NAME:

1 /11	2 /12	3 /10	4 /12	5 /5	T /50

MATH 251 (Fall 2010) Exam III, Nov 23rd

No calculators, books or notes! Show all work and give **complete explanations**. This 70 min exam is worth 50 points.

(1) [11 pts]
Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}$ is the vector field $\mathbf{F}(x, y, z) = (x + yz)\mathbf{i} + 2x\mathbf{j} + xyz\mathbf{k}$ and C is the line segment from $(1, 0, 1)$ to $(2, 3, 1)$.

(b) Carefully state Green's Theorem (a picture might help!).

(2) [12 pts]
(a) Calculate $\iint_D y^3\,dA$, where D is the triangle with vertices $(0,2)$, $(1,1)$, and $(3,2)$.

(b) Calculate $\iint_D \cos(x^2+y^2)\,dA$, where D is the region above the x-axis and within the circle $x^2+y^2=9$.

(3) [10 pts] Consider the two vector fields

$$\begin{aligned} \mathbf{F}_1(x,y) &= (3x-2y)\mathbf{i} + (-4x+3y-8)\mathbf{j} \\ \mathbf{F}_2(x,y) &= (2x-3y)\mathbf{i} + (-3x+4y-8)\mathbf{j} \end{aligned}$$

One of these vector fields is conservative.

(a) Which vector field is conservative and which is not? Why?

(b) For the vector field that is conservative, evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is any curve from $(0,0)$ to $(2,0)$.

(4) [12 pts]
(a) Use the method of Lagrange Multipliers to find the absolute maximum and absolute minimum of the function $f(x, y) = x^2 + y^2$ on the ellipse $(x - 1)^2 + 4y^2 = 4$.

(b) By sketching the ellipse and some appropriately chosen level curves, $f(x, y) = k$, determine the approximate locations of the absolute maxima and minima of f on the ellipse, and compare to the answer you found in (a).

(5) [5 pts] Sketch a vector field $\mathbf{F}$ in the plane so that $\mathbf{F}(0, 0) = (0, 0)$, $(\nabla \times \mathbf{F})(0, 0) = (0, 0)$, and $(\nabla \cdot \mathbf{F})(0, 0) > 0$.

Pledge: *I have neither given nor received aid on this exam*

Signature: _____