Assessment of Simple and Alternative Bayesian Ranking Methods Utilizing Parallel Computing

UMBC REU Site: Interdisciplinary Program in High Performance Computing (www.umbc.edu/hpcf)

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Problem Statement

The U.S. Census Bureau (USCB) assists the federal government in distributing over $400 billion of aid by ranking the states according to certain criteria, such as average poverty level. The current ranking algorithm is based on sample estimates which are associated with a certain amount of error. Klein and Wright of the USCB have compared the performance of non-informative Bayesian techniques to the USCB's current method. We expand on this work to add informed Bayesian and regression models to the comparison. By employing moderation techniques, we obtain excellent probabilities of correct rankings.

The Model

Suppose x_i is sample estimate, and σ_i is a known standard deviation. The population model used is:

$$x_i|\boldsymbol{\theta} \overset{\text{ind}}{\sim} N(\theta_i, \sigma_i^2), i = 1, \ldots, k,$$

where θ_i is an unknown population mean. The current method, SI, sorts the estimates and ranks them accordingly. The alternative Bayesian ranking methods include: P1EB, P2EB, PMEB [Klein and Wright, 2011], and MXPR [Tech Report HPCF–2011–11]. These alternative procedures incorporate the following prior distributions:

- Non-informative:

$$\theta_1, \ldots, \theta_k|\mu, \tau \overset{\text{iid}}{\sim} N(\mu, \tau^2).$$

- Regression Informed Prior (**RIP**):

$$\theta_i = \beta_0 + \beta_1 m_i + \varepsilon_i$$

  where $\beta_0, \beta_1, \varepsilon_i$ are random, m_i is the previous data.

- Fully Informed Prior (**FIP**):

$$\theta_i \sim N(m_i, \tau_i^2)$$

Comparing Various Priors

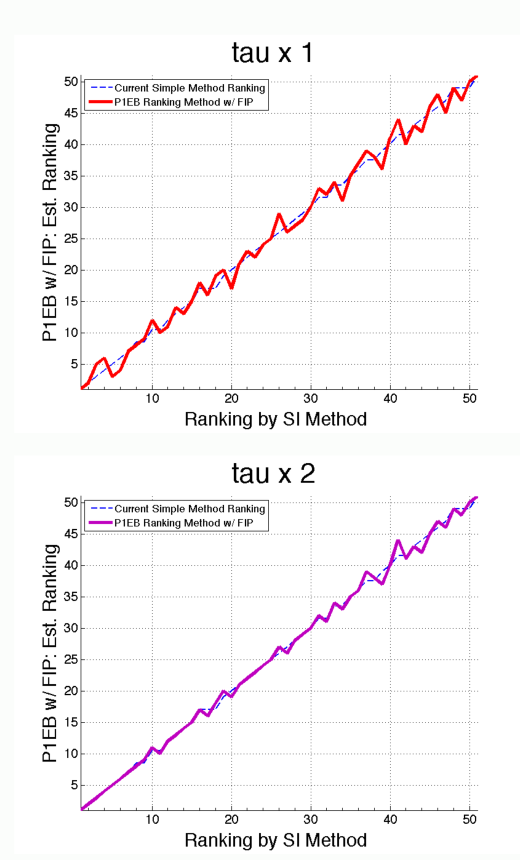
Simulation Comparison: Below is a comparison of the SI method and the two versions of the informed P1EB method: **FIP** and **RIP**. These informed methods obtain comparable results until $\boldsymbol{\theta}_{prev}$ and $\boldsymbol{\theta}$ have dissimilar orderings. With small shifts in the previous data, FIP does slightly better, but when shifts are dramatic, RIP produces much better probabilities of a correct ranking and consistently outperforms the SI method.

Data Set	$\boldsymbol{\theta}$	$\boldsymbol{\sigma}$
	10.0, 10.5, 10.7, 11.0, 11.2	0.1, 0.3, 0.3, 0.1, 0.5
Prev. Set	$\boldsymbol{\theta}_{prev}$	$\boldsymbol{\tau}$
(1)	10.1, 10.3, 10.4, 10.7, 10.9	0.1, 0.3, 0.3, 0.1, 0.5
(2)	10.1, 10.2, 10.8, 10.4, 11.0	0.1, 0.3, 0.3, 0.1, 0.5
(3)	10.7, 10.5, 10.3, 10.1, 9.9	0.1, 0.3, 0.3, 0.1, 0.5

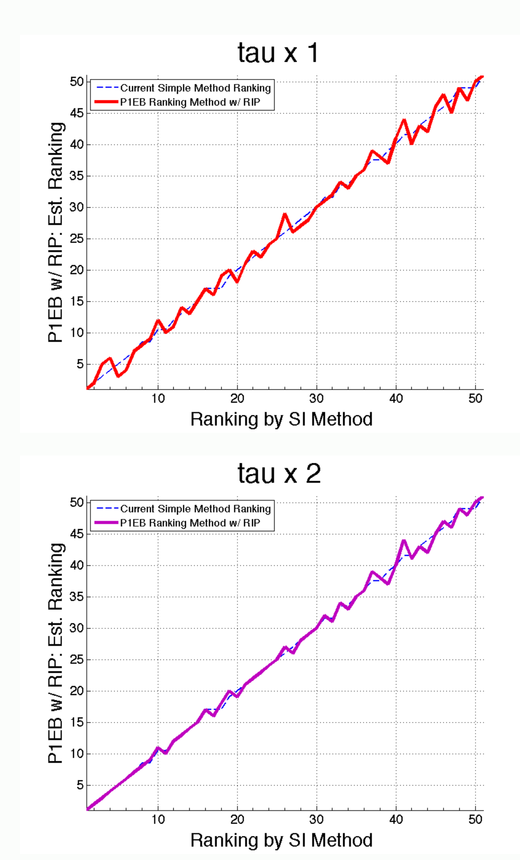
Probability results, P represents $P(\hat{r}_i = r_i)$, as compared to current SI ranking method, using each of the 3 Prev. Settings above.

