NAME:

1 /15	2 /15	3 /17	4 /18	5 /10	T /75

MATH 251H (Fall 2006) Exam 1, Sept 27th

No calculators, books or notes!
Show all work and give **complete explanations** for all your answers.
This is a 75 minute exam. It is worth a total of 75 points.

(1) [15 pts] Suppose that

$$\mathbf{r}(s,t) \;=\; (1+2s-3t, 5+s, -3+4s-t)$$

is a parametrization of a plane. Find a level set equation for this plane, *i.e.*, an equation of the form

$$ax + by + cz = d.$$

(2) [15 pts] Consider the parametrized curve $\mathbf{r}(t) = t\mathbf{i} + \frac{\sqrt{2}}{2}t^2\mathbf{j} + \frac{1}{3}t^3\mathbf{k}$.
(a) Find a parametrization for the tangent line to this curve at $t = 1$.

(b) Calculate the arclength function of the curve $\mathbf{r}$ starting from $t = 0$.

(3) [17 pts] Show that the parametrized curve $\mathbf{r}(t) = (\cos t, \sin t, 1)$ lies on the following two surfaces:

(i) $\rho = \sqrt{2}$ (in spherical coordinates)
(ii) $z = r$ (in cylindrical coordinates).

Also sketch both surfaces and the curve in the same figure.

(4) [18 pts] Find the traces (*i.e.*, slices) of the surface

$$-x^2 + 4y^2 - z^2 = 4$$

in the planes $x = 0$, $z = 0$, and $y = k$ for $k = 0$, $\pm\frac{1}{2}$, ± 1, ± 2, and ± 3,
Also sketch the surface and name it.

(5) [10 pts] Use the geometric definitions of the dot product and the cross product to show that the volume of the parallelipiped determined by the three vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ is $|\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})|$.

Pledge: *I have neither given nor received aid on this exam*

Signature: _____