NAME:

1 /15	2 /15	3 /18	4 /15	5 /12	T /75

MATH 251 (Spring 2008) Exam 1, Feb 20th

No calculators, books or notes! Show all work and give **complete explanations**.
This 65 minute exam is worth a total of 75 points.

(1) [15 pts] Let $\mathbf{a}$ and $\mathbf{b}$ be two vectors so that $|\mathbf{a}| = 2$, $|\mathbf{b}| = 5$ and the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\pi/6$.
(a) Find $\mathbf{a} \cdot \mathbf{b}$.

(b) Calculate the scalar projection of $\mathbf{b}$ onto $\mathbf{a}$.

(c) Find the area of the parallelogram determined by $\mathbf{a}$ and $\mathbf{b}$.

(2) [15 pts]
(a) Find a *parametric equation* for the plane through the points $(3, -1, 2)$, $(8, 2, 4)$, and $(-1, -2, -3)$.

(b) Find a *parametric equation* for the line through the point $(0, 1, 2)$ that is parallel to the plane $x+y+z = 2$ and perpendicular to the line $x = 1 + t$, $y = 1 - t$, $z = 2t$.

(3) [18 pts] Consider the quadric surface

$$x^2 + \left(\frac{y}{2}\right)^2 - \left(\frac{z}{3}\right)^2 = -1.$$

Find equations for the traces of this surface in the planes $x = k$, $y = k$, and $z = k$ for a few appropriately chosen values of k. Sketch each of these traces in a plane. Then sketch the surface in space.

(4) [15 pts]
(a) Sketch the image in the xy-plane of the parametrized curve $\mathbf{r}(t) = (3\cos t, 4\sin t)$, where $0 \leq t \leq \pi$.

(b) Calculate $\mathbf{r}'(\pi/3)$, where $\mathbf{r}$ is the parametrized curve in (a).

(c) State the limit definition of the derivative $\mathbf{r}'(t)$ of a parametrized curve $\mathbf{r}$. Using a picture and an English sentence explain why $\mathbf{r}'(t)$ is called the *tangent vector* to the curve at $\mathbf{r}(t)$.

(5) [12 pts]
(a) Suppose the spherical coordinates of a point P are $(\rho, \theta, \phi) = (4, \pi/6, \pi/3)$. Find the cylindrical coordinates of P.

(b) Sketch and describe in words the surface whose equation in spherical coordinates is $\phi = \pi/3$.

Pledge: *I have neither given nor received aid on this exam*

Signature: _____