NAME:

1 /10	2 /10	3 /8	4 /14	5 /8	T /50

MATH 251 (Fall 2010) Exam I, Sept 23rd

No calculators, books or notes! Show all work and give **complete explanations**. This 65 min exam is worth 50 points.

(1) [10 pts] Let $\mathbf{u} = (6, 0, 8)$ and $\mathbf{v} = (1, -2, 3)$ be two vectors in space.
(a) Calculate the vector projection, $\text{Proj}_{\mathbf{v}}(\mathbf{u})$, of the vector $\mathbf{u}$ onto the vector $\mathbf{v}$.

(b) Find a vector that is perpendicular to both $\mathbf{u}$ and $\mathbf{v}$.

(2) [10 pts]
(a) Find a parametrization of the plane that contains both the point $(2, 4, 6)$ and the line $x = 7 - 3t$, $y = 3 + 4t$, $z = 5 + 2t$.

(b) Find a level set equation (i.e., an equation of the form $ax + by + cz = d$) for the plane in (a).

(3) [8 pts] Let P be the point in space with spherical coordinates $(\rho, \theta, \phi) = (3, \frac{\pi}{4}, \frac{2\pi}{3})$. Sketch P and convert P to both rectangular and cylindrical coordinates.

(4) [14 pts] Find the traces (i.e., slices) of the surface

$$x^2 \;=\; 1 + \frac{y^2}{4} + \frac{z^2}{9}$$

in the planes $y = 0$, $z = 0$, and $x = k$, for $k = 0, \pm 1, \pm 2, \pm 3$. Then sketch the surface and name it.

(5) [8 pts] The Parallelogram Law states that, for any vectors $\mathbf{u}$ and $\mathbf{v}$,

$$|\mathbf{u}+\mathbf{v}|^2 \;+\; |\mathbf{u}-\mathbf{v}|^2 \;=\; 2|\mathbf{u}|^2 \;+\; 2|\mathbf{v}|^2.$$

(a) Give a geometrical interpretation of the Parallelogram Law.

(b) Prove the Parallelogram Law using vector algebra. [Hint: Use $|\mathbf{u}+\mathbf{v}|^2 = (\mathbf{u}+\mathbf{v})\cdot(\mathbf{u}+\mathbf{v})$ together with the distributive law for the dot product.]

Pledge: *I have neither given nor received aid on this exam*

Signature: _____