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Automatic determination of parameters in photoelasticity

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Abstract

This paper presents an automatic approach for the evaluation of isochromatics and isoclinics in photoelasticity using complementary phase shifting. First, the phase values of the isoclinics are obtained from four images in the plane polarizer arrangement by rotating the polarizer and analyzer simultaneously. Then, through use of the isoclinics, a full-field description of the first principal stress orientation with respect to the horizontal axis is determined. Phase maps for the isochromatics are then achieved at eight discrete orientations through sequential analyzer rotations. With the first principal stress orientation known from the isoclinic phase map, a whole-field description of the phase values of the isochromatics is then constructed. The proposed method was validated for the problem of a ring under diametrical load.

Keywords: Phase shifting; Isochromatics; Isoclinics

1. Introduction

Photoelasticity has served as a valuable experimental method for performing stress analysis for decades. Since its inception, it has been applied in solving a tremendous number of engineering problems. One of the most attractive qualities of photoelasticity is that it provides a simple method for direct examination of the whole-field stress distribution. However, due to the cumbersome process of data reduction and the development of numerical methods for structural analysis, it is potentially less popular today than in the past. Despite some apparent disadvantages, photoelasticity often serves as a useful complement to numerical analyses, especially for validation of complex stressed models or when the boundary conditions are difficult to model.

In response to recognized drawbacks, recent efforts have focused on the development of simplistic methods for reduction of isochromatics and isoclinics to achieve complete description of the principal stresses and their orientation. In order to simplify data reduction, new methods based on image processing techniques have been proposed. For instance, the two-wavelength method [1,2] was developed to determine the isochromatic fringe orders in a whole-field analysis. By comparing the grayscales from isochromatic fringe patterns using different light sources, fringe orders are estimated according to

$$|N_1 f_1 - N_2 f_2| = \min, \tag{1}$$

where N_1, N_2 and f_1, f_2 are the fringe number and stress optical constants for the two different light sources. But when the difference of the fringe orders reaches 0.5, Eq. (1) is no longer effective [3]. Consequently, the multi-wavelength method [4] was proposed and overcomes the limitation of the two-wavelength approach. Nevertheless, it requires relatively complex instrument design and time-consuming data processing. Load stepping [5–7] was also developed to introduce phase shifting of the isochromatics and is achieved by changing the magnitude of applied load without any additional change to the traditional polariscope. While serving as an effective approach for simple loading arrangements, it can prove troublesome if the loading condition is complex. Also, it cannot be applied for frozen-stress photoelasticity. Other methods, such as the

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use of isopachics [8], have also been shown to be useful in determining the internal stress components in real time.

In the present paper, the phase shifting technique has been applied to both isoclinics and isochromatics through rotation of the appropriate optical elements. The process provides an unambiguous description of the principal stress directions and a phase map for the isochromatics that is defined for the whole field. In an effort to validate the process and illustrate application, an example of the proposed method is presented using a circular ring loaded in diametrical compression.

2. Methods

2.1. Determination of isoclinic parameter

In photoelastic experiments conducted using the plane polariscope, the intensity distribution emitted from the analyzer represents a superposition of isochromatic and isoclinic fringes. In this optical arrangement the angle between the principal stress and the horizontal axis (α) and the angle between the analyzer and horizontal axis (β) are defined (Fig. 1) to develop relationships that describe the transmission of light. When the polarizer and analyzer are perpendicular to each other (crossed-axes), the intensity emerging from the analyzer from an arbitrary point in a loaded model can be described as

$$I = k\sin^2\frac{\phi}{2}\sin^2 2(\alpha-\beta) = \frac{1}{2}k\sin^2\frac{\phi}{2}[1-\cos 4(\alpha-\beta)], \quad (2)$$

where k is the square of the amplitude of the light vector and ϕ is the relative phase difference between the waves traveling parallel to the two principal optical axes.

If the polarizer and analyzer are rotated simultaneously to orientations of β equal to 0, $\pi/8$, $\pi/4$, and $3\pi/8$, then the intensity distribution of images recorded at each of the four orientations are determined according to (Eq. (2)) and given by

$$I_1 = \frac{1}{2}k\sin^2\frac{\phi}{2}[1-\cos 4\alpha],$$

$$I_2 = \frac{1}{2}k\sin^2\frac{\phi}{2}\left[1-\cos\left(4\alpha-\frac{\pi}{2}\right)\right],$$

$$I_3 = \frac{1}{2}k\sin^2\frac{\phi}{2}[1-\cos(4\alpha-\pi)],$$

$$I_4 = \frac{1}{2}k\sin^2\frac{\phi}{2}\left[1-\cos\left(4\alpha-\frac{3\pi}{2}\right)\right]. \quad (3)$$

