MATH 251 (Fall 2010) Hwk on Lines and Planes (12.5)

Recall the following defintions:

(i) A **vector parametrization** of the line through the endpoint of the vector $\mathbf{a}$ in the direction of the vector $\mathbf{b}$ is given by $\mathbf{r}(t) = \mathbf{a} + t\mathbf{b}$, where $t \in \mathbf{R}$.

(ii) A **scalar parametrization** of the line in (i) is

$$\begin{aligned} x &= a_1 + tb_1 \\ y &= a_2 + tb_2 \\ z &= a_3 + tb_3 \end{aligned}$$

where $\mathbf{a} = (a_1, a_2, a_3)$ and $\mathbf{b} = (b_1, b_2, b_3)$.

(iii) A **level set equation** of a plane is an equation of the form $ax + by + cz = d$, where a, b, c, d are real numbers.

(iv) A **parametrization** of a plane through the endpoint of the vector $\mathbf{u}$ that contains the vectors $\mathbf{v}$ and $\mathbf{w}$ is of the form $\mathbf{r}(s,t) = \mathbf{u} + s\mathbf{v} + t\mathbf{w}$, where $s, t \in \mathbf{R}$.

(1) Find a vector parametrization and a scalar parametrization for the line passing through the point $(3, -4, 5)$ in the direction of the vector $\mathbf{v} = -2\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}$.

(2) Find a vector parametrization for the line passing through the points $(1, 2, 3)$ and $(9, 8, 7)$.

(3) Find a vector parametrization for the line through the point $(2, 5, 6)$ and perpendicular to the plane $2x - 4y + 5z = 9$.

(4) Find a vector parametrization for the line through the point $(2, 5, 6)$ and parallel to the line with scalar parametric equations $x = -1 - 2t$, $y = 3t + 7$, $z = 6t - 2$.

(5) Find the level set equation and a parametrization of the plane through the point $(1, 2, 3)$ with normal vector $(4, 5, 6)$.

(6) Find the level set equation of the plane through the point $(1, 2, 3)$ parallel to the plane $3x - 5y + 7z = 8$.

(7) Find the level set equation and a parametrization of the plane through the points $(1, 0, -1)$, $(3, 3, 2)$, and $(4, 5, -1)$.

(8) Find a parametrization of the plane that contains both the point $(2, 4, 6)$ and the line $x = 7 - 3t$, $y = 3 + 4t$, $z = 5 + 2t$.

(9) Does the line $x = 3 + 2t$, $y = 6 - 5t$, $z = 2 + 3t$ intersect the plane $3x + 2y - 4z = 1$?