M251 DIAGNOSTIC QUIZ SOLUTIONS FALL 2004

1) (a) $f'(x) = 2x, \quad f'(3) = 6$

(b) $f'(3) = \lim\limits_{h \to 0} \dfrac{f(3+h) - f(3)}{h}$

(c) $f'(3) =$ Slope of tangent line to $f(x)$ at $x = 3$

2) (a) $\displaystyle\int_{1/2}^{2} \frac{dx}{x} = \Big[\ln|x|\Big]_{1/2}^{2} = \ln 2 - \ln \tfrac{1}{2} = 2\ln 2$
$= \ln 4$

(b) $\displaystyle\int_{x=0}^{x=\infty} x e^{-x^2}\, dx = \int_{u=0}^{u=\infty} \tfrac{1}{2} e^{-u}\, du$ $\qquad u = x^2$, $du = 2x\, dx$

$= \tfrac{1}{2}\Big[-e^{-u}\Big]_0^{\infty} = \tfrac{1}{2}$

3) (a) $f(x) = 2x^3 + 3x^2 - 12x$.

$f'(x) = 6x^2 + 6x - 12 = 6(x+2)(x-1) = 0$ at $x = 1, -2$.

f': $+$ (-2) $-$ (1) $+$

So f' is increasing on $(-\infty, -2)$ and $(1, \infty)$

(b) Absolute Max on $[0, 3]$ is largest of

$f(0)$, $f(3)$, $f(1)$ which is 45 at $x = 3$

as $f(0) = 0$, $f(1) = -7$, $f(3) = 45$

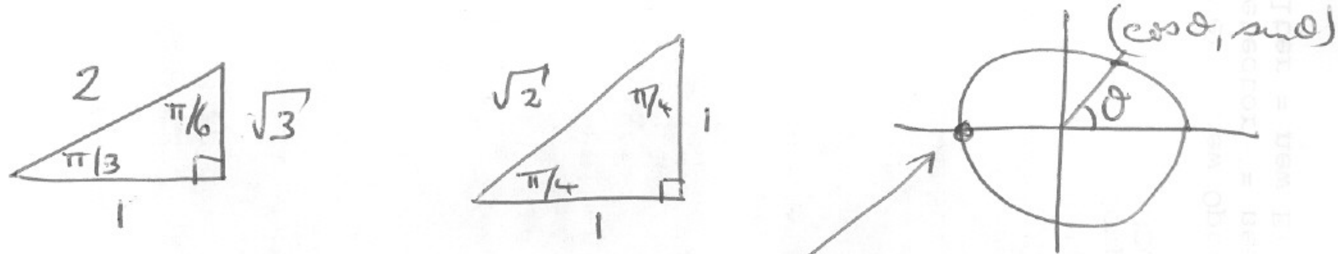
(4) (I) Suppose f is continuous on $[a, b]$, and F is an antiderivative of f, ie $F' = f$. Then $\int_a^b f(x)\, dx = F(b) - F(a)$

(II) If f is continuous on $[a, b]$ and $g(x) = \int_a^x f(t)\, dt$ then $g'(x) = f(x)$.

(5) $f'(x) = \sin(x^3)$ by FTC (II) above

So $f'(10) = \sin(1000)$

(6)



$\cos \pi/6 = \sqrt{3}/2$ $\sin \pi = 0$ $\tan \pi/4 = 1$

$\cot \pi/3 = 1/\sqrt{3} = \sqrt{3}/3$

(7) $f(x) = \cos(x^2)$

$f'(x) = -\sin(x^2) \cdot 2x = -2x \sin(x^2)$

$f(\pi/3) = \cos(\pi^2/9)$

$f'(\pi/3) = -\frac{2\pi}{3} \sin(\pi^2/9)$

Eqn of T. L.

$y = f(\pi/3) + f'(\pi/3)(x - \pi/3)$

$y = \cos(\pi^2/9) - \frac{2\pi}{3} \sin(\pi^2/9)(x - \pi/3)$