If $\sin^2(\phi/2)\neq 0$, from these four images, one can derive the phase map for the isoclinics according to

$$\alpha = \frac{1}{4}\tan^{-1}\frac{I_4-I_2}{I_3-I_1}. \quad (4)$$

According to the range of the arctangent function (arctan2) over the ratio of intensities $(-\pi,\pi)$, the angle of principal stress (α) lies in the range of $-\pi/4\leqslant\alpha\leqslant\pi/4$. For convenience, the angle is then defined with range between 0 and $\pi/2$ according to Eq. (4), by substituting β with $\pi/4$, $3\pi/8$, 0, and $\pi/8$ in turn in Eq. (3). For general planar problems, two principal stresses exist in plane and one cannot distinguish whether the angle determined in Eq. (4) is made by the first principal stress (σ_1) or the second principal stress (σ_2). Yoneyama [8] described this phenomenon as ambiguity of the isoclinic angle, which leads to a mismatch in determining the sign of isochromatic parameters. Thus, a treatment of the isoclinic angle is needed to avoid ambiguity.

Isostatics are the trajectories of the principal stresses, whose tangent at any point coincides with that of the principal stresses at the corresponding location. They can be directly obtained from the isoclinics [9]. The basic method of drawing stress trajectories is relatively well established and a description can be found in Ref. [10]. In the plane-stress state, since the two principal stresses at any point are mutually perpendicular, it follows that a system of isostatics exists which consists of two orthogonal groups of curves. One set represents the directions of the first principal stress (σ_1), and the other indicates the second principal stress (σ_2) direction. Thus, the orientation of the first principal stress can be identified at any location, as long as these two groups of curves are distinguished from each other. This can be achieved by interpreting the nature of stress near the free boundary (tension or compression) and ranking the principal stress with respect to the free-edge component. On the isoclinic phase map derived from Eq. (4), the orientation of principal stresses within the range $(0,\pi/2)$ are displayed with grayscales (0, 255), which means in some areas (e.g. Area A) the grayscale indicates the correct orientation for σ_1, whereas in other areas the grayscale does not (Area B). In other words, the resulting isoclinic phase map must be further treated so that the grayscale in all areas (including Area B) represents the correct orientation for σ_1. As the orientations of σ_1 and σ_2

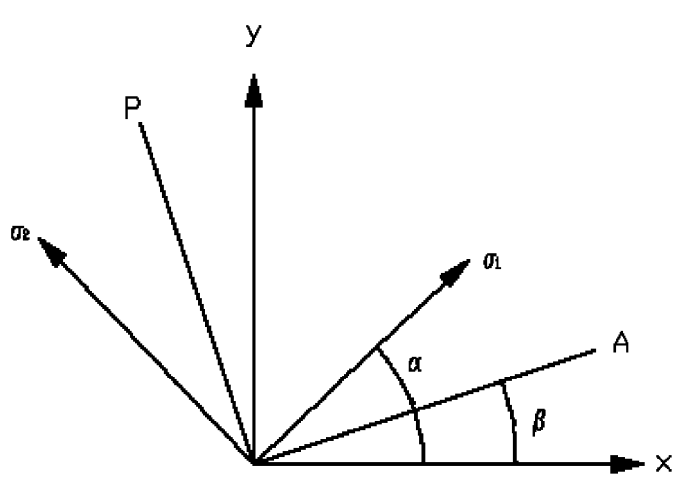

Fig. 1. Optical arrangement of the plane polariscope with crossed configuration.

are mutually perpendicular, the grayscale in Area B plus or minus 255 will extend the orientation of σ_1 to either $(0, \pi)$ or $(-\pi/2, \pi/2)$. In order to specifically indicate the first principal stress orientation, a process similar to a generalized phase unwrapping is required to suppress the range of grayscale to (128, 255) for Area A, and convert the grayscale range to (0, 127) for Area B. As a consequence, the new phase map will represent the orientation of σ_1 with range $(-\pi/2, \pi/2)$ and with grayscale range of (0, 255).

