MATH 251H (Fall 2003) Exam 3, Nov 26th

No calculators, books or notes! Show all your work. This 65 minute exam is worth 75 points.

(1) [8 pts] A contour map is shown for a function f on the square $R = [0, 4] \times [0, 4]$. Use the midpoint rule with $\Delta x = \Delta y = 2$ to estimate the value of $\iint_R f(x, y)\, dA$.

(2) [12 pts] Let D be the domain in the plane bounded by $y = 0$, $y = x^2$, and $x = 1$. Evaluate $\iint_D xe^y \, dA$.

(3) [10 pts] Evaluate $\int_0^1 \int_x^1 e^{x/y}\, dy\, dx$.

(4) [12 pts] Let E be the solid tetrahedron with vertices $(0,0,0)$, $(2,0,0)$, $(2,1,0)$, and $(0,1,1)$. Set up, but do NOT evaluate, an iterated triple integral for $\iiint_E xz\,dV$.

(5) [10 pts] Find the volume of the solid that lies within the sphere $x^2 + y^2 + z^2 = 4$, above the xy-plane, and below the cone $z = \sqrt{x^2 + y^2}$.

(6) [15 pts]
(a) State the Change of Variables Theorem for Double Integrals, and, using a picture, explain the geometric meaning of the Jacobian, $|\frac{\partial(x,y)}{\partial(u,v)}|$.

(b) Use the transformation $u = x - y$, $v = x + y$, to evaluate $\iint_D \frac{x-y}{x+y}\, dA$, where D is the square with vertices $(0, 2)$, $(1, 1)$, $(2, 2)$, and $(1, 3)$.

(7) [8 pts] Sketch the vector field $\mathbf{F} = \frac{x\mathbf{i} - y\mathbf{j}}{\sqrt{x^2+y^2}}$.

Pledge: *I have neither given nor received aid on this exam*

Signature: _____