Math 251H, Fall 2003

Multivariable Calculus
Matlab Assignment for Chapter 14

The goal of this assignment is to study a curve which is not smooth everywhere, by expressing it as a limit of smooth curves.[1] For the original curve, and for each of the smooth approximating curves, you will plot the curve, and also plot the speed and curvature as functions of time. To make the computations easy in Matlab, you will learn how to do define functions symbolically, compute their derivatives and graph them.

Before beginning the assignment, carefully read and run the demo program SymDemo.m I wrote to introduce you to symbolic calculations in Matlab. You are to use the same circle of ideas to do the assignment.

(1) Let C be the curve parametrized by

$$\mathbf{r}(t) \ = \ ((\cos t)^3, (\sin t)^3), \qquad 0 \le t \le 2\pi. \tag{1}$$

(a) Plot the curve using *ezplot*. Is it smooth? Does it have cusps? (Read Stewart page 879 to remind your self what the terms "smooth" and "cusp" mean.) Where do you expect the speed to be zero?
(b) Calculate by hand the speed of the curve. Is the speed zero where you expect it to be from (a)?
(c) Calculate the curvature by hand using Theorem 10 on page 896. Where is the curvature undefined? Does this agree with your graph in (a)?

(2) To obtain a smooth approximation to C we are going to modify $\mathbf{r}$. Let C_δ be parametrized by

$$\mathbf{r}_\delta(t) \ = \ \frac{1}{1+\delta}((\cos t)^3 + \delta \cos t, (\sin t)^3 + \delta \sin t). \tag{2}$$

(a) Plot this curve for $\delta = 0.5$, 0.2, and 0.1. See how as $\delta \to 0$, C_δ is a smooth approximation to C.
(b) Use matlab's symbolic tools as discussed in SymDemo.m to calculate the speed and curvature of $\mathbf{r}_\delta$, and graph both speed and curvature on the interval $[0, 2\pi]$ using *ezplot3*. For this let t and δ be symbolic variables and define the x and y coordinates of $\mathbf{r}_\delta$ in terms of t and δ. You can use *diff* to compute the velocity of the curve, and to get the speed you'll need to use something like

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Speed = (Xvelocity^2 + Yvelocity^2)^0.5
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[1] This assignment is a minor modification of Problems 3,4, and 8 in Chapter 4 of "A MATLAB Companion for Multivariable Calculus", by Jeffery Cooper.

since Matlab's symbolic toolbox doesn't understand the matlab function *norm*. To graph, you'll need to use *subs* to substitute in particular values for δ to get functions X and Y of time t. Also set $Z(t) = 0$. Then use a command of the form *ezplot3(X,Y,Z,'animate')* to make a movie of position as a function of time for this parameterization for the different values of δ, and also for $\delta = 0$. You can rotate the viewing direction of the 3D plot so that you are looking down onto the xy-plane from the positive z-axis. Notice how the position slows at those values of t that correspond to the cusps of $\mathbf{r}$.

(3) Finally, we are going to construct a "helix" over the curve C in (1). Assume $\alpha \geq 0$ and let Γ be the curve parametrized by

$$\mathbf{q}(t) \;=\; ((\cos t)^3, (\sin t)^3, \alpha t), \qquad 0 \leq t \leq 2\pi. \tag{3}$$

Plot this curve using *ezplot3* for 3 values of α. Is the curve smooth whenever $\alpha > 0$? Can you explain this by a calculation of the speed of $\mathbf{q}$?