The treatment process should be conducted as follows. To begin, a position with known σ_1 orientation should be selected within Area A on the isoclinic phase map that was obtained from Eq. (4). The true phase values (unwrapped phase) are calculated on horizontal and vertical lines which pass through this point. On these lines, the current location is defined as the one being processed, and the former location is the one that has been calculated and neighbors the current. If the grayscale difference between the current and former locations is within a certain range (i.e. resulting in a continuous map), no special process is needed. Otherwise, a discontinuous phase map is realized due to abrupt change in phase. Then, assuming the σ_1 orientation at the current location is in Area A, stress trajectories should be constructed for σ_1 through this point and the former point. If these two isostatics are parallel to each other, no processing is needed. If not, the current point is in Area B and a value of 255 must be subtracted from the grayscale. The aforementioned process must be conducted over the whole image. At last, the grayscale values at all points for this image are all divided by 2, and a value of 127 is added to the grayscale, to form an 8-bit unwrapped isoclinic phase map, which extends the range of α to $-\pi/2 \leqslant \alpha \leqslant \pi/2$. As a result, the orientation of σ_1 with respect to the horizontal axis is fully determined.

2.2. Determination of isochromatic parameter

The dark field of the circular polariscope can be arranged such that the polarizer coincides with the direction of the principal stresses (σ_1, σ_2) at regions of interest in the photoelastic model. As shown in Fig. 2, the polarizer (P) and analyzer (A) coincide with the orientation of principal stresses at point O. The quarter wave plates are placed such that they generate a dark-field circular polarizer arrangement. For this arrangement, if the analyzer A is rotated an angle β to a new position A′ counterclockwise, the light intensity emerging from the analyzer can be derived as [11]

$$I = k\sin^2\left(\frac{\phi_\alpha}{2} - \beta\right) = \frac{1}{2}k[1 - \cos(\phi_\alpha - 2\beta)], \tag{5}$$

where $\phi_\alpha = 2\pi\delta/\lambda = (2\pi d/f)(\sigma_1 - \sigma_2)$ is the phase difference due to the retardation (δ) in directions of σ_1 and σ_2, d is the model thickness and f is the stress-optic constant of the model material. If the rotation angles of the analyzer

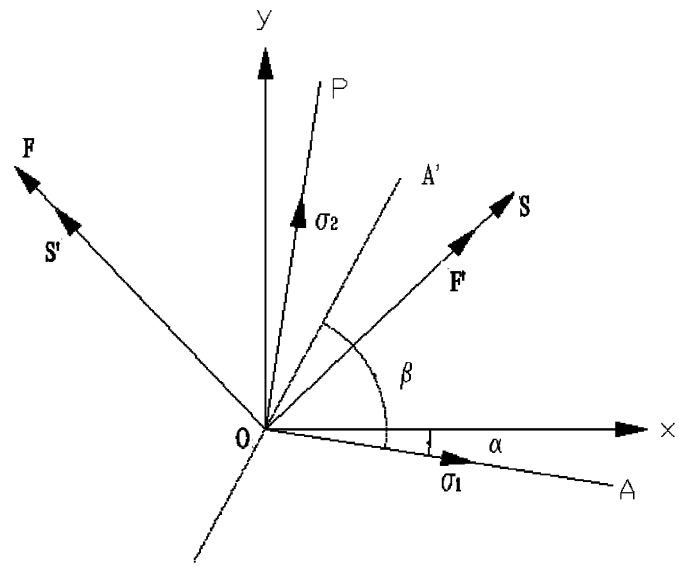

Fig. 2. Optical arrangement of the circular polariscope with crossed configuration.

are set at 0, $\pi/4$, $\pi/2$, and $3\pi/4$, the corresponding intensity of light at each point in the model is described at each of the four different analyzer rotations as

$$I_1 = \tfrac{1}{2}k[1 - \cos\phi_\alpha],$$

$$I_2 = \frac{1}{2}k\left[1 - \cos\left(\phi_\alpha - \frac{\pi}{2}\right)\right],$$

$$I_3 = \tfrac{1}{2}k[1 - \cos(\phi_\alpha - \pi)],$$

$$I_4 = \frac{1}{2}k\left[1 - \cos\left(\phi_\alpha - \frac{3\pi}{2}\right)\right]. \tag{6}$$

The phase value (ϕ_α) at each point in the model is then calculated according to

$$\phi_\alpha = \tan^{-1}\frac{I_2 - I_4}{I_3 - I_1}. \tag{7}$$

Note that Eq. (7) is only valid in the region where the direction of the first principal stress (σ_1) agrees with that of the polarizer. Since the direction of σ_1 ranges from $-\pi/2$ to $\pi/2$, the preceding phase shifting process should be carried out for all possible orientations (i.e. $-\pi/2 \leqslant \alpha \leqslant \pi/2$). Yet, for simplicity, phase maps of the isochromatics are constructed at eight discrete polarizer orientations using the circular polariscope arrangement with angle between the polarizer and horizontal axis of $-3\pi/8$, $-\pi/4$, $-\pi/8$, 0, $\pi/8$, $\pi/4$, $3\pi/8$ and $\pi/2$. At each of the aforementioned orientations, four images are acquired at the four distinct analyzer orientations. Using the processing steps described by Eqs. (6) and (7), eight unique phase maps of the isochromatics are obtained. In each individual image, the phase values (ϕ_α) are obtained in specific regions that are restricted to where the polarizer orientation coincides with the direction of the first principal stress. Since the directions of σ_1 were obtained using phase shifting of the

