NAME:

1 /6	2 /12	3 /10	4 /12	5 /10	T /50

MATH 251 (Fall 2010) Exam II, Oct 21st

No calculators, books or notes! Show all work and give **complete explanations**. This 65 min exam is worth 50 points.

(1) [6 pts] Suppose that

$$x = 3u + 2v, \qquad y = 4u - 5v$$

and let $z = f(x, y)$ be a function so that

(a,b)	$f(a,b)$	$\frac{\partial f}{\partial x}(a,b)$	$\frac{\partial f}{\partial y}(a,b)$
$(1,2)$	5	6	4
$(7,-6)$	-1	-5	7

Find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ at $(u, v) = (1, 2)$.

(2) [12 pts]
(a) Sketch the parametrized curve $(x, y) = r(t) = (2\sin t, 3\cos t)$ for $0 \leq t \leq \pi$.

(b) Sketch the level curves of the function $z = f(x, y) = x - e^y$ at levels $k = -1$, $k = 0$, and $k = 1$. Also calculate the gradient of f at the origin, add it to your sketch, and explain how it is related to the level curve that passes through the origin.

(3) [10 pts] Let $z = f(x, y) = x^2 + y^3 + 4xy$.

(a) Suppose that the function $z = f(x, y)$ is temperature at the point (x, y) in the plane. Suppose that a stink bug is walking at constant speed in this plane. In what direction should the stink bug walk from the point $(x, y) = (-1, 2)$ to decrease its temperature the fastest?

(b) Find the rate of change of f at the point $(x, y) = (-1, 2)$ in the direction of the vector $2\mathbf{i} + 3\mathbf{j}$.

(c) Find a vector that is *tangent* to the level curve $x^2 + y^3 + 4xy = 1$ at the point $(x, y) = (-1, 2)$.

(4) [12 pts] Let S be the surface parametrized by

$$\mathbf{r}(u,v) \;=\; (1+\cos u, \sin u, v) \qquad 0 \le u \le \frac{\pi}{2}, \quad 0 \le v \le 4.$$

(a) Find a level-set equation of the form $F(x,y,z) = 0$ that is satisfied by all points on S.

(b) Calculate the tangent vectors to the grid curves $u = \pi/4$ and $v = 2$ at the point $\mathbf{r}(\pi/4, 2)$.

(c) Sketch S together with the grid curves $u = \pi/4$ and $v = 2$ and their tangent vectors at $\mathbf{r}(\pi/4, 2)$.

(5) [10 pts] Find all local maxima, local minima, and saddle points of the function $z = f(x, y) = xye^y$.

Pledge: *I have neither given nor received aid on this exam*

Signature: ______________________________