

MATH 251 (Spring 2004) Diagnostic Quiz

This quiz does not count towards your grade. No books, notes, or friends!
Show all work. Use extra paper if you need!

(1) Let $f(x) = x^2$.

(a) Compute $f'(3)$ $f'(x) = 2x$ $f'(3) = 6$

(b) What is the definition of $f'(3)$? $f'(3) = \lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h}$

(c) What does $f'(3)$ mean geometrically?

$f'(3)$ is the SLOPE of the tangent line to $y = f(x)$ at $x = 3$.

(2) Find (a) $\int_{1/2}^2 \frac{1}{x} dx$

$$= \left[\ln|x| \right]_{1/2}^2 = \ln|2| - \ln\left|\frac{1}{2}\right| = 2\ln 2 = \boxed{\ln 4}$$

(b) $\int_0^\infty x e^{-x^2} dx$

$$u = x^2 \quad du = 2x dx$$

$$= \frac{1}{2} \int_{u=0}^{u=\infty} e^{-u} du = \left[-\frac{1}{2} e^{-u} \right]_{u=0}^{u=\infty} = \boxed{\frac{1}{2}}$$

(3) On what intervals is $f(x) = x^3 - 3x^2 + 7x$ increasing?

$$f'(x) = 3x^2 - 6x + 7 = 3(x-1)^2 + 4 \quad \text{by completing the square}$$

> 0 So f is increasing on $(-\infty, \infty)$

(4) State the Fundamental Theorem of Calculus.

Let $y = f(x)$ be a continuous function on an interval $[a, b]$. The $\int_a^b f(x) dx = F(b) - F(a)$, where $F' = f$

(5) Let $f(x) = \int_5^x \sin(t^3) dt$. What is $f'(10)$?

By another form of the Fundamental Theorem of Calculus [if $g(x) = \int_a^x f(t) dt$, then $g'(x) = f(x)$]

$$f'(x) = \sin(x^3)$$

$$f'(10) = \sin(1000)$$