isoclinics, a new image of the phase map for the isochromatics can be reconstructed for the whole field according to the following discipline:

$$\phi = \begin{cases} \phi_{\pi/2} & \text{if } \alpha < -7\pi/16 \text{ or } \alpha \geqslant 7\pi/16, \\ \phi_{-3\pi/8} & \text{if } -7\pi/16 \leqslant \alpha < -5\pi/16, \\ \phi_{-\pi/4} & \text{if } -5\pi/16 \leqslant \alpha < -3\pi/16, \\ \phi_{-\pi/8} & \text{if } -3\pi/16 \leqslant \alpha < -\pi/16, \\ \phi_{0} & \text{if } -\pi/16 \leqslant \alpha < \pi/16, \\ \phi_{\pi/8} & \text{if } \pi/16 \leqslant \alpha < 3\pi/16, \\ \phi_{\pi/4} & \text{if } 3\pi/16 \leqslant \alpha < 5\pi/16, \\ \phi_{3\pi/8} & \text{if } 5\pi/16 \leqslant \alpha < 7\pi/16. \end{cases} \tag{8}$$

In regions where the differences of isoclinic parameters are within $\pm\pi/16$, the true phase is replaced by one of eight images as previously described. Consequently, there is a possibility for errors to be introduced in the phase values. In contrast to the description of light intensity in the circular polariscope arrangement given by Eq. (5) (for $\alpha = 0$), the light intensity emerging from the analyzer for $\alpha = \pi/16$ is given by

$$I = k\sin^2\left(\frac{\phi_\alpha}{2} - \beta\right) + 0.05k\left[\sin^2\left(\frac{\phi_\alpha}{2} + \beta\right) - \sin^2\left(\frac{\phi_\alpha}{2} - \beta\right)\right]. \tag{9}$$

Eq. (9) indicates the maximum possible error introduced by the misalignment of principal stress and the polarizer which results from intermediate isoclinics (within $\pm\pi/16$). In comparison to Eq. (5), the only difference in apparent intensity is the second term, which has a factor of 0.05 and can be neglected. Therefore, using the direction of σ_1 obtained from the isoclinics, the phase values of the isochromatics can be described completely with limited error.

3. Experiments

A plane photoelastic model was prepared from a transparent polycarbonate and consisted of a ring with inside diameter of 20 mm, outside diameter of 40 mm and thickness of 8 mm. The model was placed within the polariscope and subjected to diametral compressive load of 490 N. A video camera was placed behind the analyzer (Fig. 3) to acquire images, which were stored in the computer. In the plane polariscope arrangement with crossed analyzer and polarizer, isoclinics and isochromatics were recorded for specific orientations of the polariscope (i.e. with analyzer and polarizer rotated simultaneously) with the use of a white light source. Images were obtained for β (Fig. 1) equal to 0, $\pi/8$, $\pi/4$, and $3\pi/8$, and are shown in Fig. 4. Phase shifting of the isoclinics was performed according to the protocol described by Eq. (4).

In the circular polariscope with dark field arrangement, a monochromatic light source was used and phase shifting

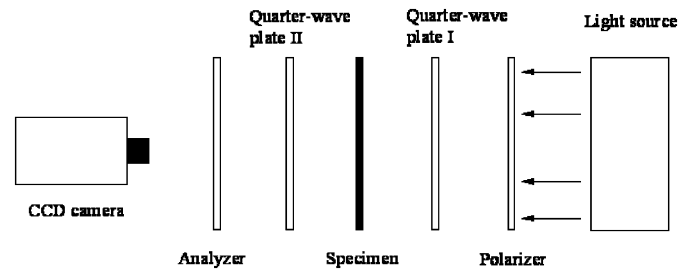

Fig. 3. Optical arrangement used in the experimental investigation.

was performed in each of eight unique orientations. For the first group, the polarizer was oriented with polarizing axis coinciding with the vertical axis as shown in Fig. 2. The first image was captured under the dark field. The second image was captured after the analyzer was rotated $\pi/4$ counterclockwise, and the third image was obtained after rotating the analyzer another $\pi/4$ until the light field emerged (i.e. $\pi/2$). The last image was acquired following an additional $\pi/4$ counterclockwise rotation of the analyzer (total angle of $3\pi/4$). Then the polariscope was reset in dark field arrangement once again, and then the optical axis of the arrangement was rotated $\pi/8$ counterclockwise. Four more images were acquired using the same procedures adopted for the first group of images. The process was repeated for each of the eight different polariscope orientations.

