NAME:

1 /10	2 /10	3 /10	4 /10	5 /10	T /50

MATH 251 (Fall 2009) Exam III, Nov 25th

No calculators, books or notes! Show all work and give **complete explanations**. This is 65 min exam is worth 50 points.

(1) [10 pts] Calculate $\iint_D x\,dA$, where D is the triangle in the xy-plane with vertices $(0,0)$, $(1,0)$, and $(1,2)$.

(2) [10 pts] Evaluate the integral

$$\int_{x=0}^{x=2} \int_{y=-\sqrt{4-x^2}}^{y=+\sqrt{4-x^2}} x \, dy \, dx,$$

by converting it to polar coordinates.

(3) [10 pts] Let $\mathbf{F}$ be the vector field $\mathbf{F}(x, y, z) = xy\mathbf{i} + 3z\mathbf{j} + y\mathbf{k}$ and let C be the curve parametrized by $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + \mathbf{k}$, where $0 \leq t \leq 1$. Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

(4) [10 pts] Consider the two vector fields

$$\begin{aligned}\mathbf{F}_1(x,y) &= (2xy - 2y^2 \sin x)\mathbf{i} + (x^2 + 4y\cos x)\mathbf{j} \\ \mathbf{F}_2(x,y) &= (2xy^2 - 2y \sin x)\mathbf{i} + (x^2 + 4y^2\cos x)\mathbf{j}\end{aligned}$$

One of these vector fields is conservative.

(a) Which vector field is conservative and which is not? Why?

(b) For the vector field that is conservative, evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is any curve from $(0,0)$ to $(0,1)$.

(5) [10 pts] Find a double integral equal to the volume of the solid bounded by the surfaces $y = x$, $x = 2$, $z = 0$, and $z = y$, and evaluate this integral.

Pledge: *I have neither given nor received aid on this exam*

Signature: _____