

MATH 251 (Fall 2009) Hwk on Arc Length and Curvature (11.2)

(1) Consider the curve $\mathbf{r}(t) = t\mathbf{i} + \frac{2}{3}t^{3/2}\mathbf{k}$ for $0 \leq t \leq 8$. Find the unit tangent vector to this curve at $t = 2$. Also find the length of the curve.

(2) Calculate the arclength function

$$s(t) = \int_0^t |\mathbf{r}'(\tau)| d\tau$$

for the curve

$$\mathbf{r}(t) = (1 + 2t)\mathbf{i} + (1 + 3t)\mathbf{j} + (6 - 6t)\mathbf{k} \quad 0 \leq t \leq 1$$

and use it to calculate the length of the curve.

(3) Calculate the unit tangent vector and the curvature of the following curves.

(a) $\mathbf{r}(t) = (2t + 3)\mathbf{i} + (5 - t^2)\mathbf{j}$

(b) $\mathbf{r}(t) = (3 \sin t)\mathbf{i} + (3 \cos t)\mathbf{j} + 4t\mathbf{k}$

(c) $\mathbf{r}(t) = (\cos^3 t)\mathbf{i} + (\sin^3 t)\mathbf{j}$