4. Results

The phase map for the isoclinics was obtained from the images in Fig. 4 using Eq. (4) and is shown in Fig. 5(a) with range from 0 to $\pi/2$. As previously described, though the grayscale provides the principal stress orientation, there is ambiguity in determining which principal stress this isoclinic parameter represents. However, by drawing the stress trajectories (isostatics) and using the free boundary information, the orientations of both the principal stresses are defined. For example, if the slope of the isostatics agrees with the grayscale indicated in the isoclinic phase map, no change is needed. If it does not agree, $\pi/2$ will be subtracted from the phase values. Through this process, the slope of the first principal stress is achieved and used to extend the phase value of the isoclinics from $(0, \pi/2)$ to $(-\pi/2, \pi/2)$. As a result, the orientation of σ_1 is determined in full field, as shown in Fig. 5(b).

Phase maps were obtained for the isochromatics in the eight specific orientations $(-3\pi/8, -\pi/4, -\pi/8, 0, \pi/8, \pi/4, 3\pi/8$ and $\pi/2)$ as shown in Fig. 6. According to Eq. (8), phase maps of the isochromatics resulted from the information of the σ_1 orientation as shown in Fig. 7(a). The unwrapped phase map is shown in Fig. 7(b) by means of a robust unweighted least-squares phase unwrapping algorithm [12].

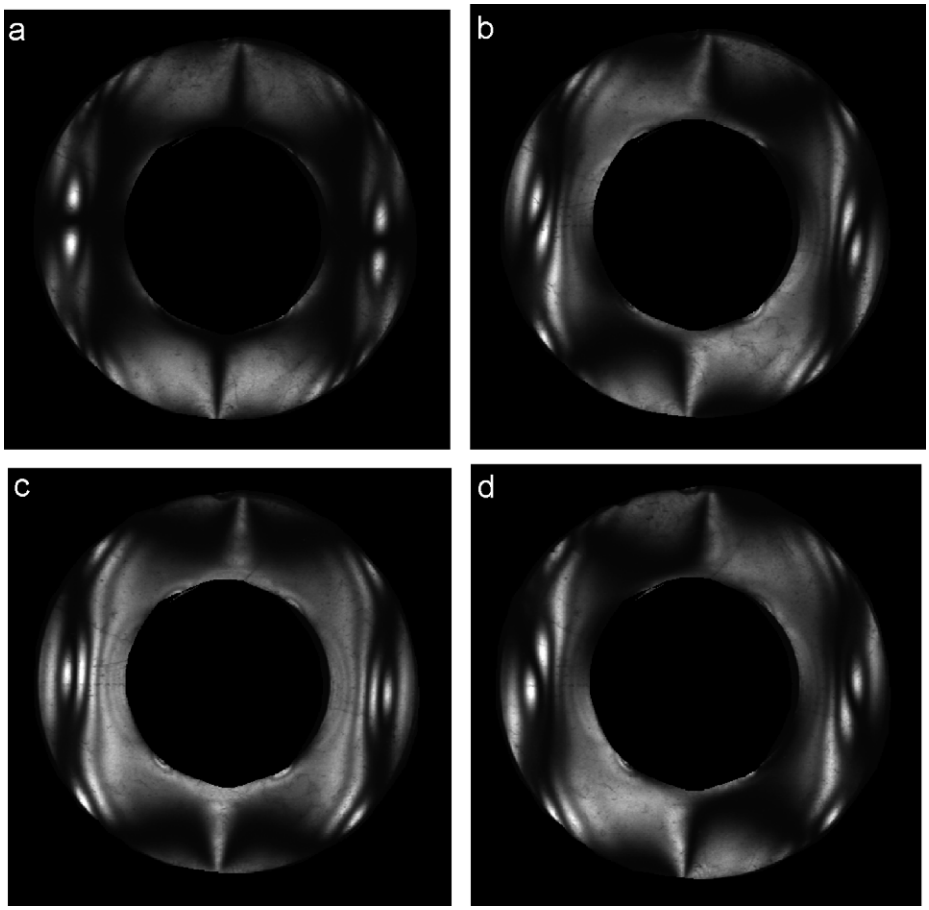

Fig. 4. The fringe patterns of a ring under diametral compression captured at four different β orientations in planer polariscope: (a) 0, (b) $\pi/8$, (c) $\pi/4$, (d) $3\pi/8$.

