NAME:

1 /8	2 /10	3 /10	4 /12	5 /10	T /50

MATH 251 (Fall 2011) Exam III, Nov 22nd

No calculators, books or notes! Show all work and give **complete explanations**. This 65 min exam is worth 50 points.

(1) [8 pts] Let C be the straight line segment in the xy-plane from the point $(1, 2)$ to the point $(5, 3)$. Let $\mathbf{F}$ be the vector field in the plane defined by $\mathbf{F}(x, y) = \frac{1}{2}(x\mathbf{i} + y\mathbf{j})$.

(a) Make a sketch showing the vector $\mathbf{F}(x, y)$ at three points (x, y) on C. Using your sketch, determine whether $\int_C \mathbf{F} \cdot d\mathbf{r}$ is positive, negative, or zero. Explain!

(b) Now calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

(2) [10 pts]
(a) Let D be the half-disc in the xy-plane given by $x^2 + y^2 \leq 9$ and $x \geq 0$. Calculate $\iint_D e^{-(x^2+y^2)}\, dA$.

(b) Let D be the region in the *first quadrant* (*i.e.*, $x \geq 0$ and $y \geq 0$) of the xy-plane that is bounded by the y axis and the curves $y = \sin x$ and $y = \cos x$, and such that $x \leq \pi/4$. Calculate $\iint_D y\, dA$.

(3) [10 pts] Let $\mathbf{r}(t) = (2\cos t, 3\sin t)$, for $0 \leq t \leq 2\pi$, and let $(u, v) = F(x, y) = (3x + 2y, x^2 + 5y^2)$. The composition $\mathbf{s}(t) = F(\mathbf{r}(t))$ is a curve in the plane. Use the *Chain Rule from Multivariable Calculus* to answer the following two questions.

(a) At which times, t, is the tangent vector to the curve $(u, v) = \mathbf{s}(t)$ vertical?

(b) For each of the times you found in (a), is the tangent vector pointing in the $+\mathbf{j}$ or $-\mathbf{j}$ direction?

(4) [12 pts] Let $z = f(x, y) = x^3 - 12xy + 8y^3$.

(a) Find a tangent vector to the level curve $f(x, y) = 5$ at the point $(x, y) = (1, -1)$.

(b) Find all local maxima, local minima, and saddle points of f.

(5) [10 pts] Let $z = f(x, y)$ be a function such that

(x, y)	$(2, 1)$	$(-2, -1)$	$(0, \sqrt{3})$	$(\sqrt{3}, 0)$
$\frac{\partial f}{\partial x}$	-10	10	0	4
$\frac{\partial f}{\partial y}$	-2	4	0	-3

Which of the (x, y) values in this table are candidates for the absolute maximum and absolute minimum of f on the curve $2x^2 - 3xy + 4y^2 = 6$? *Carefully justify your answers!*

Pledge: *I have neither given nor received aid on this exam*

Signature: _____