NAME:

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MATH 251 (Fall 2009) Final Exam, Dec 18th

No calculators, books or notes! Show all work and give **complete explanations**. This 120 min exam is worth 100 points.

(1) [8 pts] Find the point in which the plane $2x + 3y + z = 1$ intersects the line segment from $(-1, -2, -3)$ to $(4, 5, 2)$.

(2) [8 pts] Calculate the following limits or show they do not exist.

(a) $\lim_{(x,y)\to(0,0)} \frac{x^3}{x^2+y^2}$

(b) $\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}$

(3) [10 pts] Find the (x, y) values of all local minima, local maxima, and saddle points of the function

$$z = f(x, y) = 3x^2y + y^3 - 3x^2 - 3y^2 + 4.$$

(4) [12 pts] This problem concerns the surface S in space that is parametrized by

$$(x, y, z) = \mathbf{r}(u, v) = (\sin(u)\cos(v), \sin(u)\sin(v), 2\cos(u)), \qquad 0 \le u \le \pi/2,\ 0 \le v \le 2\pi.$$

(a) Calculate a normal vector to the surface at $(u, v) = (\pi/2, \pi/4)$.

(b) Find an equation of the form $F(x, y, z) = 0$ that is satisfied by all points on S.

(c) Sketch S. Be sure to label your axes, include a scale on each axis, and sketch the surface to scale. Draw some grid curves ($i.e.$, curves of the form $u = u_0$ or $v = v_0$) on S. [Recall from (a) that $0 \leq u \leq \pi/2$, $0 \leq v \leq 2\pi$.]

(5) [10 pts] Evaluate $\int_0^1 \int_{3y}^3 e^{x^2}\, dxdy$.

(6) [10 pts]
(a) Sketch the contours of the function $z = f(x, y) = x^2 - 4y^2$ at levels $z = -1$, $z = 0$, and $z = 1$. Be sure to include a scale on both axes, sketch the curves to scale, and label each contour with its z-value.

(b) For the function in (a), calculate a unit vector in the (x, y)-plane that gives the direction of greatest increase of f at the point $(2, 1)$. How is this vector related to the level curve of f through $(2, 1)$?

(7) [10 pts] Let $\mathbf{F}(x, y, z) = e^{xy}\mathbf{i} + \cos(z)\mathbf{j} + (\sin(x) + e^{z})\mathbf{k}$ be the velocity vector field of a fluid flowing in space.

(a) On average, is fluid flowing in or out of the point $(0, 0, 0)$? Why?

(b) Suppose a small paddlewheel is released into the fluid. Find a vector that represents the axis of rotation of the paddlewheel when the paddlewheel is at the origin.

(8) [10 pts]

(a) Carefully state the Divergence Theorem. You may find it helpful to draw a picture and refer to it in your written explanation.

(b) Carefully explain how the Divergence Theorem is related to the Fundamental Theorem of Calculus in single-variable calculus.

(9) [10 pts] Let E be the solid region enclosed by the half-cylinder $x^2 + y^2 = 9$ with $x \geq 0$, and the planes $x = 0$, $z = 1$, and $z = 5$. Sketch E and calculate $\iiint_E xe^z \, dV$.

(10) [12 pts] Let S be the surface that is the portion of the cylinder $x^2 + z^2 = 1$ between the planes $y = 0$ and $y = 2$. We choose the unit normal on S to be the one that points away from the y-axis (outward). Let $\mathbf{F}(x, y, z) = xz\mathbf{i} + z\mathbf{j} + \frac{y}{z}\mathbf{k}$. Calculate $\iint_S \mathbf{F} \cdot d\mathbf{S}$. [Hint: (θ, y) are parameters on S, where θ is the angle in the (x, z)-plane from the x-axis.]

Pledge: *I have neither given nor received aid on this exam*

Signature: ________________________________