5. Discussion

A new approach was presented for whole-field determination of the phase values for the principal stress difference from isochromatic fringe patterns. For demonstration, the process was applied to isoclinic and isochromatic fringe patterns obtained from a circular ring subjected to diametral compression. Using the phase values resulting from the four photoelastic fringe patterns, the orientations of both principal stresses were determined with the range of $-\pi/4 \leqslant \alpha \leqslant \pi/4$. However, without additional information it was impossible to assess whether the phase at each location refers to σ_1 or σ_2. In Ref. [5], multiple load steps were introduced to remove such ambiguity occurring in the phase map of the isochromatics. That approach could also be applied in the evaluation of the principal stresses within the circular ring. However, it is not as applicable if the photoelastic model is subjected to complex loading conditions or a frozen stress model is analyzed. In contrast, the proposed approach provides a relatively convenient way of identifying the inclination angle of σ_1 with respect to the horizontal axis using the σ_1 trajectory, which can be obtained from the same isoclinic phase map. With the help of a special unwrapping process, the range in phase was extended to $-\pi/2 \leqslant \alpha \leqslant \pi/2$, which reflects the true range in principal stress orientations. The extended range is also helpful for post-processing stress separation using the shear-difference method as it requires the orientation of σ_1 with respect to horizontal axis.

With the orientation of the first principal stress fully established, phase shifting can then be performed with the isochromatic fringe patterns. Using the phase maps of the circular ring from each of the eight different orientations (Fig. 6) and the orientation of σ_1, an overall phase map was constructed (Fig. 7) by applying the correction process described by Eq. (8). Due to the error described in Eq. (9), there are some discontinuities in the grayscale distribution, particularly at the junctions where two isochromatic phase maps with close orientation of σ_2 are combined. Nevertheless, the resulting error distribution in phase values is relatively small and very reasonable for most engineering problems. Note that the error can be further reduced by using a division of 16 phase maps (rather than eight) if more accurate analysis is desired.

Although the two-wavelength method [2] provides an alternate approach for determining the isochromatic fringe patterns, it is uncertain when the difference of fringe orders caused by retardation exceeds 0.5. The multiple-wavelength

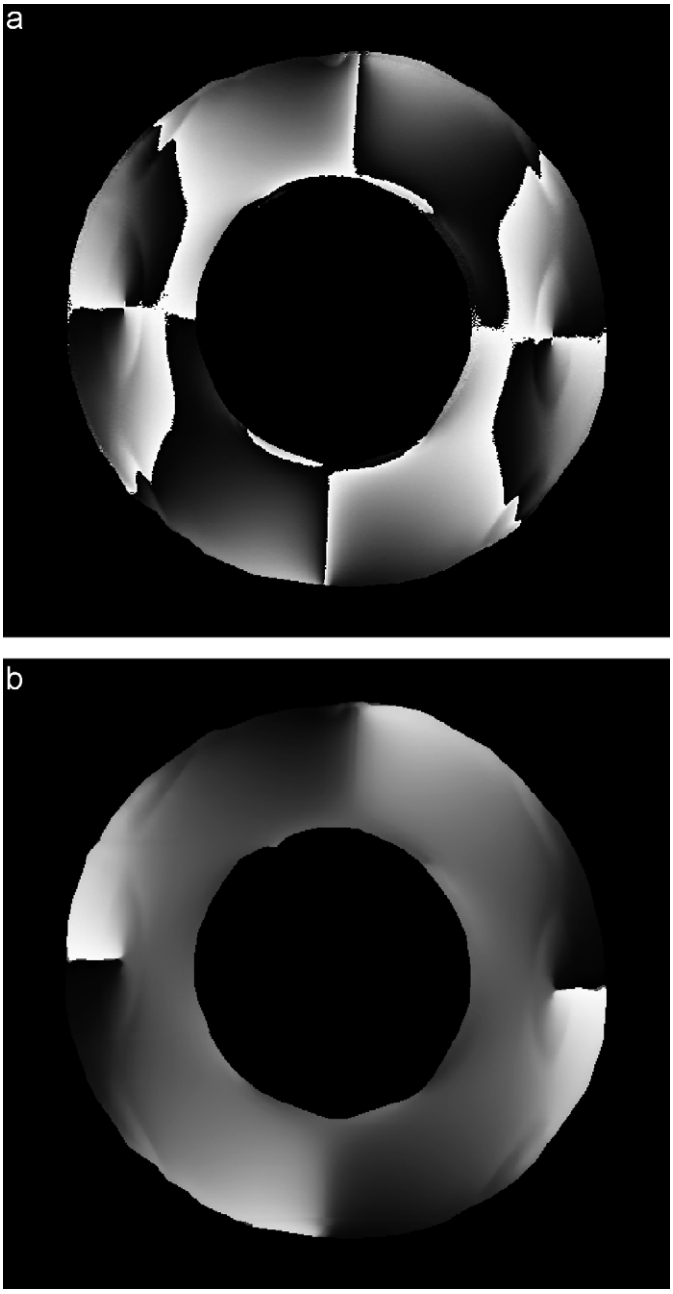

Fig. 5. Phase maps for the isoclinics: (a) original map ranging from 0 to $\pi/2$, (b) extended phase map with range from $-\pi/2$ to $\pi/2$.

