NAME:

1 /15	2 /15	3 /15	4 /20	5 /10	T /75

MATH 251 (Spring 2008) Exam 2, Mar 31st

No calculators, books or notes! Show all work and give **complete explanations**.
This 65 minute exam is worth a total of 75 points.

(1) [15 pts]
(a) Find the curvature of the unit speed curve

$$\mathbf{r}(s) = (1 + \cos(s/2), \sqrt{3}\cos(s/2), 2\sin(s/2)).$$

(b) The curve in (a) is a circle. What is the radius of this circle, and why?

(2) [15 pts] Carefully sketch the level curves of the function $z = f(x, y) = x^2 - 4y^2$ at levels $z = 0$, ± 1, ± 2. Each level curve should be labeled and all should be drawn to scale *on the same set of axes.*

(3) [15 pts] Find all local maxima, local minima and saddle points of the function

$$z = f(x, y) = x^4 + y^4 - 4xy + 2.$$

(4) [20 pts]

(a) Show that the following limit does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{xy\cos y}{x^2+3y^2}.$$

(b) Calculate the equation of the tangent plane to the function $z = f(x,y) = x^3e^{2y}$ at the point $(x,y,z) = (2,0,8)$.

(5) [10 pts] A mouse walks around a circle in the xy-plane. Suppose that the position of the mouse at time t is given by the parametrized curve $(x, y) = \mathbf{r}(t) = (\cos t, \sin t)$. Let $z = T(x, y)$ be the temperature function in the plane. Suppose that when the mouse is at the point $(x, y) = (\frac{\sqrt{3}}{2}, \frac{1}{2})$ it experiences a rate of change of temperature of 5 degrees Fahrenheit per second. Suppose that an ant is also walking at speed 1 centimeter per second, but that unlike the mouse it can walk whereever it wants to in the xy-plane. If the ant is at the same point $(\frac{\sqrt{3}}{2}, \frac{1}{2})$ as the mouse, in what direction should it walk to decrease the temperature T the fastest if $\frac{\partial T}{\partial x} = -2$ at $(\frac{\sqrt{3}}{2}, \frac{1}{2})$.

Pledge: *I have neither given nor received aid on this exam*

Signature: ________________________________