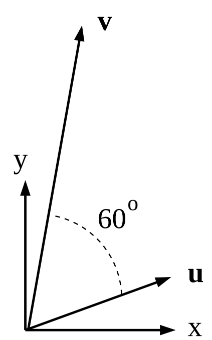
MATH 251H (Fall 2003) Exam 1, Oct 6th

No calculators, books or notes!
Show all work and give **complete explanations** for all your answers.
This is a 65 minute exam. It is worth a total of 75 points.

(1) [10 pts] Let $\mathbf{u}$ and $\mathbf{v}$ be two vectors in the xy-plane, with lengths $|\mathbf{u}| = \mathbf{5}$ and $|\mathbf{v}| = \mathbf{10}$, and



(a) Find $\mathbf{u} \bullet \mathbf{v} \ =$

(b) Find $\mathbf{u} \times \mathbf{v} \ =$

(c) Find the length of the projection of $\mathbf{u}$ onto $\mathbf{v}$

(d) How are dot products useful?

(2) [15 pts]
(a) Find a parameterization of the plane through the points
$P = (1, 2, 3)$, $Q = (4, -1, 2)$, and $R = (2, 0, -5)$.

(b) Does the line through the points $(2, -2, -1)$ and $(3, -1, 0)$ intersect the plane that goes through the point $(0, 0, 2)$ and is perpendicular to the vector $(2, -3, 1)$?

(3) [16 pts]
(a) Find the traces (slices) of the surface $z = 3x^2 + y^2$ in the xz-plane, the yz-plane, and the planes $z = 0$ and $z = 1$. Then sketch the graph of the surface.

(b) Identify the surface $\rho = 4 \sin \phi \cos \theta$.

(4) [12 pts] Let $\mathbf{r}(t)$ be the helix $\mathbf{r}(t) = (\cos(2t), \sin(2t), 5t)$.

(a) Compute the parametric equation of the tangent line to this helix at $t = \pi$.

(b) Compute the arclength of the helix from $t = 0$ to $t = \pi$.

(c) Compute the curvature of $\mathbf{r}$

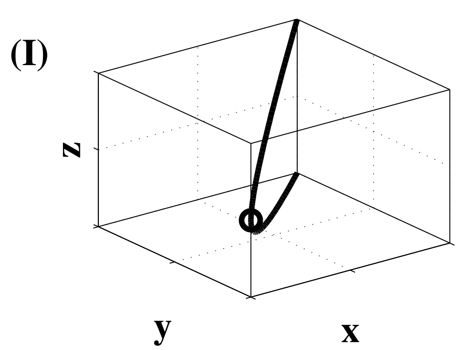
(5) [12 pts] Match the equations (a)-(d) with the graphs labeled (I)-(IV). Give reasons for your choices.

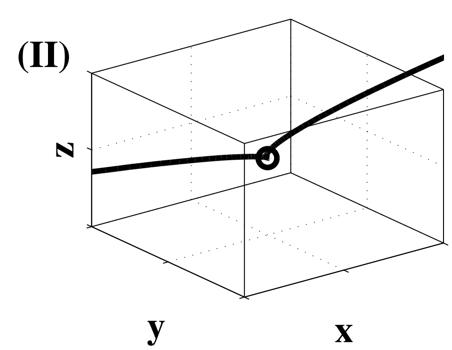
(a) $x = t^3, \quad y = t, \quad z = t^2$

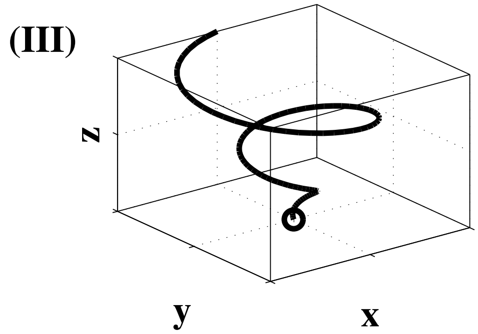
(b) $x = t\sin(2t), \quad y = t\cos(2t), \quad z = t$

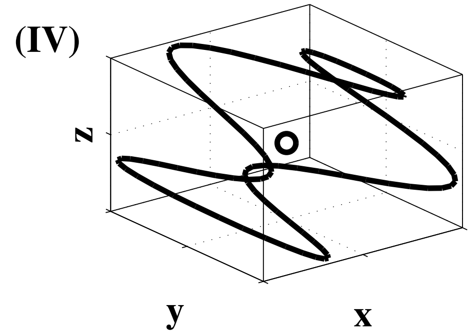
(c) $x = \cos(t), \quad y = \sin(4t), \quad z = \sin(t)$

(d) $x = t^2, \quad y = t^2, \quad z = t$









(6) [10 pts] Suppose $\mathbf{u} \neq \mathbf{0}$. Are the following statements true or false? For each part, either give an example of vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ for which the statement is false, or prove that the statement is true.

(a) If $\mathbf{u} \bullet \mathbf{v} = \mathbf{u} \bullet \mathbf{w}$ then $\mathbf{v} = \mathbf{w}$.

(b) If $\mathbf{u} \times \mathbf{v} = \mathbf{u} \times \mathbf{w}$ then $\mathbf{v} = \mathbf{w}$.

(c) If $\mathbf{u} \bullet \mathbf{v} = \mathbf{u} \bullet \mathbf{w}$ and $\mathbf{u} \times \mathbf{v} = \mathbf{u} \times \mathbf{w}$ then $\mathbf{v} = \mathbf{w}$.

Pledge: *I have neither given nor received aid on this exam*

Signature: ______________________________