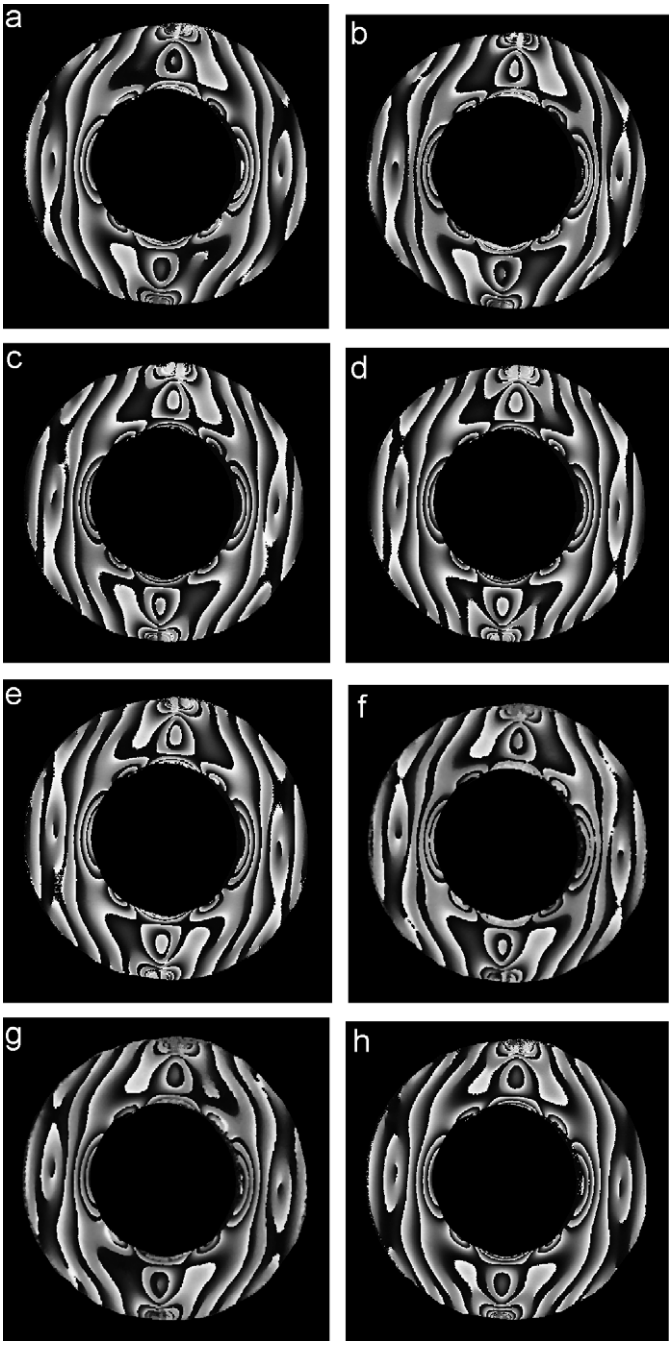

Fig. 6. Phase maps of the isochromatics in eight orientations: (a) $\pi/2$, (b) $-3\pi/8$, (c) $-\pi/4$, (d) $-\pi/8$, (e) 0, (f) $\pi/8$, (g) $\pi/4$, (h) $3\pi/8$.

method was introduced to overcome this problem and while relatively simple in practice, the method is not readily applicable due to the need for additional light sources. One of the largest benefits of the method described herein is that it requires no additional equipment and is suitable to either standard or frozen stress photoelasticity. In the current approach a series of images are taken with the quarter-wave plates, polarizer and analyzer rotated according to a simple scheme. These operations are not unlike the traditional operations used in other aspects of fringe analysis (e.g. Tardy Compensation, etc.) [13]. Furthermore, it requires no interactive communication between an experienced experimentalist and the computer and it provides the isochromatic phase map over a region, rather than a point, where the orientation of σ_1 is a constant. As such it facilitates an automatic process for generating isochromatic phase maps in eight discrete σ_1 orientations and reconstructing the full-field isochromatic phase map. Our next step is to develop an automatic motorized system that allows the computer to control image acquisition and rotation of the optical components simultaneously.

