NAME:

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MATH 251 (Spring 2008) Final Exam, May 21st

No calculators, books or notes! Show all work and give **complete explanations**.
This 120 minute exam is worth a total of 120 points.

(1) [14 pts]
(a) Find a parametrization of the line through the points $(1, 0, 1)$ and $(2, 1, 0)$.

(b) Find the point of intersection of the line in (a) with the plane $x + 2y - 2z + 4 = 0$.

(2) [15 pts] Consider the parametrized curve $\mathbf{r}(t) = (\sin t, 2t, \cos t)$, where $0 \leq t \leq 2\pi$.
(a) Show that this curve lies on a cylinder.

(b) Carefully sketch the curve and the cylinder on which it lies.

(c) Reparametrize the curve $\mathbf{r}$ with respect to arclength measured from the point where $t = 0$ in the direction of increasing t.

(3) [12 pts] Calculate $\iint_D y^2 dA$, where D is the region in the xy-plane that is bounded by the parabolas $x = y^2$ and $x + y^2 = 8$.

(4) [12 pts] Calculate the following:
(a) What is the rate of change of the function $f(x, y, z) = e^{3xy} + x^3z^4$ in the direction $(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ at the point $(1, 0, 2)$?

(b) Let $z = f(x, y, z)$ be a function with $f(1, 0, 0) = 5$ and $\nabla f(1, 0, 0) = (2, 6, -1)$ and let $\mathbf{r}(t) = (\cos t, t^5, 4\sin t)$ be a parametrized curve. If $g(t) = f(\mathbf{r}(t))$ find $g'(0)$.

(5) [12 pts] (a) Let $\mathbf{F}$ be the vector field $\mathbf{F}(x, y, z) = xz\mathbf{i} + y^2\mathbf{j} + y\cos z\mathbf{k}$. Calculate $\nabla \times \mathbf{F}$.

(b) Suppose that the vector field $\mathbf{F}(x, y) = x^2y\mathbf{i} + (x^2 - y^2)\mathbf{j}$ is the velocity vector field of a fluid flowing in the xy-plane. On average is the fluid flowing in or out of a small disk centered at the point $(-1, 2)$? Why?

(6) [15 pts] Suppose that $z = f(x, y)$ is a function whose second partial derivatives are continuous and that

(a, b)	$f(a, b)$	$\nabla f(a, b)$	$f_{xx}(a, b)$	$f_{xy}(a, b)$	$f_{yy}(a, b)$
$(1, 2)$	0	(0,0)	5	3	1
$(7, -2)$	0	(0,1)	5	3	1
$(3, 4)$	7	(0,0)	-5	-3	-2
$(5, -3)$	68	(0,0)	8	-4	2
$(2, 1)$	35	(0,0)	5	3	2

Identify any local maxima, minima, and saddle points of f. Explain the reasons for your answers.

(7) [10 pts] Let S be the surface $z = 9 - \frac{1}{4}(x^2 + y^2)$, where $x^2 + y^2 \leq 36$, with the upward orientation. Set up an iterated double integral of the form $\int_a^b \int_c^d g(u, v)dudv$ for the flux of the vector field $\mathbf{F} = y\mathbf{i} + \mathbf{j} + z\mathbf{k}$ through the surface S. In particular find a formula for the integrand $g(u, v)$ and find the endpoints of integration, a, b, c, and d. [*Do not evaluate this double integral.*]

(8) [10 pts] Calculate $\iiint_E x^2y^2dV$, where E is the solid region bounded by the parabolic cylinder $z = 1-y^2$ and the planes $z = 0$, $x = 1$ and $x = -1$.

(9) [10 pts]
(a) Carefully state Stokes' Theorem.

(b) Use Stokes' Theorem to evaluate $\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = x^2 z\mathbf{i} + yz^2\mathbf{j} + z^3 e^{xy}\mathbf{k}$, S is the part of the sphere $x^2 + y^2 + z^2 = 5$ that lies above the plane $z = 1$, and S is oriented upwards.

(10) [10 pts]
(a) Carefully state the Change of Variables Theorem for Double Integrals.

(b) Use the Change of Variables Theorem to calculate $\iint_R x dA$, where R is the region in the xy-plane where $x \geq 0$ and $9x^2 + 4y^2 \leq 36$. [Hint: Use the change of variables $x = 2u$, $y = 3v$.]

Pledge: *I have neither given nor received aid on this exam*

Signature: ________________________________