	True Rank	SI		P1EB w/ FIP				P1EB w/ RIP			
				$\tau \times 1$		$\tau \times 2$		$\tau \times 1$		$\tau \times 2$	
		$\hat{r}_i$	P	$\hat{r}_i$	P	$\hat{r}_i$	P	$\hat{r}_i$	P	$\hat{r}_i$	P
PrevSet(1)	r_1	1	0.93	1	0.99	1	0.96	1	0.99	1	0.96
	r_2	2	0.56	2	0.75	2	0.63	2	0.78	2	0.65
	r_3	3	0.43	3	0.71	3	0.54	3	0.74	3	0.56
	r_4	4	0.56	4	0.77	4	0.64	4	0.80	4	0.67
	r_5	5	0.63	5	0.79	5	0.67	5	0.79	5	0.68
PrevSet(2)	r_1	1	0.93	1	0.98	1	0.96	1	0.96	1	0.95
	r_2	2	0.56	2	0.94	2	0.74	2	0.86	2	0.71
	r_3	3	0.43	4	0.33	3	0.51	4	0.31	3	0.47
	r_4	4	0.56	3	0.38	4	0.62	3	0.36	4	0.55
	r_5	5	0.63	5	0.89	5	0.72	5	0.80	5	0.69
PrevSet(3)	r_1	1	0.93	1	0.56	1	0.88	1	0.99	1	0.92
	r_2	2	0.56	3	0.23	2	0.51	2	0.84	2	0.58
	r_3	3	0.43	2	0.22	3	0.39	3	0.79	3	0.48
	r_4	4	0.56	5	0.42	4	0.50	4	0.79	4	0.63
	r_5	5	0.63	4	0.39	5	0.57	5	0.79	5	0.70

Application: Figures (a) and (b) compare FIP and RIP, respectively, applied to 2008 state data using the 2007 estimates to inform the prior distribution; a multiplier on $\boldsymbol{\tau}$ moderates the influence of the previous year's data on the rankings.



(a)

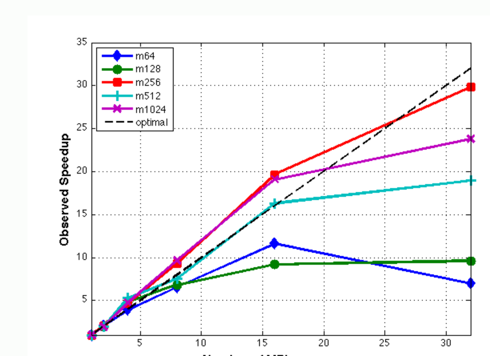


(b)

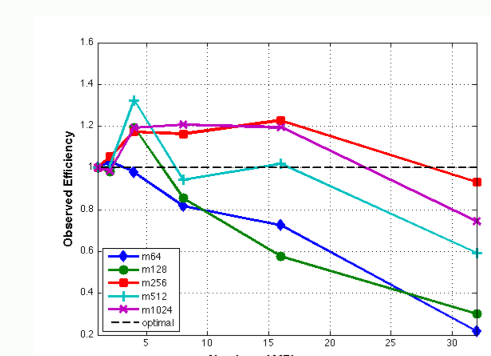
Parallel Bootstrap

In practice, the accuracy of the estimated ranks are computed by a bootstrap. We implement this computationally expensive procedure in parallel.

m	$p = 1$	$p = 2$	$p = 4$	$p = 8$	$p = 16$	$p = 32$
64	9.77 min	4.75 min	2.50 min	1.50 min	0.84 min	1.41 min
128	17.97 min	9.17 min	3.76 min	2.63 min	1.95 min	1.87 min
256	38.11 min	18.07 min	8.13 min	4.09 min	1.94 min	1.28 min
512	70.20 min	35.15 min	13.31 min	9.35 min	4.32 min	3.72 min
1024	141.00 min	71.4 min	29.54 min	14.59 min	7.37 min	5.91 min



(c)



(d)

Time study for parallelized bootstrap for increasing bootstrap sizes (m) using the P1EB ranking procedure and p processes. Each processor is computing $l_m = m/p$ length chunks of data. Figure (c) shows speedup and figure (d) shows efficiency.

Conclusions

- We are most confident in the P1EB method with RIP, which produces higher probabilities of correct rankings, even with varied $\boldsymbol{\theta}_{prev}$ settings.
- Utilizing previous data allows us to specify a more realistic model for the prior distribution of the current data.

References

For more information, refer to:

- Tech Report HPCF–2011–11
  www.umbc.edu/hpcreu/2011/projects/team1.html
- Martin Klein and Tommy Wright, *International Journal of Statistical Sciences*, 2011

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