NAME:

1 /10	2 /12	3 /15	4 /10	5 /14	6 /14	T /75

MATH 251H (Fall 2006) Exam 3, Nov 22nd

No calculators, books or notes!
Show all work and give **complete explanations** for all your answers.
This is a 75 minute exam. It is worth a total of 75 points.

(1) [10 pts]
(a) Find the divergence of the vector field $\mathbf{F}(x, y, z) = e^x \sin y\,\mathbf{i} + e^x \cos y\,\mathbf{j} + z\,\mathbf{k}$.

(b) Let $\mathbf{F}$ be the vector field $\mathbf{F}(x, y) = x^2 \cos(y)\,\mathbf{i} + y \sin(x)\,\mathbf{j}$ and let C be the curve in the plane given by $y = x^3$ from $(0, 0)$ to $(2, 8)$. Find a formula for a function g so that $\int_C \mathbf{F} \cdot d\mathbf{r} = \int_0^2 g(t)\,dt$.

(2) [12 pts] Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x,y) = (2xy - \sin x \sin y + e^x)\,\mathbf{i} + (x^2 + \cos x \cos y)\,\mathbf{j}$ and where C is any curve from $(0,0)$ to $(2,3)$.

(3) [15 pts] Calculate the volume of the solid enclosed by the parabolic cylinders $z = x^2$, $y = x^2$ and the planes $z = 0$ and $y = 4$.

(4) [10 pts] Calculate the integral $\iint_D e^{-x^2-y^2}\,dA$, where D is the region bounded by the semicircle $x = \sqrt{4-y^2}$ and the y-axis.

(5) [14 pts] Use the Method of Lagrange Multipliers to find the absolute maximum of the function $f(x, y) = (x - y)^3$ subject to the constraint $x^2 + y^2 = 1$.

(6) [14 pts] Carefully state Green's Theorem and use it to calculate the integral

$$\int_C (y + e^{\sqrt{x}})dx + (2x + \cos(y^2))dy,$$

where C is the positively-oriented boundary of the region enclosed by the parabolas $y = x^2$ and $x = y^2$.

Pledge: *I have neither given nor received aid on this exam*

Signature: ______________________________