

MATH 251 (Spring 2004) Exam 1, Feb 25th

No calculators, books or notes!

Show all work and give **complete explanations** for all your answers.

This is a 65 minute exam. It is worth a total of 75 points.

(1) [10 pts] Let  $\mathbf{u}$  and  $\mathbf{v}$  be the vectors shown in the sketch and suppose that  $|\mathbf{u}| = 2$ .

Use the geometric definition of the dot and cross products to find

(a)  $\mathbf{u} \times \mathbf{v} =$

(b)  $\mathbf{u} \bullet \mathbf{v} =$

(2) [15 pts] Let  $\mathbf{r}(t)$  be the parametrized curve  $\mathbf{r}(t) = (t, \cos t, 2 \sin t)$ .

(a) Sketch the curve. [Hint: First find and sketch a surface on which the curves lies.]

(b) Compute the velocity vector of the curve  $\mathbf{r}$  at  $t = 0$ .

(c) Compute the parametric equation of the tangent line to the curve at  $t = 0$ .

(3) [20 pts]

Consider the plane through the points

$P = (1, 2, 3)$ ,  $Q = (1, -2, -5)$ , and  $R = (3, 0, 7)$ .

(a) Find a vector that is perpendicular to this plane.

(b) Write down an equation of the form  $ax + by + cz = d$  for this plane.

(c) Find a parametrization of this plane.

(d) Using a sketch and a couple of sentences, explain why  $\mathbf{r}(s, t) = \mathbf{r}_0 + s\mathbf{u} + t\mathbf{v}$  is a parametrization of the plane through the endpoint of the vector  $\mathbf{r}_0$  containing the vectors  $\mathbf{u}$  and  $\mathbf{v}$ .

(4) [10 pts] Let  $P$  be the point with spherical coordinates  $\rho = 4$ ,  $\theta = \pi$ ,  $\phi = \pi/6$ . Find the rectangular and cylindrical coordinates of  $P$ .

(5) [20 pts] Match the equations (a)-(e) with the graphs labeled (I)-(VI). [Note that there are more graphs than equations!] *Give reasons for your choices.*

(a)  $y^2 = x^2 + z^2$

(b)  $x^2 + y^2 + z^2 = 2y$

(c)  $\rho = 1$

(d)  $y^2 = x^2 + z^2 + 1$

(e)  $r = 2$

Pledge: *I have neither given nor received aid on this exam*

Signature: \_\_\_\_\_