MATH 251 (Fall 2011) Exam I, Sept 29th

No calculators, books or notes! Show all work and give **complete explanations**. This 65 min exam is worth 50 points.

(1) [8 pts] Let $\mathbf{u}$ be a unit vector in the xy-plane. Think of $\mathbf{u}$ as a vector that starts at the origin. Let $\mathbf{v}$ be the vector obtained by rotating $\mathbf{u}$ clockwise about the origin by 60°. Let $\mathbf{w} = \mathbf{v} - \mathbf{u}$.

(a) Draw a sketch that illustrates how the vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are related.

(b) Use the geometric (physics) definitions of the dot product and cross product to find (i) $\mathbf{u} \cdot \mathbf{v}$ (ii) $\mathbf{u} \cdot \mathbf{w}$ and (iii) $\mathbf{u} \times \mathbf{v}$.

(2) [10 pts]

(a) Find a vector parametrization of the line obtained by intersecting the planes $x + 2y + 3z = 1$ and $x - y + z = 2$.

(b) Find a vector parametrization of the plane $x + 2y + 3z = 6$.

(3) [12 pts] Let L_1 and L_2 be lines in space with parametrizations

$$x = 1 + 2t \qquad y = 2 + t \qquad z = -4 + t$$

and

$$x = 1 + 5t \qquad y = 2 + 2t \qquad z = -4.$$

(a) Using a schematic diagram and an English sentence, explain why L_1 and L_2 lie in a plane, P.

(b) Find a vector parametrization of the plane P.

(c) Find a level set equation of the plane P.

(4) [12 pts] Find the traces (i.e., slices) of the surface

$$-x^2 + 4y^2 - 9z^2 = 4$$

in the planes $x = 0$, $z = 0$, and $y = k$, for $k = 0, \pm 1, \pm 2$. Then sketch the surface.

(5) [8 pts] Which of the following expressions are meaningful? Which are meaningless? Explain! (Here **a**, **b**, **c**, and **d** are vectors in space.)

(a) $(\mathbf{a} \cdot \mathbf{b}) \cdot \mathbf{c}$

(b) $(\mathbf{a} \cdot \mathbf{b}) + \mathbf{c}$

(c) $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$

(d) $(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d})$

Pledge: *I have neither given nor received aid on this exam*

Signature: _____