NAME:

1 /12	2 /16	3 /16	4 /16	5 /15	T /75

MATH 251 (Spring 2008) Exam 3, Apr 23rd

No calculators, books or notes! Show all work and give **complete explanations**.
This 65 minute exam is worth a total of 75 points.

(1) [12 pts] Let C be the curve in the plane which is parametrized by $\mathbf{r}(t) = (3t, 4t+1)$, where $0 \le t \le 1$ and let $z = f(x, y) = xy$. Calculate $\int_C f\, ds$.

(2) [16 pts]
(a) Let $\mathbf{F}$ be the vector field

$$\mathbf{F}(x, y) = (2x + e^x \cos y)\mathbf{i} + (3y^2 - e^x \sin y)\mathbf{j}.$$

Show that $\mathbf{F}$ is conservative on the domain $D = \mathbf{R}^2$.

(b) Let C be the curve in the plane which is parametrized by $\mathbf{r}(t) = (\cos t, \sin t)$, where $0 \leq t \leq \pi$ and let $\mathbf{F}$ be the vector field $\mathbf{F}(x, y) = y\mathbf{i} + e^x\mathbf{j}$. Find a formula for a function $g(t)$ so that

$$\int_C \mathbf{F} \cdot d\mathbf{r} \;=\; \int_0^\pi g(t)\, dt.$$

(3) [16 pts] (a) Let D be the region in the plane bounded by the curves $y = 1 + x^2$, $y = 2x^2$. Calculate $\iint_D x^2 \, dA$.

(b) Let D be the region in the first quadrant (i.e., $x \geq 0$ and $y \geq 0$) that is between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$. Calculate $\iint_D y\, dA$.

(4) [16 pts] Let S be the surface in $\mathbf{R}^3$ that is parametrized by

$$\mathbf{r}(u,v) = (v\cos u, v\sin u, v),$$

where $0 \le u \le 2\pi$ and $v > 0$.
(a) Show that S is the cone $z = \sqrt{x^2+y^2}$.

(b) Use the parametrization $\mathbf{r}(u,v)$ above to calculate a *parametrization* of the tangent plane to S at the point $\mathbf{r}(\pi/4, 1)$.

(5) [15 pts] Use the Method of Lagrange Multipliers to find the maximum and minimum values of the function $f(x, y) = 6x + 8y$ subject to the constraint $x^2 + y^2 = 1$.

Pledge: *I have neither given nor received aid on this exam*

Signature: ______________________________