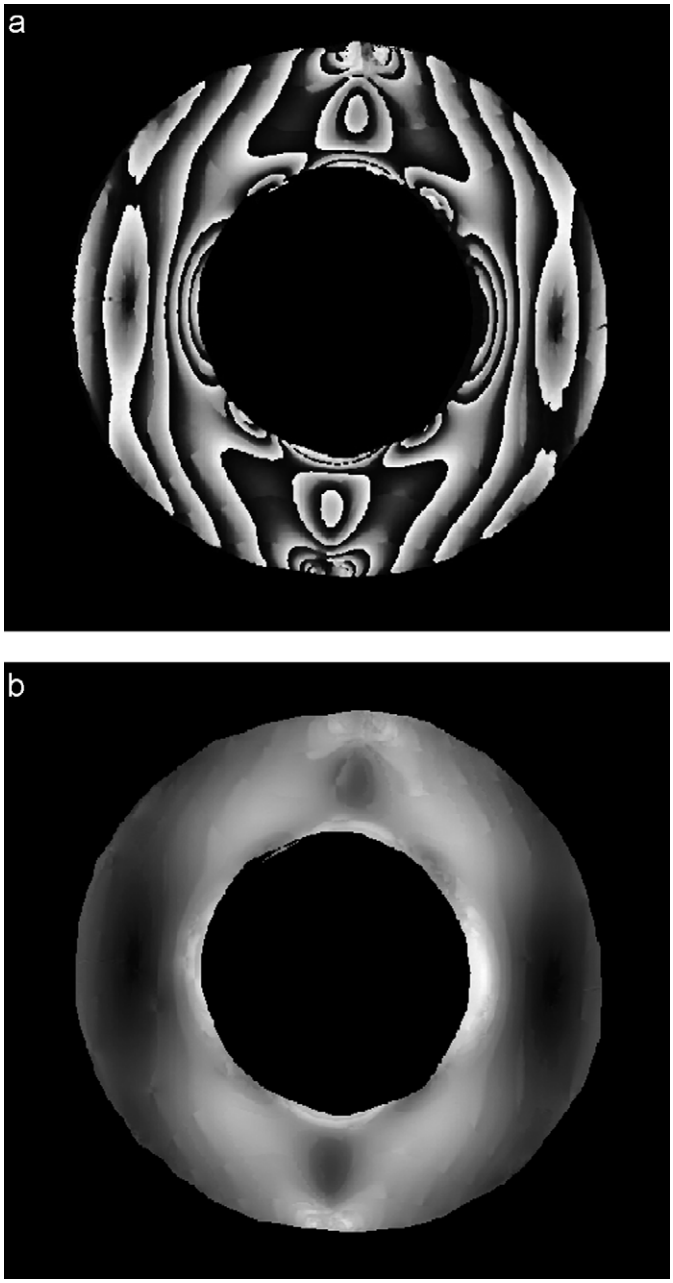

Fig. 7. The reconstructed phase map of the isochromatics for a ring under diametrical compression: (a) wrapped phase map, (b) unwrapped phase map.

A determination of the principal stress orientation must be conducted with precision, as this parameter is used as a control factor to construct phase maps of the isochromatics. Failure to obtain isoclinic phase maps with good quality will cause discontinuities and "saw-like" phase maps of the isochromatics. But isoclinics can only be observed in the plane polariscope arrangement with simultaneous interference of the isochromatics. Except in regions where the magnitudes of the two principal stresses are equal, the isochromatics are colorful fringe patterns. This quality enables application of a couple different methods for reducing the extent of interference between the isoclinics and isochromatics, and potential errors. With increasing fringe order, the isochromatic fringe intensity decreases. Also, the use of a monochromatic video camera and a white light source is recommended to capture the isoclinics with minimum disturbance from the isochromatics.

6. Conclusions

A novel method for determining the parameters required for stress separation in photoelasticity was presented. First, the phase map of the isoclinic fringe patterns is obtained using a plane polariscope. With the knowledge of the principal stress trajectories from the family of isoclinics, a whole-field description for the first principal stress orientation (σ_1) is determined with a range of $(-\pi/2, \pi/2)$. Then, phase shifting is conducted with the isochromatic fringe patterns at eight discrete polariscope orientations. Using the knowledge of the first principal stress orientation and the eight orientation-dependent phase maps, a composite phase map is developed for the first principal stress. Finally, an application to a circular ring loaded in diametral compression was presented. Results of the investigation have shown that the method is easy to conduct and applicable to engineering problems.

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