) + \sin(t)(\frac{1}{\sqrt{3}}\mathbf{i} + \frac{1}{\sqrt{3}}\mathbf{j} + \frac{1}{\sqrt{3}}\mathbf{k})$$

describes the motion of a particle moving in the circle radius 1 centered at the point $(2, 2, 1)$ and lying in the plane $x + y - 2z = 2$. Hints: (1) This curve is of the form $\mathbf{r}(t) = \mathbf{r}_0 + \cos(t)\mathbf{v} + \sin(t)\mathbf{w}$. First explain why $\mathbf{r}(t)$ is in the plane through the point $\mathbf{r}_0$ that contains the vectors $\mathbf{v}$ and $\mathbf{w}$. (2) Verify that $\mathbf{v}$ and $\mathbf{w}$ are unit length vectors that are perpendicular to each other. (3) Use vector algebra to calculate $|\mathbf{r}(t) - \mathbf{r}_0|$. (4) Understand why these calculations solve the problem!

(5) A particle moves with constant speed along a curve in space. Show that its velcoity and acceleration vectors are always perpedicular.