NAME:

1 /15	2 /20	3 /15	4 /8	5 /10	6 /7	T /75

MATH 251H (Fall 2006) Exam 2, Oct 27th

No calculators, books or notes!
Show all work and give **complete explanations** for all your answers.
This is a 75 minute exam. It is worth a total of 75 points.

(1) [15 pts] Either compute the following limits or show they do not exist.

(a) $\lim_{(x,y)\to(0,0)} \frac{x^2-3y^2}{4x^2+5y^2}$

(b) $\lim_{(x,y)\to(0,0)} \frac{x^3}{x^2+y^2}$

(2) [20 pts] Let f be the function $z = f(x, y) = x^2 - xy + y^2 + 3x$.

(a) Calculate an equation of the form $z = ax + by + c$ for the tangent plane to the surface $z = f(x, y)$ at the point $(x, y, z) = (2, 3, 13)$.

(b) Find and classify all critical points of f.

(c) Find the absolute maximum and minimum of f on the region in the xy-plane bounded by the lines $x = 0$, $y = 1$, and $x + y = -4$.

(3) [15 pts]
(a) Use equations to explain why $\mathbf{r}(u,v) = (u\cos v, u\sin v, u)$, where $u \geq 0$ and $0 \leq v \leq 2\pi$, is a parametrization of a cone.
(b) Plot the $u =$ constant and $v =$ constant grid curves on a picture of the cone and label them.
(c) Carefully describe what happens to the parametrization and the surface when $u = 0$.
(d) Using the parametrization in (a), calculate a parametrization of the tangent plane to the cone at $(u,v) = (2, \pi/6)$.

(4) [8 pts] Suppose that the directional derivative of a function $w = f(x, y, z)$ at a point P is greatest in the direction of the vector $\mathbf{v} = (1, 1, -1)$, and that in this direction the value of the directional derivative is $2\sqrt{3}$.

(a) What is ∇f at P, and why?

(b) What is the directional derivative of f in the direction of the vector $(1, 1, 0)$?

(5) [10 pts] Suppose that $g(t) = f(\mathbf{r}(t))$, where $\mathbf{r}$ is the curve $\mathbf{r}(t) = (\cos t, \sin t, t)$ and

$$\frac{\partial f}{\partial x} = x \qquad \frac{\partial f}{\partial y} = y \qquad \frac{\partial f}{\partial z} = z - 2.$$

Find any local maxima and minima of g. (Do *not* find a formula for f.)

(6) [7 pts] If $z = f(x, y)$, where $x = r\cos\theta$ and $y = r\sin\theta$, use the Chain Rule to prove that

$$\frac{\partial z}{\partial \theta} = x\,\frac{\partial z}{\partial y} - y\,\frac{\partial z}{\partial x}$$

and

$$r\,\frac{\partial z}{\partial r} = x\,\frac{\partial z}{\partial x} + y\,\frac{\partial z}{\partial y}$$

Pledge: *I have neither given nor received aid on this exam*

Signature: ______________________________