MATH 251H (Fall 2003) Exam 2, Oct 31st

No calculators, books or notes! Show all your work. This 65 minute exam is worth 75 points.

(1) [8 pts] [p 995 #8] Find the limit if it exists, or show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{3xy}{x^2 + 4y^2}$$

(2) [8 pts] [15.4 #2] Find the equation of the tangent plane to the surface $z = f(x, y) = 9x^2 + y^2 + 6x - 3y + 5$ at the point $(1, 2, 18)$.

(3) [10 pts] [15.5 #33] The temperature at a point (x, y) is $T(x, y)$ measured in degrees Celsius. A bug crawls so that its position after t seconds is given by $x = \sqrt{1+t}$, $y = 2 + \frac{1}{3}t$, where x and y are measured in centimeters. The temperature function satifies $\frac{\partial T}{\partial x}(2, 3) = 4$ and $\frac{\partial T}{\partial y}(2, 3) = 3$. How fast is the temperature rising on the bug's path after 3 seconds?

(4) [10 pts] [17.6 #21] Find a parametrization for that part of the sphere $x^2 + y^2 + z^2 = 4$ that lies above the cone $z = \sqrt{x^2 + y^2}$.

(5) [15 pts] [p997 #50] Find the locations of any local maxima, minima, and saddle points of the function

$$z = f(x, y) = x^3 - 6xy + 8y^3$$

(6) [12 pts] [15.8 #4] Use Lagrange Multipliers to find the maximum and minimum values of $z = f(x, y) = 4x + 6y$ subject to the constraint $x^2 + y^2 = 13$.

(7) [12 pts]
(a) [Theory from Class] Prove that if $z = f(x, y)$ is differentiable at (x_0, y_0) and $\mathbf{u}$ is a vector in the xy-plane then

$$(D_{\mathbf{u}}f)(x_0, y_0) \;=\; \bigtriangledown f(x_0, y_0) \bullet \mathbf{u} \tag{1}$$

(b) [p994 #14c] Explain the geometric significance of the gradient.

Pledge: *I have neither given nor received aid on this exam*

Signature: ______________________________