MATH 603 Homework on
Quadratic Forms and Positive Definite Matrices

The starred problems will be graded.

(1) 7.6.1

(2*) 7.6.4

(3*) Let $\mathbf{A}$ be symmetric positive definite. Prove that there is a $c > 0$ so that $\mathbf{x}^T\mathbf{A}\mathbf{x} \geq c\|\mathbf{x}\|^2$ for all $\mathbf{x}$.

(4*) Prove that every inner product on $\mathcal{R}^n$ is of the form $\langle \mathbf{x} \,|\, \mathbf{y} \rangle = \mathbf{x}^T\mathbf{A}\mathbf{y}$ for some symmetric positive definite matrix $\mathbf{A}$.

(5*) Let $\mathbf{A}$ be symmetric positive definite. Prove that there is a unique symmetric positive definite $\mathbf{B}$ so that $\mathbf{B}^2 = \mathbf{A}$.

(6) Express the following quadratic forms as a sum/difference of squares. Which are positive definite? [Use linear algebra!]
(a) $x^2 + 8xy + y^2$
(b) $x^2 - 4xy + 7y^2$
(c) $x^2 + 6xy$

(7*) Suppose $\mathbf{A}$ and $\mathbf{B}$ are symmetric positive definite. Which of the following are symmetric positive definite? (Prove or provide a counter example.)
(a) $\mathbf{A} + \mathbf{B}$
(b) $\mathbf{A}\mathbf{B}$
(c) $\mathbf{A}^{-1}$.